

Efficiency analysis and optimization of four coil magnetic resonance wireless power transmission system

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Abstract. Four coils magnetic resonant wireless power transmission system was constructed, and the efficiency model was established by using the theory of mutual inductance. The analysis of the maximum efficiency is based on two part factors, which are driving source and coils. The adjustment strategy of load coil and driving coil under the optimal efficiency was pointed out, which provided the theoretical guidance for the optimization of coil configuration mode and coil design. So the overall efficiency of the transmission system was improved effectively. At the same time, the experimental setup was built to test and analyze the change of efficiency under different transmission distance and different load resistance, and the correctness of theoretical analysis was verified. Based on the theoretical analysis, the maximum overall transmission efficiency of the experimental setup is 66.4%.

1. Introduction

Magnetically-coupled resonant wireless power transfer (MCR-WPT) technology has gradually become a research highlight in China and abroad, since it was proposed by MIT research team in 2007. It has a wide range of applications, and can be used in wireless charging for electric vehicle and mobile, power supply for electrical equipment, etc. However, as an energy transmission technology, it is more important to reduce the loss and improve the transmission efficiency of the whole system. At present, this technology is still in its infancy. Therefore, It is necessary to analyze and study the efficiency and transmission characteristics of wireless power transfer system so as to achieve high efficiency wireless power transmission.

There are many papers on the efficiency analysis of magnetically coupled resonant wireless transmission system at home and abroad. In[2], the mutual inductance model is used to analyze the effect of the coil in the resonant state on the efficiency of the two coil system. The maximum efficiency of the coupling mechanism and the optimum design of the system are studied, and the experimental verification is carried out. In[3], The influence of the system impedance on the transmission power and efficiency is investigated, and the effects of system parameters on the system impedance and efficiency are investigated experimentally. In[4], the magnetic circuit mechanism of the system is optimized. However, most papers focus on the transmission efficiency of the coils, ignoring the effect of the coils on the efficiency of the drive source and the overall efficiency of the system.

In this paper, the influence of high frequency drive source and coil transmission is considered synthetically to analyze the transmission efficiency characteristics of the four coils WPT system by using the mutual inductance theory. The effect of high frequency drive source maintaining high efficiency on coil configuration is considered. For the coils system, the adjustment strategy of the load coil and the exciting coil under different transmission distance and different load resistances are analyzed based on the maximum transmission efficiency. According to the analysis of the maximum transmission efficiency, the theoretical guidance for the optimal design of the resonant coil is provided. On this basis, Magnetically-coupled resonant wireless power transfer device is built, and the theoretical analysis is verified by experiments.

2. Analysis of Transmission Efficiency of The System

2.1 System modeling and efficiency calculation

There are two basic structures of magnetically-coupled resonant wireless power transfer system, two coil and four coil. In contrast, the four coil system adds excitation and load coils to the resonant coil to reduce the influence of the power and load on the resonant coil and to facilitate impedance regulation[5]. The traditional two coils system must rely on external matching circuit or change the system parameters to adjust the system performance.

Fig.1 shows the four coils system connecting the typical SSSS structure. The energy transmitter consists of the excitation coil and the transmitting coil, the energy receiver consists of the receiving coil and the load coil. The transmitting coil and the receiving coil are resonant coils. The transmitter generates a high frequency alternating magnetic field under the excitation of the high-frequency class-E inverter, and the receiver obtains energy from the magnetic field and supplies power to the load. Because of the single turn structure of the excitation and load coils, the inter coupling coefficients k_{13} , k_{14} and k_{24} of the coil can be neglected, considering the coupling coefficients k_{12} , k_{23} and k_{34} of adjacent coils only.

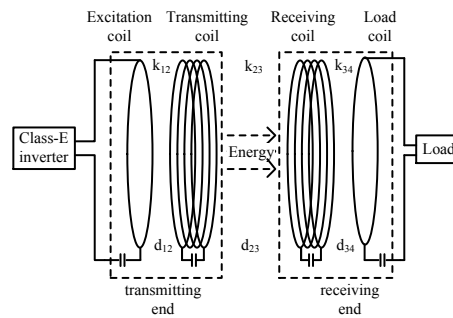


Fig.1 Schematic diagram of four coils SSSS structure

Fig.2 shows the four coil SSSS topological equivalent model, according to the working principle of magnetically-coupled resonant wireless power transfer system. The excitation coil and the load coil can be converted to the transmit and receive coils as the reflected resistance. Thus, the four coils system can be simplified to the traditional two coils model, as shown in Fig.2.

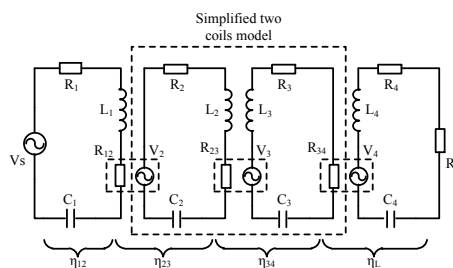


Fig.2 Equivalent model of four coils SSSS topology

V_S is a high frequency driving source, R_L is the resistance load, R_i ($i=1,2,3,4$) is the equivalent resistance in high frequency coil, L_i and C_i ($i=1,2,3,4$) are coil self inductance and compensation capacitance. The reflected resistance R_{34} reflects the energy of the load coil provided by the receiving coil, R_{23} reflects the energy of the receiving coil provided by the transmitting coil, R_{12} reflects the energy of the transmitting coil provided by the exciting coil:

$$R_{34} = \frac{(\omega M_{34})^2}{R_4 + R_L} \quad (1)$$

$$R_{23} = \frac{(\omega M_{23})^2}{R_3 + R_{34}} \quad (2)$$

$$R_{12} = \frac{(\omega M_{12})^2}{R_2 + R_{23}} \quad (3)$$

The transmission efficiency of the four coils system consists of two parts: the efficiency of the high frequency drive source V_s and the transmission efficiency of the coil system:

$$\eta = \eta_s \cdot \eta_{coil} \quad (4)$$

According to the impedance characteristics of the system, the transmission characteristics of the magnetically-coupled resonant system are analyzed. As shown in Fig.2, η_{12} , η_{23} , η_{34} and η_L represent the transmission efficiency from the incentive coil to the transmitting coil, from the transmitting coil to the receiving coil, from the receiving coil to coil load and from the load coil to the load respectively, which form the transmission efficiency of coil system η_{coil} :

$$\begin{aligned} \eta_{coil} &= \eta_{12} \cdot \eta_{23} \cdot \eta_{34} \cdot \eta_L \\ &= \frac{R_{12}}{R_1 + R_{12}} \cdot \frac{R_{23}}{R_2 + R_{23}} \cdot \frac{R_{34}}{R_3 + R_{34}} \cdot \frac{R_L}{R_4 + R_L} \end{aligned} \quad (5)$$

The reflected impedance of the system:

$$R_{re} = R_1 + R_{12} \quad (6)$$

The class-E inverter is used as the driving source of the WPT system, which can gain high efficiency at high frequency when the load resistance of the power source is equal to R_{eq} :

$$R_{eq} = R_{re} \quad (7)$$

2.2 System analysis and efficiency optimization

1) Transmission efficiency of coil system. The excitation coil and the load coil adopt a single turn coil structure, so the internal resistance of the coil is small. According to formula (5), by adjusting the parameter of coil system, η_{12} and η_L will be equal to 1 when $R_{12} \gg R_1$, $R_L \gg R_4$. Therefore, the transmission efficiency of the coil system is mainly affected by η_{23} and η_{34} . η_{23} and η_{34} show the energy transfer status between the transmitting coil and receiving coil, so The transmission efficiency between the resonant coils can be expressed as:

$$\eta_r = \eta_{23} \cdot \eta_{34} = \frac{R_{23}}{R_2 + R_{23}} \cdot \frac{R_{34}}{R_3 + R_{34}} \quad (8)$$

2) Matching adjustment of single turn coil. The transmission distance and load change of the magnetically coupled resonant WPT system are the two main factors affecting the transmission efficiency of the system. By adjusting the exciting coil and the load coil, the maximum efficiency transmission can be achieved. Substituting (2) to (8), η_r can be expressed as:

$$\eta_r = \frac{(\omega M_{23})^2 R_{34}}{(R_3 + R_{34}) [R_2 (R_3 + R_{34}) + (\omega M_{23})^2]} \quad (9)$$

When R_{34} is small, η_r tends to zero; When R_{34} is large, η_r tends to zero. So there is an optimal R_{34} , which can make η_r reach the maximum value. Assuming $\partial \eta_r / \partial R_{34} = 0$, the optimal R_{34} and optimum efficiency η_{ropt} can be obtained as follows:

$$R_{34opt} = \sqrt{1 + \frac{(\omega M_{23})^2}{R_2 R_3}} \cdot R_3 = \sqrt{1 + k_{23}^2 Q_2 Q_3} \cdot R_3 \quad (10)$$

$$\eta_{ropt} = \frac{k_{23}^2 Q_2 Q_3}{\left(1 + \sqrt{1 + k_{23}^2 Q_2 Q_3}\right)^2} \quad (11)$$

$Q_i (i=1,2,3)$ represent the unloaded quality factor of the corresponding coil respectively, Q_4 represent loaded quality factor of load coil. From (1) we can get:

$$k_{34}^2 Q_3 Q_4 = \frac{R_{34}}{R_3} \quad (12)$$

According to (10) and (12), the optimum coupling coefficient of load can be obtained:

$$k_{34opt} = \sqrt{\frac{\sqrt{1+k_{23}^2 Q_2 Q_3}}{Q_3 Q_4}} \tag{13}$$

Substituting (3) to (6), with (2) (7) (10), the optimal coupling coefficient of exciting coil k_{12opt} can be obtained as:

$$k_{12opt} = \sqrt{\frac{\sqrt{1+k_{23}^2 Q_2 Q_3}}{Q_1 Q_2} \cdot \left(\frac{R_{Eq}}{R_1} - 1\right)} \tag{14}$$

(13) and (14) are the basis of adjusting the exciting coil and load coil when the working condition of the system changes. This adjustment method not only realizes the maximum transmission efficiency of the system under the corresponding conditions, but also maintains the high efficiency of the class-E power source.

k_{34opt} is used to adjust the distance d_{34} when the transmission distance or load changes, so as to achieve the maximum transmission efficiency between the resonant coil and the load coil, k_{12opt} is used for power impedance matching, so as to adjust the distance d_{12} to meet $R_{12} \gg R_1$ and $R_{eq} = R_{re}$ and achieve high efficiency η_{12} and power efficiency η_s .

(1) The transmission distance d_{23} changes. In the case of no change in the loop structure, the variation of the transmission distance results in a change in the coupling coefficient k_{23} between the transmitting and receiving coils. If the excitation and the load coil not adjusted accordingly, the optimal transmission efficiency will not be realized and the transmission efficiency of the system will decrease. When the k_{34} is adjusted according to (13), the η_r will maintain the optimal value. When the k_{12} is adjusted according to (14), the impedance matching of the power source will be achieved and the whole system can achieve the optimal transmission efficiency.

(2) Load R_L change. The change of load R_L impacts loaded quality factor Q_4 of load coil. There are two ways to realize the optimal transmission efficiency of WPT systems: 1. Adjusting k_{23} and k_{12} according to (13) and (14); 2. Adjusting k_{34} to keep k_{34} equal to k_{34opt} according to (13), so as to counteract the effects of load changes on the system. The first adjustment method is more complex, and the adjustment of k_{23} will bring about the change of the optimal efficiency value in this case[7]; and the second methods are more simple and feasible.

3. Optimization of resonant coils

3.1 The target of coil optimization

According to previous analysis, the efficiency of coil transmission system of the WPT system is mainly affected by η_r . As shown in Fig.3, the variation curve of the optimal efficiency of η_r can be obtained by (11).

It can be seen from the fig.3, the larger $k_{23}^2 Q_2 Q_3$, the higher efficiency of η_r . Therefore, the system's transmit and receive coil should be independently designed and keep Q_2 , Q_3 large enough to meet the high transmission efficiency. Especially in the case of a change of k_{23} , Q_2 and Q_3 remain large enough to reduce the influence of k_{23} on efficiency as well as improve the transmission distance.

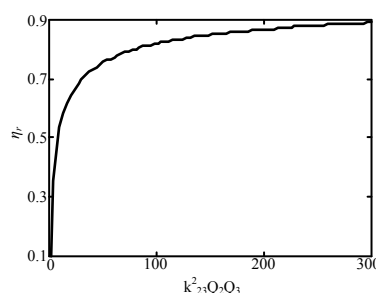


Fig.3 Change curve of efficiency η_r on $k_{23}^2 Q_2 Q_3$

3.2 Optimization of resonant coils

The resonant coil adopts a closely wound hollow spiral coil structure, because it has a more uniform magnetic field than the planar disk coil, and the magnetic field varies little with the distance[9]. The main parameters of closely wound helical coils are coil radius r , turns N , wire radius a . Considering the space size and magnetic field intensity, the basic condition of the designed system is that the operating frequency f_s is 3MHz, and the coil radius r is 15cm. The resonance coil is optimized on this basis.

1) *Optimization of quality factor Q* . The quality factor Q is related to the angular frequency ω , coil inductance L and loop parasitic resistance R_i , the formulas for each parameter are as follows[1][12]:

$$L = \mu_0 r N^2 \left(\ln \frac{8r}{a} - 2 \right) \quad (15)$$

$$R_i = R_L + R_C = \sqrt{\frac{\omega \mu_0}{2\sigma}} \cdot \frac{N \cdot r}{a} + \sqrt{\frac{\mu_0}{\epsilon_0}} \left[\frac{\pi}{12} N^2 \left(\frac{\omega r}{c} \right)^4 \right] + 1 \quad (16)$$

R_C represents the parasitic resistance ESR of the lumped parameter ceramic capacitors.

Using (15) (16), the curve of quality factor Q on the number of turns N and wire radius a can be obtained, as shown in Fig.4. Within the diagram, the more number of turns N , the larger the radius of the wire a , as well as the greater the quality factor Q of the coil. And the more turns, the larger the trend of Q increases, while the trend of Q increases with the wire radius a gradually slows down. When a exceeds 1.5mm, the difference can be ignored. Therefore, the coil turns and wire radius are selected as follows: $N=10$, $a=1.5\text{mm}$

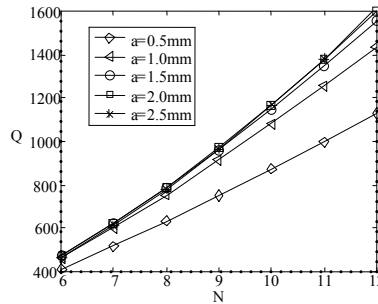


Fig.4 Change curve of quality factor Q on N and a

2) *Optimization of $k_{23}^2 Q_2 Q_3$* . The independent optimization of the resonant coil quality factor Q is the guarantee of high efficiency transmission of the system. The aim of synthesis optimization of $k_{23}^2 Q_2 Q_3$ is to keep the coupling coefficient k_{23} small in the premise of high quality factor, so that the transmission distance d_{23} can be increased[10]. The mutual inductance M between coils and coupling coefficient k are expressed as follows:

$$M = \mu_0 N_1 N_2 \sqrt{r_1 r_2} \cdot b \cdot \int_0^{\pi/2} \frac{2 \sin^2 \theta - 1}{\sqrt{1 - b^2 \sin^2 \theta}} d\theta \quad (17)$$

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad (18)$$

$$b = \sqrt{4r_1 r_2 / [(r_1 + r_2)^2 + d^2]} \quad (19)$$

Using (15) (17) (18), the curve of $k_{23}^2 Q_2 Q_3$ on the number of turns N and coil-span d can be obtained, as shown in Fig.5. $k_{23}^2 Q_2 Q_3$ increases with the increase of number of turns N , and decreases sharply with the increase of the coil-span d . When $N=10$, $d=0.3\text{m}$, the value of $k_{23}^2 Q_2 Q_3$ is 763. According to the relationship between the coil efficiency η_{ropt} and $k_{23}^2 Q_2 Q_3$ in Fig.3, the high

transmission efficiency is maintained when the transmission distance equals to coil diameter. Therefore, the parameters of resonant coils are determined as follows:

$$f_s=3\text{MHz}, N=10, r=15\text{cm}, a=1.5\text{mm}, d=30\text{cm}$$

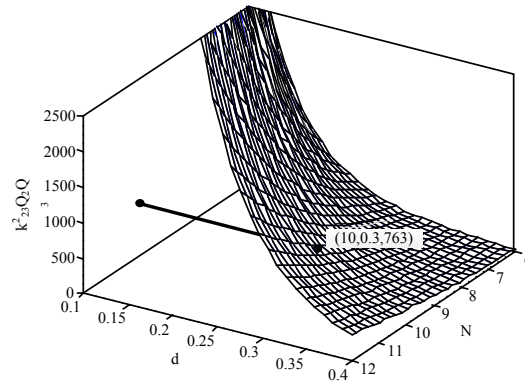


Fig.5 Change curve of comprehensive factor $k_{23}^2 Q_2 Q_3$ on D and N

4. Experiment and analysis

The radius of the exciting coil and the load coil is equal to the radius of the resonant coil. According to the whole system efficiency analysis and resonant coil optimization design, the coil parameters and system parameters measured by LCR meter IM3536 are shown in Tab.1.

Tab.1 Coil parameters and operating parameters

Coil	Inductance /(μH)	Resistance /(Ω)	Q
Resonant Coil	60.65	2.35	487
Exciting(Load) coil	0.86	0.11	147
f /(MHz)	DC Supply /(V)	Distance /(cm)	Load /(Ω)
3	30	30	50

The magnetically-coupled resonant wireless power transfer device is constructed as shown in Fig.6. The excitation power source is the class-E inverter and its rated working efficiency is 72%. The load is an 48v/40w incandescent bulb, and the system is working properly. The operating waveforms of the system are shown in Fig.7. The switch voltage waveform V_{ds} of the class-E inverter maintains zero voltage switching characteristics to improve the efficiency of the excitation source. The load voltage V_O and load current I_O are sinusoidal wave, which realize the effective transmission of electric energy. It is proved by experiments that this magnetically-coupled resonant wireless power transfer device can achieve the highest overall transmission efficiency of 66.4% and the maximum transmission power of 28.5w.

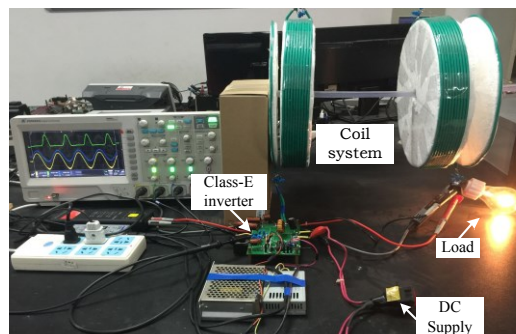


Fig.6 Experimental setup of magnetically-coupled resonant WPT

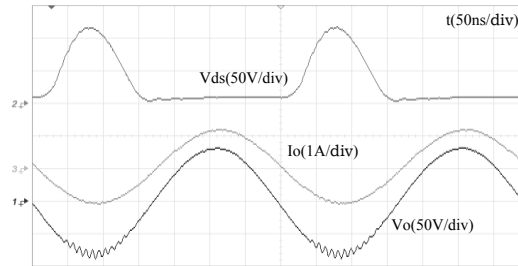


Fig.7 Operating waveforms of the system

4.1 The influence of transmission distance d_{23} variation on system efficiency

The transmission efficiency of a WPT system varies with the transmission distance d_{23} . According to the analysis in the paper [3], with the increase of transmission distance d_{23} , the transmission efficiency of coil η_{coil} increases first and then decreases. However, in the process of change in transmission distance d_{23} , by properly adjusting k_{12} and k_{34} in order to meet the changing relation of (13) and (14), the optimal transmission efficiency can be maintained in a large range.

Fig. 8 is the variation curve of the whole efficiency of the system under the condition of the fixed ($d_{12}=d_{34}=3\text{cm}$) coupling state and the varied coupling state between the exciting coil (the load coil) and the resonant coil. By meeting optimal coupling state, the efficiency is maintained at about 65% within the transmission distance of 25cm. However, when the coupling state is fixed, the transmission efficiency increases first and then decreases, and the maximum efficiency is 63.9% at the optimal distance 18cm. The transmission efficiency of the two cases were decrease significantly when the transmission distance is beyond the critical distance. The efficiency under optimal coupling state is higher than the efficiency under fixed coupling state, the largest difference between them is 22%. This shows that the optimal adjustment of coupling state can keep the overall efficiency of the system unchanged within the certain transmission distance. And in the process of increasing transmission distance, the system efficiency is improved obviously by the optimal adjustment.

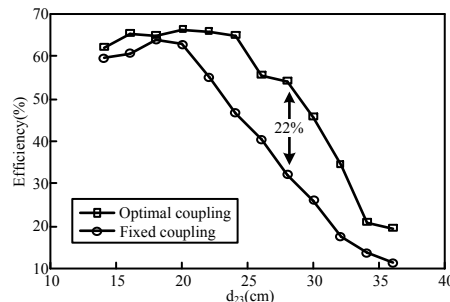


Fig.8 Change curve of system's overall efficiency on the transmission distance

4.2 The influence of load R_L variation on system efficiency

When the system load R_L changes, the four coils system also has good regulating effect. Q_4 changes with the variation of load R_L , the transmission efficiency of the system will eventually be affected without adjustment. According to the analysis, by keeping k_{34} equal to the k_{34opt} in (13), the influence of load R_L variation on system efficiency can be counteracted. And the adjustment of k_{34} can be realized by adjusting d_{34} without changing the coil structure.

Fig. 9 is the variation curve of the whole efficiency of the system under the condition of the fixed ($d_{34}=5\text{cm}$) coupling state and the optimal coupling state between the load coil and receiving coil. Here, $d_{12}=4\text{cm}$, $d_{23}=20\text{cm}$ are constant values, load R_L increases from 20Ω to 100Ω . Excluding the influence of measurement error, the optimal coupling of k_{34} can keep the system efficiency at 60% within the range of load resistance changes, and eliminate the influence of the change of load R_L . However, for the case of k_{34} fixed coupling, as the load resistance increases, the transmission

efficiency of the system increases first and then decreases gradually. The larger the load resistance, the greater the reduction in efficiency, and when the load is 100Ω , the efficiency is 13% lower. This reflects that the four coils system has flexible load regulation ability and greatly reduces the adverse effects of load changes.

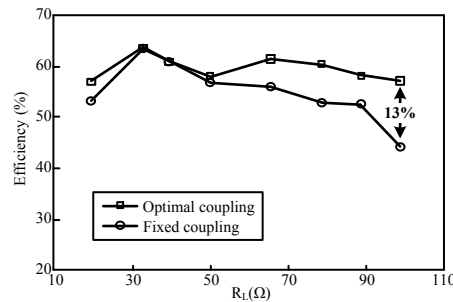


Fig.9 Change curve of system's overall efficiency on the load resistance

5. Conclusion

In this paper, the analysis and optimization for the transmission efficiency of the four coils magnetically-coupled resonant wireless power transfer system are studied. The efficiency model is established by mutual inductance coupling theory. In order to maintain the system transmission efficiency is high, in consideration of the influence of the driving source's efficiency, the adjustment strategy between the excitation coil (the load coil) and the resonant coil is analyzed when the transmission distance d_{23} and the load R_L change.

The experiment proves that the impedance adjustment of excitation and load coil can significantly improve the transmission efficiency of WPT system in four coils system. When the transmission distance varies, compared with the fixed coupling condition, the optimal coupling adjustment between coils can increase the efficiency by 22%. When the load R_L changes, the optimal adjustment between coils can keep the system efficiency basically at 60% within the range of load resistance changes, and eliminate the influence of the change of load R_L . This reflects that the four coils system has the flexible and convenient load regulation ability.

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