Advances in Intelligent Systems Research, volume 156
7th International Conference on Management, Education and Information (MEICI 2017)

The Air Defense Missile Weapon System Based on Queuing Theory Mixed Deployment Effectiveness Evaluation

Haitao Yao and Liang Ma*
Department of Surface Ship Command, Dalian Naval Academy, Dalian, 116018,China
446982051@qq.com

Keywords: Queuing Theory; Missile Weapon System; Effectiveness Evaluation

Abstract. Using the queuing theory knowledge, using the methods of quantitative analysis, the mixed deployment of air defense missile weapon system combat effectiveness to make an objective evaluation. The method can assess the performance analysis of air defense missile weapon system, and contribute to the decision-making to be scientific.

Introduction

In the modern air defense combat, defense is the threat from each position. Target threat degree and the number is far more than the original model of air defense combat, make both sides are in a state of "supersaturated" work. For defenders to correctly evaluate the operational effectiveness of air defense missile weapon system, and through the reasonable deployment of air defence missile weapon system to improve the air defense missile weapon system combat effectiveness is a very critical problem. In this paper, we study a kind of mixed air-defense missile weapon system deployment to fight against the enemy air defense combat effectiveness evaluation method of air.

Effectiveness Evaluation Method Based on the Queuing Theory

Due to evaluation of air defense missile weapon system based on dealing with the future direction and batch, big density, total height of air combat, you must evaluate missile weapon system under the condition of the anti air combat capability against multiple targets. So you can use the idea and method of queuing theory offers a concise and solve the problem. It is based on the following reasons:

(1)The targets can be flow hypothesis for the simplest flow

The intensity of a large number of small, independent of each other random flow close to the sum of the simplest flow (poisson flow). The simplest flow (poisson flow) has a stability, normal and no aftereffect. For service system, the most difficult to adapt to the simplest flow. So, according to the simplest flow design service system, does not consider the question from the most difficult conditions. The mathematical expressions for the simplest flow:

$$F(t) = 1 - e^{-\lambda t} \tag{1}$$

in this type: λ is flow and intensity $\lambda > 0$, per unit of time into the air defense system on the number of the targets.

(2)Air defense missile weapon system service (shooting) time can be as meet the index distribution law

Distribution function form of exponential distribution law are as follows:

$$F(t) = 1 - e^{-\mu t} \tag{2}$$

Type: $\mu = \frac{1}{t_{serve}}$ is constant, t_{serve} is the service time of mathematical expectation.

(3)Air defense missile weapon systems of air attack target shooting (services) conforms to the queuing rules of service

Effectiveness Evaluation Method based on the Queuing Theory

In the implementation of air defense missile weapon system combat deployment, on the one hand to make the different models of air defense weapons mixed deployment, to play to their maximum effectiveness, to implement the overall efficient against enemy, ensure the safety of the secured target; On the other hand, want to consider the survival of their own air defense forces itself, eventually can be the most effective to destroy the enemy formation, ensure the safety of the region and preserve their own purposes. In this paper, based on the queuing theory method, the construction of air defense missile weapon system adopt double line configuration mode of air defense combat effectiveness evaluation model, and as for three or more different types of operational effectiveness evaluation of missile strikes method can so on.

State Analysis

Resistance to air missile weapon system adopt double line configuration mode, in every line of defense configured on the same type and different quantity of air defense weapons. The first line of defense in the number of air defense weapon; The second line of defense in air defense weapons. The incoming target group similar to poisson flow. Firepower unit on a target shooting time obey negative exponential distribution of parameters for. At the same time have a firepower unit on a target kill probability . The targets have to after the first line of defense and direct access to the second line of defense.

The model of the air defense missile weapon system is shown in figure 1.

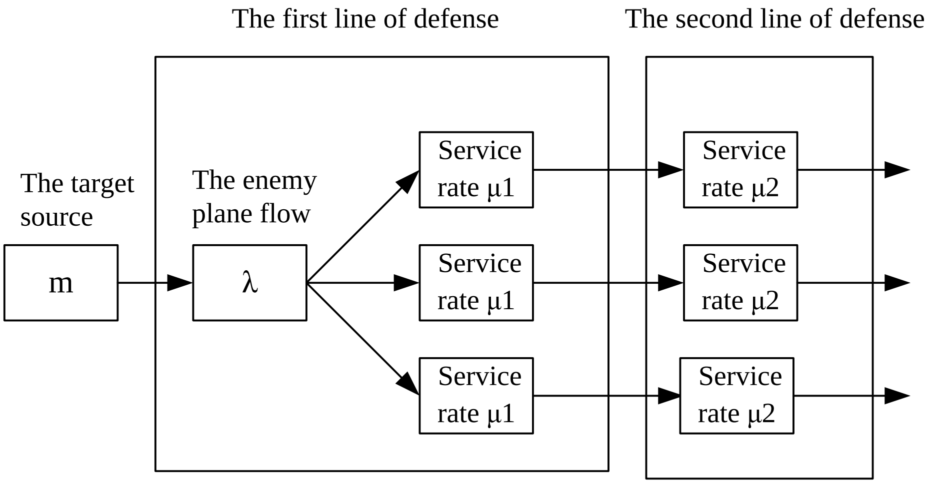

Figure 1 two lines of defence configuration model of air defense

The t time system of each layer of defense on fire in the fire unit number to the status of random service system. You can tell it is concluded that: each line may be in the state has the following three:

(1) $A_0(t)$——all on the line of defense weapons are not shooting;

(2) $A_{k_m}(t)$——on the line with arms are shooting ($k_m = 1, 2, ..., n_m - 1$), n_m is for all on the line of defense firepower unit number, $m = 1\text{、}2$ and represent the first line of defense, the second line of defense;

(3) $A_{n_m}(t)$—— on the line all the weapons in the shooting. The moment may be in the state of the double defense system have nine, state combination number can be expressed as follows:

$$A_{\begin{pmatrix} 0 \\ k_1 \\ n_1 \end{pmatrix} \begin{pmatrix} 0 \\ k_2 \\ n_2 \end{pmatrix}}(t) \tag{3}$$

Type: $k_1 = 1, 2, ..., n_1 - 1; k_2 = 1, 2, ..., n_2 - 1; n_1, n_2$—— on the first and second line of defense weapons.

By (3), the possible states of the system with 9 kinds, namely:

$A_{00}(t)$、$A_{0k_2}(t)$、$A_{0n_2}(t)$、$A_{k_10}(t)$、$A_{k_1k_2}(t)$、$A_{k_1n_2}(t)$、$A_{n_10}(t)$、$A_{n_1k_2}(t)$、$A_{n_1n_2}(t)$.

$A_{k_1k_2}(t)$ said the system of on line 1, 2, respectively k_1、k_2 firepower unit in shooting, similar to the rest of the definition. $P_{ij}(t)(i=0,1,2,...,n_1;\ j=0,1,2,...,n_2)$ in said time system in the above the probability of each state. Q_1,Q_2 respectively on the line of defense firepower unit 1, 2 kill probability of target (the same defense firepower unit to target the kill probability of the same). 1, 2 on the line of defense firepower unit for target shooting intensity μ_1,μ_2 respectively (the same defense firepower unit for the same target shooting strength).

When $t\to\infty$, can get double line disparate weapons defence system stability equations:

$$\begin{aligned}
&\lambda P_{00}=\mu_1Q_1P_{10}+\mu_2P_{01}\\
&(\lambda+k_2\mu_2)P_{0k_2}=\mu_1Q_1P_{1k_2}+(k_2+1)\mu_2P_{0k_2+1}\\
&(\lambda+n_2\mu_2)P_{0n_2}=\mu_1P_{1n_2}+\mu_1(1-Q_1)P_{1n_2-1}\\
&(\lambda+k_1\mu_1)P_{k_10}=\lambda P_{k_1-10}+(k_1+1)\mu_1Q_1P_{k_1+10}+\mu_2P_{k_11}\\
&(\lambda+k_1\mu_1+k_2\mu_2)P_{k_1k_2}=\lambda P_{k_1-1k_2}+(k_1+1)\mu_1(1-Q_1)P_{k_1+1k_2-1}+\\
&(k_1+1)\mu_1Q_1P_{k_1+1k_2}+(k_2+1)\mu_2P_{k_1k_2+1}\\
&(\lambda+k_1\mu_1+n_2\mu_2)P_{k_1n_2}=\lambda P_{k_1-1n_2}+(k_1+1)\mu_1(1-Q_1)P_{k_1+1n_2-1}+\\
&(k_1+1)\mu_1P_{k_1+1n_2}\\
&(\lambda+n_1\mu_1)P_{n_10}=\lambda P_{n_1-10}+\mu_2P_{n_1+1n_2}\\
&(\lambda+n_1\mu_1+n_2\mu_2)P_{n_1k_2}=\lambda P_{n_1-1k_2}+\lambda P_{n_1k_2-1}+(k_2+1)\mu_2P_{n_1k_2+1}\\
&(n_1\mu_1+n_2\mu_2)P_{n_1n_2}=\lambda P_{n_1n_2-1}+\lambda P_{n_1-1n_2}
\end{aligned} \tag{4}$$

$\sum_{j=0}^{n_2}\sum_{i=0}^{n_1}P_{ij}=1, k_1=1,2,...,n_1-1; k_2=1,2,...,n_2-1$. Type. By the equation (4) solution:

$$P_{ij}=f(\lambda,\mu_1,\mu_2,Q_1,n_1,n_2)P_{00}; i=1,2,...,n_1; j=1,2,...,n_2 \tag{5}$$

in this type, $P_{00}=\dfrac{1}{\sum_{j=0}^{n_2}\sum_{i=0}^{n_1}f_{ij}(\lambda,\mu_1,\mu_2,Q_1,n_1,n_2)}$.

Parameter Analysis

when $\mu_1=\mu_2=\mu, Q_1=1$, the system is simplified to the same kind of weapons and weapons to target kill probability of 1 two lines of defence defense system, the penetration probability of the targets are as follows:

$$p_{n_1+n_2}=\frac{\dfrac{\rho^{n_1+n_2}}{(n_1+n_2)!}}{\sum_{k=0}^{n_1+n_2}\dfrac{\rho^k}{k!}} \tag{6}$$

In this type, $\rho=\lambda/\mu$.

when $\mu_1=...=\mu_i=...=\mu, Q_1=...=Q_{i-1}=...=1$, the system is simplified as the same kind of weapons and weapons to target kill probability of 1 multiple lines of defence defense system, the penetration probability of the targets are as follows:

$$p_{n_1+n_2+...+n_i} = \frac{\dfrac{\rho^{n_1+n_2+...+n_i}}{(n_1+n_2+...+n_i)!}}{\sum_{k=0}^{n_1+n_2+...+n_i} \dfrac{\rho^k}{k!}} \quad (7)$$

when $n_1 = n_2 = 1$, the system is simplified to each line of defense is only one firepower unit, penetration probability is:

$$p_{n_1 n_2} = \frac{\lambda^2[\lambda+\mu_2+\mu_1(1-Q_1)]}{(\lambda+\mu_1)[\mu_1\mu_2+\lambda\mu_1(1-Q_1)+(\lambda+\mu_2)^2]} \quad (8)$$

Here, also analyze a situation: when $Q_1! = Q_2! = 1$, the probability of target penetration event is P_t. This event for the following 4 kinds of events and event: (1) the target was not shot, namely into the first line of defense and the second line of defense, each line of defense firepower unit were assigned to the original target, no firepower unit shooting for the new target; (2) the target is only the first line of defense weapon shooting, but not destroyed; (3) the target is only the second line of defense weapon shooting, but not destroyed; (4) on the target by two lines of defence weapon shooting, but not destroyed. P_t is For more than the sum of four events probability:

$$P_t = P_{n_1 n_2} + (\sum_{i=1}^{n_1-1} P_{in_2})(1-Q_1) + (\sum_{j=0}^{n_2-1} P_{n_1 j})(1-Q_2) + (\sum_{j=0}^{n_2-1}\sum_{i=0}^{n_1-1} P_{ij})(1-Q_1)(1-Q_2) \quad (9)$$

At this point, as long as calculated by equations (4), (9) can be used type for double line of heterogeneous weapon defense penetration probability. The above idea can be extended to more lines of defence defense penetration probability calculation.

Penetration of mathematical expectation of target number calculation formula is: $M = M_0 \bullet P_{ij}$, M_0 is the total number of incoming target.

The probability of target destroyed: $P = 1 - P_{ij}$.

Destroyed the target number of mathematical expectation of the calculation formula is: $M^{'} = M_0 \bullet P$.

Verify

(1) The similar air defense missile weapon the operational effectiveness of single line and multi-layer line configuration

If incoming $\lambda = 2$ target/min flow intensity, incoming target for total frame $M_0 = 6$, air defense missile weapon service rate $\mu = 1$ per minute, each line of air defense missile weapon to destroy rate for target, is similar to air defense missile weapon combat effectiveness when single line and multi-layer line configuration, as shown in table 1.

Table 1 the similar air defense missile weapon combat efficiency of the single line and multi-layer line configuration

parameter / indicators	Single line	Multilayer defense	Multilayer defense
	$n_1 = 2$	$n_1 = 2, n_2 = 3$	$n_1 = 2, n_2 = 3, n_3 = 1$
Penetration probability	0.4	0.037	0.012
Penetration goals expected number	2.4	0.222	0.072
Destroy probability	0.6	0.963	9.988
Destroy target expectations	3.6	5.778	5.928

Can be seen from table 1: in the process of air defense combat of air defense missile weapon configuration, should determine the main attack direction in the incoming target, the multi-layer defensive line configuration ways on this direction, can significantly reduce the penetration probability of enemy planes.

(2) Heterogeneous the operational effectiveness of air defense missile weapon multi-layer line configuration

If incoming $\lambda = 2$ target/min flow intensity, incoming target for total frame $M_0 = 6$, each line of air defense missile weapon to the target on the destruction rate was $Q = 1$ but different service rate is μ, can be to heterogeneous the operational effectiveness of air defense missile weapon multi-layer line configuration, as shown in table 2.

Table 2 heterogeneous air-defense missile weapon combat efficiency of the multi-layer line configuration

parameter / indicators	Configuration mode $\mu_1 = 2, \mu_2 = 4$	Configuration mode $\mu_1 = 4, \mu_2 = 2$
Penetration probability	0.137	0.111
Penetration goals expected number	0.822	0.666
Destroy probability	0.863	0.889
Destroy target expectations	5.178	5.334

By the table shows that when the first configuration mode is adopted in operational effectiveness than the second mode of configuration of operational effectiveness is lower. Generally missile weapon of the service rate is higher, and anti-aircraft artillery service rate is low, so the air defense combat, in the process of air defense weapon system configuration is usually "outside the bullet gun".

Conclusion

Paper using queuing theory knowledge, combined with the quantitative analysis method, mixed deployment of air defense missile weapon system combat effectiveness to make an objective evaluation, the assessment method is validated by an example of accuracy and objectivity, the resulting conclusions are accurate and reliable. This evaluation method can be used in the system of cost - effectiveness analysis, can be used for the scheme selection, decision-making for weapon equipment development and provide quantitative basis for system performance of tactical and technical argumentation, contribute to the decision-making to be scientific.

Reference

[1]Palmer D, Kirschenbaum M, Murton J, et al. Decentralized cooperative auction for multiple agent task allocation using synchronized random number generators[C]. Proc. of the International Conference on Intelligent Robots and Systems, 2003: 1963-1968.

[2]Dolgov D, Durfee EH. Satisfying Strategies for Resource-Limited Policy Search in Dynamic Environments[C]. The First International Joint Conference on Autonomous Agents and Multi-Agent Systems: Part 3 Table of Contents. Bologna, Italy. 2002:1325-1332.

[3]Bead RW, TimothyW M, GoodrichM A. Coordinated Target Assignment and Intercept for Unmanned Air Vehicles[J]. IEEE Transactions on Robotics and Automation. 2002.18(6):911-922.