

Outage Capacity and Energy Efficiency trade-off for Distributed MIMO Systems

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Keywords: Outage capacity, Energy efficiency, Distributed MIMO systems.

Abstract. In this paper, we investigate the relationship between the outage capacity and the transmit power. Finally, we derived the energy efficiency (EE) based on outage capacity for the distributed multiple-input multiple-output (MIMO) systems under the idealistic power model and the realistic power model, respectively. Then we evaluate the EE performance under three parameters such as the cell radius, path loss exponent and outage probability. From theoretical analysis, we obtain that the outage capacity and the transmit power are not a simple linear relationship. Numerical results show that the performance of the EE is increased with the increase of the cell radius, the performance of the EE is decreased with the increase of the outage probability, and the performance of EE increase with the decrease of the path loss exponent.

Introduction

Due to improve system capacity, increase communication link stability and reduce the power consumption of the system, distributed antenna systems (DAS) has a great advantage and attracted more and more attention, and has been introduced in an earlier paper [1][2]. Compared to the co-located multiple-input multiple-output (C-MIMO) technology [3][4], Distributed multiple-input multiple-output (D-MIMO) have micro diversity and macro diversity [5].

Outage probability and channel capacity are two important performance indexes in the upper level network planning of D-MIMO systems. The capacity can be divided into ergodic capacity and outage capacity. In the early work [6][7], the definition of ergodic mutual information were given. The expression of the approximated maximum rate and outage capacity have been derived in [8].

The research of energy efficiency (EE) is becoming more and more important [9]. The ergodic capacity for energy efficiency research have been discussed in [10]. The outage probability of D-MIMO systems have been introduced in [11]. The reference [10][12] discussed the EE based on the ergodic capacity, and didn't discuss EE based on outage capacity.

On the basis of the above work [6][10][13], in this paper we first investigate the relationship between the outage capacity and transmit power, the performance of EE based on mutual information of outage capacity. Secondly discuss the three parameters such as the cell radius, the path loss and the outage probability for the EE which hope to find some factors that affect EE.

In this paper, there have some notations as follows. The I_M denotes the identity matrix of size $k \times k$. The operator $E(\cdot)$ denotes expectation, $V(\cdot)$ denotes variance, and the $\text{erfc}(\cdot)$ denotes the error function.

System Model

We consider a D-MIMO systems with N remote access units (RAUs). For the convenience, we only consider a single user scenario, the mobile stations (MSs) has M antennas and each RAUs are equipped with Q antennas. In this paper, the D-MIMO systems can be denoted by (M, N, Q) . When $N=1$, the D-MIMO systems becomes a C-MIMO systems. Specially, the central unit (CU) can be considered as a special RAU and is denoted by RAU 1 when the $Q=1$. Besides, assuming that the transmitter does not know the channel state information (CSI), but the receiver knows. The received signal at the BS of the downlink D-MIMO systems can be written as

$$\mathbf{Y} = \mathbf{H}(\mathbf{d})\mathbf{X} + \mathbf{n}, \quad (1)$$

where \mathbf{Y} is the $M \times 1$ received signal vector, $\mathbf{X} = [x_1, x_2, \dots, x_{NL}]^T$ is the transmitted signal vector, $\mathbf{d} = [d_1, d_2, \dots, d_Q]^T$ indicates the distance vector from the N -th RAU to MS, where \mathbf{n} is the additive white zero-mean complex Gaussian noise with unit variance, \mathbf{H} is the $M \times LN$ random matrix, which consist of the small-scale fast fading and large scale fading, can be expressed as [6]

$$\mathbf{H}_q(\mathbf{d}_q) = h_{sh,q} \mathbf{H}_{w,q}, 1 \leq q \leq Q, \quad (2)$$

where $\mathbf{H}_{w,q}$ is a random matrix and according to complex Gaussian distribution, which indicated the small scale fading channel. and $h_{sh,q}$ denoted the path loss and large scale fading. Besides, $\mathbf{H}_{w,q}$ are independent of $h_{sh,q}$. The large scale fading can be modelled as [6]

$$h_{sh,q} = \sqrt{\frac{cs_q}{d_q^\alpha}}, \quad (3)$$

where s_q is a log-normal shadow fading variable, and $10 \log_{10} s_q$ is a zero mean Gaussian random variable with standard deviation σ_{sh} . and $d_q = 1 \text{ km}$ is the distance between the RAU Q and the MS, c is the constant which is the median of the mean path gain at a reference distance d_q . where α represent path loss exponent, and it is a chosen parameter in later simulations [6].

In order to analyze and discuss, we suppose that the cell shape is approximated by a circle of radius K . so that (K_Q, θ_Q) can be expressed the RAUs' polar coordinates relative to the center of the cell and (ρ, θ) shows the MS's polar coordinate. we assumed the CU/RAU 1 is the center of the cell and the polar coordinates is denoted by $(0, 0)$, and the other RAU's polar coordinate are $(3 - \sqrt{3})K/2, 2\pi(q-1)/(Q-1), q = 1, 2, \dots, Q-1$ [10]. The distance d_q from the q -th RAU to the MS can be calculated as [6]

$$d_q = \sqrt{\rho^2 + K_q^2 - 2\rho \cos(\theta - \theta_q)}. \quad (4)$$

And the probability density function (PDF) of (ρ, θ) can be expressed as [6]

$$p(\rho) = \frac{2\rho}{K^2}, \quad 0 \leq \rho \leq K, \quad (5)$$

$$p(\theta) = \frac{1}{2\pi}, \quad 0 \leq \theta \leq 2\pi. \quad (6)$$

According to the works [7][13], the power consumption model have been divided the D-MIMO systems into two parts, the idealistic power consumption model and the realistic power consumption model. the idealistic power consumption model can be express as

$$P_{idealistic} = \frac{P_s}{\tau}, \quad (7)$$

where P_s is the transmit power, and the τ is the radio frequency power amplifier efficiency.

In the pratical application scenarios. we must considering the effects of the other power, so that the realistic power consumption is modeled as [13]

$$P_{realistic} = \frac{P_s}{\tau} + P_{sta} + LQP_{dyn} + P_o, \quad (8)$$

where the P_{sta} is static power, P_{dyn} is the dynamic power consumption and the P_o is the dissipated power consumption.

The first introduced for EE in [14], Then the research work in [7], so the formula of the EE can be written as

$$\eta_{EE} = \frac{C}{P_{total}}, \quad (9)$$

where C denoted the capacity, P_{total} is the the total power consumption which have two parts and the formula is equal to (7) or (8).

The Capacity and EE of the Distributed MIMO Systems

In this section, we will first give the expression of the ergodic capacity for the D-MIMO antennas systems, and then shows the expression of outage capacity for the D-MIMO antennas systems.

We consider the receiver have perfect knowledges of channel and unknown at the transmitter. Then, we can get the expression of the ergodic capacity in bits/s/Hz through the mutual information for D-MIMO systems [6]

$$I = \log_2 \det \left[I_M + \frac{P_S}{M} \mathbf{H}^H(\mathbf{d}) \mathbf{H}(\mathbf{d}) \right]. \quad (10)$$

From [6], the mutual information of the outage capacity can be written as

$$I^\delta(\mathbf{d}) = \mu_I(\mathbf{d}) + \sqrt{2\sigma_I^2(\mathbf{d})} \text{erfc}^{-1}(2(1-\delta)), \quad (11)$$

where the δ is the outage probability, $\mu_I(\mathbf{d})$ and σ_I^2 is the expectation and variance of the mutual information, respectively.

In this paper, we consider the outage capacity with $M \leq Q$. For the (M, N, Q) D-MIMO systems, the outage capacity can be written as [6]

$$C_{out} = \mu_{ID-MIMO} + \sqrt{2\delta_{ID-MIMO}^2} \text{erfc}^{-1}(2(1-\delta)). \quad (12)$$

From work [6], $\mu_I(\mathbf{d})$ and $\sigma_I(\mathbf{d})$ can be expressed as

$$\mu_{ID-MIMO}(\mathbf{d}) = \frac{M}{\ln 2} \ln \frac{\gamma}{M} + \frac{M}{\ln 2} \left(2 \ln t_1 - \frac{1}{2} \ln t_2 \right), \quad (13)$$

$$\delta_{ID-MIMO}^2(\mathbf{d}) = \left(\frac{M}{\ln 2} \right)^2 (\ln t_2 - 2 \ln t_1), \quad (14)$$

where

$$\gamma = \frac{c P_S}{D^\alpha}, \quad (15)$$

γ denoted the noise level of the distributed massive MIMO system refer to distance K . where t_1 and t_2 can be written as [6]

$$t_1 = \sum_{n=1}^N \exp \left(\mu_{wn} + \frac{1}{2} \sigma_{wn}^2 \right), \quad (16)$$

$$t_2 = \sum_{n=1}^N \exp(2\mu_{wn} + 2\sigma_{wn}^2) + 2 \sum_{n=1}^{N-1} \sum_{m=n+1}^N \exp(\mu_{wn} + \mu_{wm}) \\ \times \exp \left(\frac{1}{2} \mu_{wn} + \frac{1}{2} \sigma_{wm}^2 \right). \quad (17)$$

so, we can get the expression of EE according to (8), (9) and (14)

$$\eta_{D_MIMO_EE_outage} = \frac{\mu_{ID-MIMO} + \sqrt{2\delta_{ID-MIMO}^2} \text{erfc}^{-1}(2(1-\delta))}{P_{total}}. \quad (18)$$

The ergodic capacity and outage capacity with $M \geq NL$ case have been discussed and studied in [6], respectively. But the outage capacity with $M \leq L$ without further details. In works [7][11] discussed the EE based on ergodic capacity. Corresponding to the formula (18), we can get some characters for EE which based on outage capacity for the D-MIMO systems. It can be written as in Theorem 1 as following which is demonstrated in Appendix A.

Theorem 1: The EE decreases with the increase of transmit power for the idealistic power consumption model. Nevertheless, the EE of the D-MIMO systems with $M \leq L$ first increases and then decreases under the realistic power consumption model.

Theorem 2: The performance of the EE is increased with the increase of the cell radius, the performance of the EE is decreases with the increase of the outage probability, and the performance of EE increases with the decrease of the path loss exponent.

Numerical Results

In this section, the numerical results is provided to verify the correctness of the previous expression through Monte Carlo simulation. At first, evaluate the performance of (1,1,5) and (2,2,7) D-MIMO systems under the realistic and idealistic power consumption model, respectively.

Fig.1 As anticipated, the outage capacity is increases with the increase of the transmit power. From the simulation results shown in Fig.2(a), it is seen that the EE decreases with the increase of the idealistic power consumption model. From Fig.2(b), it shows that the EE first increases and then decreases under the realistic power consumption model. This indicates that there is a trade-off between the EE and the outage capacity or maximum transmit power.

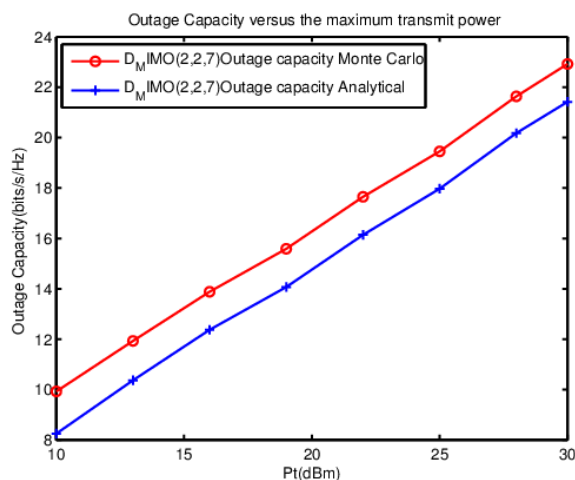
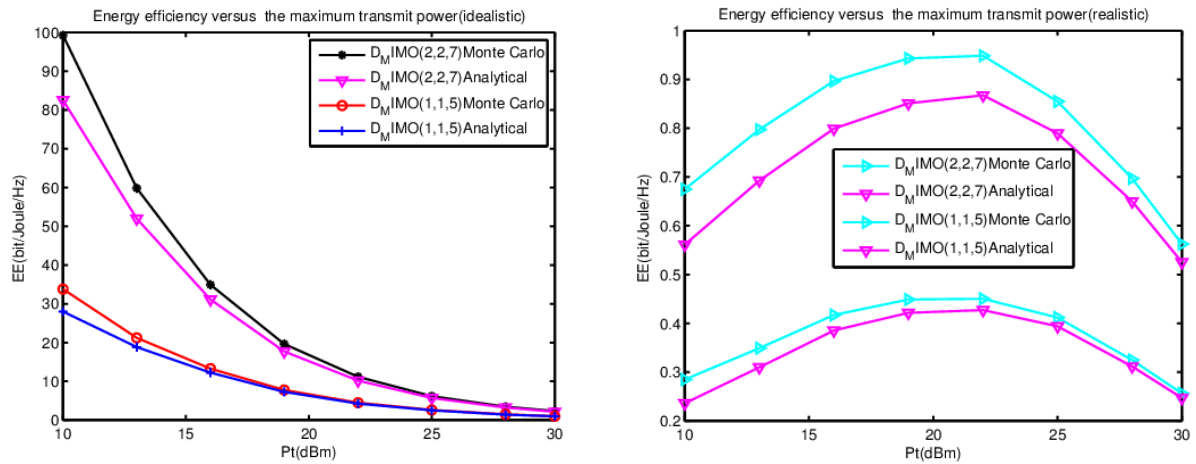


Fig.1. Outage capacity versus the maximum transmit power

From Fig.3, we can see definitely that the EE increases with the increase of the outage probability. Compare with the formula (11) and (12), the outage probability is an important factor to influence the outage capacity, and effect the performance of the energy efficient. Fig.4 shows that the performance of the EE is decreases with the increase of the cell radius. As in the simulation results, the EE of the cell radius $K=1.5$ km is approximately 30.12% lower than the EE of the cell radius $K_2=1$ km. and the EE of the cell radius $K=2$ km is approximately 53.87% lower than the EE of the cell radius $K_3=1$ km. From the result shown in Fig.6, it is seen that the performance of EE increase with the decrease of the path loss exponent α the EE of the $\alpha=4$ is approximately 8.38% lower than the EE of the $\alpha=3.7$. Meanwhile, the EE of the $\alpha=4.3$ is almost 15.139% lower than the EE of the $\alpha=3.7$. In conclusion, just agree with the Theorem 2.



(a) EE under the idealistic power consumption model (b) EE under the realistic power consumption model
Fig. 2. EE under the different power consumption model

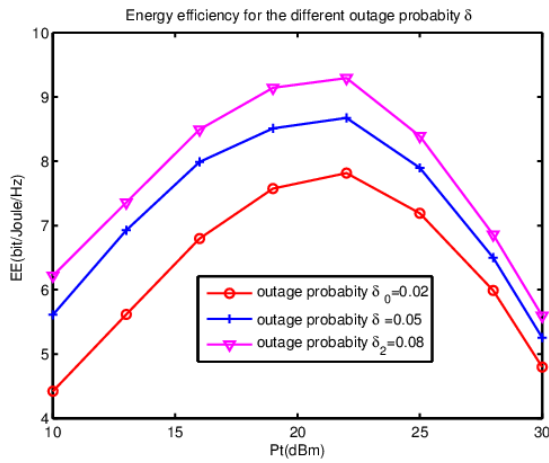


Fig.3. EE versus the maximum transmit power with different outage probability power

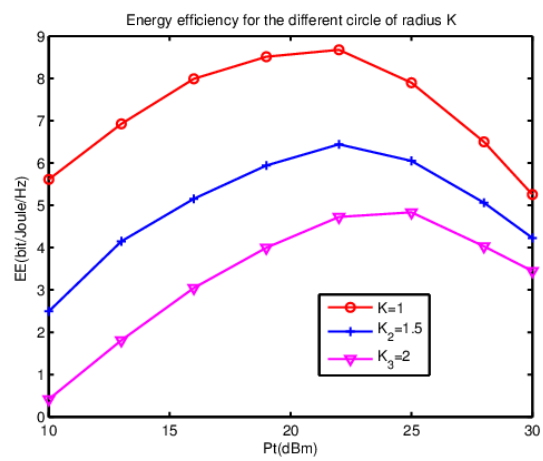


Fig.4. EE versus the maximum transmit power with different cell radius

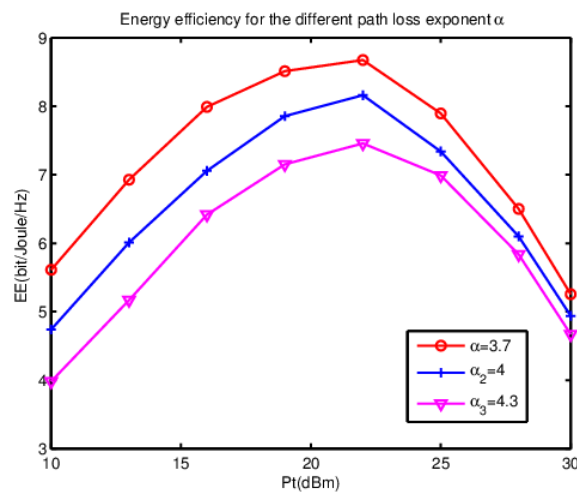


Fig.5. EE versus the maximum transmit power different path loss exponent

Summary

In this paper, we investigate the outage capacity and EE in D-MIMO systems. The outage capacity expression and numerical results shown that outage capacity is almost linear with the transmit power, and outage capacity increases with the increase of the transmit power. From the numerical results, we find outage probability, cell radius and path loss exponent will affect the performance of outage capacity. Simulation results have also demonstrated that the EE increases with the increase of the outage probability, the EE increases with the decrease of the cell radius and the EE increases with the decrease of the path loss exponent.

Appendix A: Proof of Theorem 1

According to (8), (9) and (12), substituting (8) to (12), the EE of the D-MIMO systems can be given as

$$\eta_{D_MIMO_EE_outage} = \frac{\mu_{D-MIMO} + V}{\frac{P_s}{\tau} + U}, \quad (19)$$

where $V = \sqrt{2\delta_{ID-MIMO}^2} \text{erfc}^{-1}(2(1-\delta))$, $U = P_{sta} + LQP_{dyn} + P_o$.

After we have to take the derivative of the (19) which based on maximum transmitted power P_t , we get the expression as

$$\frac{\partial \eta_{D_MIMO_EE_outage}}{\partial P_t} = \frac{\frac{M}{\ln 2} \frac{1}{P_s} \left(\frac{P_s}{\tau} + U \right) - \left(\frac{M}{\ln 2} \ln \frac{cP_s}{D^\alpha} + V \right) \left(\frac{1}{\tau} \right)}{\left(\frac{P_s}{\tau} + U \right)^2}. \quad (20)$$

Obviously, the denominator of the formula (20) is a positive quality. So we focus on the molecular researching, and then we denoted molecular be a new function

$$g(P_s) = \frac{M}{\ln 2} \frac{1}{P_s} \left(\frac{P_s}{\tau} + U \right) - \left(\frac{M}{\ln 2} \ln \frac{cP_s}{D^\alpha} + V \right) \left(\frac{1}{\tau} \right). \quad (21)$$

We have to take the first derivative of (21) based on P_s , and get the equation as

$$\frac{\partial g(P_s)}{\partial P_s} = -\frac{M}{\ln 2} \left(\frac{U}{P_s^2} + \frac{1}{\tau P_s} \right) < 0. \quad (22)$$

It is shows that the function (22) decreases with the increase of transmit power P_s , so we simply drawed $g(\infty) \leq g(P_s) \leq g(0)$. so we can get the conclusion from the (22) which $\lim_{P_t \rightarrow 0} g(P_s) > 0$ and $\lim_{P_t \rightarrow \infty} g(P_s) > 0$ is that EE of D-MIMO systems based on outage capacity first increases and then decreases with the increase of power P_s . Theorem 1 is proved.

Acknowledgement

This work is partially supported by the Natural Science Foundation of China under grants 61601300, the Science and Technology In-novation Comm-ission of Shenzhen under Grants JCYJ20150324140036835, the OpenResearch Fund through the National Mobi-le Communications Research Lab-oratory, Southeast University, Nanjing, China, under Grant 2017D10, Shenzhen University start-up funding No. 2016053.

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