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Continuous and Jump Betas : Implications for Portfolio Diversification (Evidence from Indonesia Stock Market)

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ABSTRACT

This study aimed to investigate the difference of CAPM beta for both continuous and jump components for constituent stocks market using Indonesia Stock Exchange (JKSE) and overall 45 the most active stocks from the LQ45 index for six months' period from March 2017 to August 2017. The study decomposed the time-varying beta for stocks into beta for continuous and discontinuous systematic risk. This study employed 5 min interval data set from Thompson Reuters, and the findings show that there is significance jumps components in Indonesia stock market (JKSE). However, there was no significance jumps component in individual stock market in LQ 45. We estimate individual beta small size companies have larger beta than large size companies with the highest beta is 0.00004% and the average beta is 0.0000132%. Furthermore, low volatility firms have smaller beta with the average 0.000056% as opposed to high volatility firms with the average 0,0000171%. This research reveals that continuous volatility in stock market is 0.0000123% and jump volatility is 0,0000434%. We also investigate that diverfication effect can be employed to decrease the total realized volatility by simply adding the number of stocks. Whether there is a similar pattern for continuous and jump systematic risk could not be found, but it was discovered in this study that investor can omitted jump systematic risk when they have at least 10 stocks in their portfolio.

Type of paper: Empirical

Keywords: high frequency data, continuous and jump beta, systematic risk.

1. Introduction

The optimal of portfolio size has been an important aspect of diversification since Markowitz (1959) introduced the concept of diversification by simply adding the *n* of stocks in a portfolio resulting the decrease of firm specific risk without affected average return of portfolio from the investor, and it could improve the principle of risk-return trade-off as well. So that, the investors probably better of holding many stocks than a smaller number of high-quality stocks (Vitaly Alexeev M. D., 2016)

Numerous questions then arise to address the number of stocks must be in included to create an optimal portfolio. Though, the optimal number of portfolio holdings are not consistent start from 10 stocks to 60 stocks to reduce most of the diversifiable risk, and this result cannot be generalized since there are differences such as assumptions, time-periods, the data, and the assumption of risk measurements. Nevertheless, many researchers agree that one of the measurement of risk rests upon the empirical market model is beta coefficient, in which the sensitivity of the return on an asset portfolio is a linier function of market factors common to all asset.

Another point of view, CAPM theory, said that expected return can be explained by systematic risk. This risk is undiversified so investor need to be ready to accept this risk even though they have optimal portfolio. Some substantial evidences say that asset prices evolve as a combination of Brownian motion with stochastic volatility and a jump process, we examine the differences between beta calculated for continuous and jump components of systematic risk. This study used CAPM perspective to disentangling between continuous and jump beta. We cannot separate betas from the calculation of Fama-French factors model because the lack of theory and dataset. This study results show that jump volatility is much lower than continuous volatility in both our portfolio and market.

Evidence of the existence of jumps in many financial asset classes may be found in (Anders, 2007), (Dungey *et al.*, 2009), (Jacod & Todorov, 2009), (Lahaye, Laurent, & Neely, 2011), and (Yacine Ait-Sahalia, 2016). Jump process has been shown to be an important element in improving the estimation of yield curve, estimates and forecast of daily volatility, explaining the high equity premium and optimal hedging strategies. The market itself presents evidence of significant rewards for bearing non-divertible jump risk; for example, via the expense of short-maturity options written on the market index with strikes that are far from its current level. Investors may treat rare events somewhat differently from common, more frequent events, in devising portfolio strategies.

This paper takes advantage of high-frequency data and new econometrics tools to split risk into its continuous and discontinuous components. We use the recent methodology of (Todorov V. B., 2010) to construct estimates of betas using high-frequency intraday data. We use all stocks from LQ 45 and creating random portfolio in which consist of 1, 5, 10, 15, and so on until 45 stocks. Then, we produce estimates of the extent portfolio of the continuous and discontinuous betas as well as we calculate continuous and discontinuous portfolio and market volatility. However, we find risk premium on discontinuous risk are insignificant in Indonesia stock market in period observed. Further decomposition reveals that significant risk premium exist on continuous risk in Indonesia stock market.

The remainder of this paper is organized as follows. Section 2 describes the dataset used in the empirical analysis. Section 3 introduces the modelling framework and parameter choices for estimating the continuous and jump betas. Our empirical analysis in Section 4 includes the estimation of portfolios stock betas, discussion of their properties. In Section 5, we assess the implication of the two risk components for portfolio diversification. Section 6 concludes the paper.

2. Data

This study investigated Indonesia Stock Exchange (JKSE) and overall 45 the most active stocks from the LQ45 index for six months' period from March 2017 to August 2017. The data consist of 5-min price observation between 9:00 am and 16:00 pm Indonesia west standard time (WIB) from Thompson Reuters database. This study followed the current convention in using 5-min observations as the choice of optimal sampling frequency for multiple assets. (Ait-Sahalia, 2012) suggest that 5-min observation is the most appropriate range stocks to adjust full of information from market.

Afterwards, portfolio was constructed as an equally weighted portfolio of investible stocks from LQ 45 index in each estimation window; this was chosen in preference to a value-weighted portfolio to avoid cases where one stock has a disproportionately high weight in the portfolio (Vitaly Alexeev M. D., 2016)

3. Modelling Framework

This research started by considering the *i*th log-price prosess p_i (t) from a collection of *n* processes $\{p_i(t)\}^n_{i=1}$ evolving in continuous time. We assume that $p_i(t)$ evolves as

$$dp_i(t) = \mu_i(t)\, dt + \sigma_i(t)\, dw_i(t) + \kappa_i(t)\, d\mu_i(t) \quad (1)$$

where $\mu_i(t)$ and $\sigma_i(t)$ refer to the drift $r_{i,t}$ and local volatility, respectively, $w_i(t)$ is a standard Brownian motion, and $\kappa_i(t)$ is a pure jump Levy process. A common modelling assumption for Levy process is the compound Poisson process, or rare jump process, where the jump intensity and the jump sizes are independent and identically distributed.

Denote the return on a benchmark market portfolio as $r_{0,t}$ is a standard single index model or one factor asset pricing model representation relating to $r_{0,t}$ takes the equation

$$r_{i,t} = \alpha_i + \beta_i\, r_{0,t} + \varepsilon_{i,t} \qquad i = 1,\ldots\ldots, N. \quad (2)$$

where β_i coefficient in eq 2 is the sensitivity of the expected return on the *i*-th asset to the return on the market (or systematic) factor. Following Todorov and Bollerslev (2010), the market return $r_{0,t}$ can be represented in the same way as in eq 1 consisting of the continuous and discontinuous components. Therefore, in the presence of jumps, equation (2) becomes

$$r_{i,t} = \alpha_i + \beta^c_i\, \sigma_{0,t}\, dw_{0,t} + \beta^d_i\, \kappa_{0,t}\, d\mu_{it} \quad + \varepsilon_{i,t} \qquad i = 1,\ldots\ldots, N. \quad (3)$$

where $\varepsilon_{i,t}$ in this case contains both the idiosyncratic continuous and discontinuous components in each individual stock. Eq 3 implies that the sensitivity of an asset return to the two components of systematic risk can be different, represented by β^c and β^d respectively. Overall, systematic risk is due to the continuous component of the market movement $\sigma_{0,t}\, dw_{0,t}$ and the discontinuous component $\kappa_{0,t}\, d\mu_{it.}$ Assuming that the continuous record of the asset return $r_{i,t}$ and $r_{0,t}$ are available so the value of β^c and β^d can be derived from eq 3 using covariation approach.

For any i = 0,1, ..., *N,* the quadratic variation of $r_{i,t}$ over the time interval (0,T) is defined as

$$[r_i, r_{i,}]^2_T = \int_0^t \sigma^2_{i,s}\, ds + \sum_{0<s\leq t} \kappa^2_{i,s} \quad (4)$$

it follows that quadratic covariation between $r_{i,t}$ and $r_{0,t}$ is

$$[r_i, r_o]^2_T = \beta^c \int_0^t \sigma^2_{i,s}\, ds + \beta^d \sum_{0<s\leq t} \kappa^2_{i,s} \quad (5)$$

The first term in eq 5 $[r_i, r_{i,}]^2_T$, captures the covariation in the continuous components of $r_{i,t}$ and $r_{0,t}$ whereas the second term represents the discontinuous covariation. For the continuous beta,

$$\beta^c = \frac{[ri,ro]2T}{[r0,ro]2T}, \quad i = 1,......, N, \tag{6}$$

and

$$[r_i^d, r_o^d]^2_T = (\beta^d_i)^{\tau} \sum_{0<s\leq t} \kappa^{2\tau}_{0,s} = (\beta^d_i)^{\tau}[r_i^d, r_o^d]^{2\tau}_T, \quad 1< \tau \tag{7}$$

sample -provide the basis for constructing discrete 7and 6for the discontinuous beta eq estimator of β^c and β^d.

3.1 The estimators in discrete time

Empirical applications do not have the luxury of continuously recorded asset prices and return. Instead, we assume that they are observed at every Δ, 2Δ,....., to [*T/*Δ]. Hence, there are [*T/*Δ] interval returns within (0,T), which are denoted by

$$r_{i,j} = p_{i,j} - p_{i,j-1}, i = 0, 1, \ldots, N, \quad j = 1,.........,[T/\Delta]. \tag{8}$$

the consistent estimators for β^c and β^d given by Todorov and Bollerslev (2010) are constructed as follows: let rj = ($r_{0,j}$, $r_{1,j}$, ..., $r_{N,j}$) be the (N+1) x 1 vector of the observed returns, *j* = 1,........,[*T/*Δ]. The sample counterpart of eq 6 is constructed using truncated realised covariations,

$$\hat{\beta}_i^c = \frac{\sum_{j=1}^{\lfloor T/\Delta \rfloor} r_{i,j}\, r_{0,j}\, \mathbb{1}_{\{|\mathbf{r}_j|\leq \theta\}}}{\sum_{j=1}^{\lfloor T/\Delta \rfloor} r_{0,j}^2\, \mathbb{1}_{\{|\mathbf{r}_j|\leq \theta\}}}, \quad i = 1,...... , N \tag{9}$$

where $\theta = (\alpha_0\Delta^{\omega,}\ \alpha_1\Delta^{\omega,}\ ...., \alpha_N\Delta^{\omega})'$, $\omega \in (0,1/2)$ and 1 denotes the indicator function. The truncated realised covariation extends the univariate measure first proposed by Mancini (2001) and provides a consistent estimator of $[r_{i,} r_o]^2_T$ as Δ converges to 0.

The discrete-time estimator of β^d utilises higher power covariations

$$\hat{\beta}_i^d = \text{sign}\left\{\sum_{j=1}^{\lfloor T/\Delta \rfloor} \text{sign}\{r_{i,j}\, r_{0,j}\}|r_{i,j}\, r_{0,j}|^{\tau}\right\} \times \left(\frac{\sum_{j=1}^{\lfloor T/\Delta \rfloor} \text{sign}\{r_{i,j}\, r_{0,j}\}|r_{i,j}\, r_{0,j}|^{\tau}}{\sum_{j=1}^{\lfloor T/\Delta \rfloor} r_{0,j}^{2\tau}}\right)^{\frac{1}{\tau}} \tag{10}$$

Where the power $\tau \geq 2$ so that the continuous price movements become negligible asymptotically and only large returns are retained. An alternative estimator of the discontinuous beta relies on the truncation technique. Instead of taking the "small" returns as in (9), it collects the complement set of $\{|\mathbf{r}_j| \leq \boldsymbol{\theta}\}$ and inserts this in equation 10. Both estimators are consistent and asymptotically equivalent. However, theorem 1 in Todorov and Bollerslev (2010) suggests that the convergence in probability of $\hat{\beta}^d$ only holds if there is at least one jump in the market portfolio $r_{0,t}$ over the interval (0, T]. Therefore, to calculate $\hat{\beta}^d$,we first need to test for the existence of jumps in the market portfolio.

Table 1. Descriptive Statistics

5-min sample	LQ 45 Sample	Stock Market
Mean return	0.0066%	0.0544%
Realised Volatility (RV)	0.0019%	0.0023%
Intragrated Volatility	0.0011%	0.0013%
Realised Jump Volatility	0.0007%	0.0009%

Table 2. Jump Test Statictics

H0 = no jumps	LQ 45 Sample	Stock Market
Number of detected jumps	28	46
Critical level	0.001***	0.001***
Critical Value	3.09	3.09

4. Empirical Evidence

4.1 Descriptive statistics

Fourty five of the most active stocks from the LQ45 index over six months' period from March 2017 to August 2017 was calculated. The data consist of 5-min price observation between 9:00 am and 16:00 pm Indonesia west standard time (WIB). We follow current convention in using 5-min observations as the choice of optimal sampling frequency for multiple assets. (Ait-Sahalia, 2012) suggest that 5-min observation is the most appropriate range stocks to adjust full of information from market.

Table 1 depicts descriptive statistics between LQ45 portfolio and market portfolio (JKSE). It clearly shows that both continuous and jump volatility in JKSE is higher than in LQ 45 portfolio which are 0.0013% and 0.0009% as opposed to 0.0011% and 0.0009% respectively. Higher integrated volatility means that JKSE has long-run risk instead of short-run, which is related to public information from market. While, higher realized jumps can be interpreted as the number of private information which flows in market and it could affect the stock price. The result is the same for test of the number of jump existence using significance level 99%, and the number of jumps in our LQ45 portfolio is lower than stock market (JKSE) which is 28 and 46 respectively.

The characteristics of volatility in stock market to determined jumps beta and continuous beta were also investigated. Figure 1 shows the characteristics of volatility in stock market during morning session, mid-day and after lunch session. Overall, it indicates that the most volatility was happened during mid-day or slightly before and after lunch break which was about 11.20 am to 13.55 pm. The highest volatility was happened before market close at about 15.55 pm.

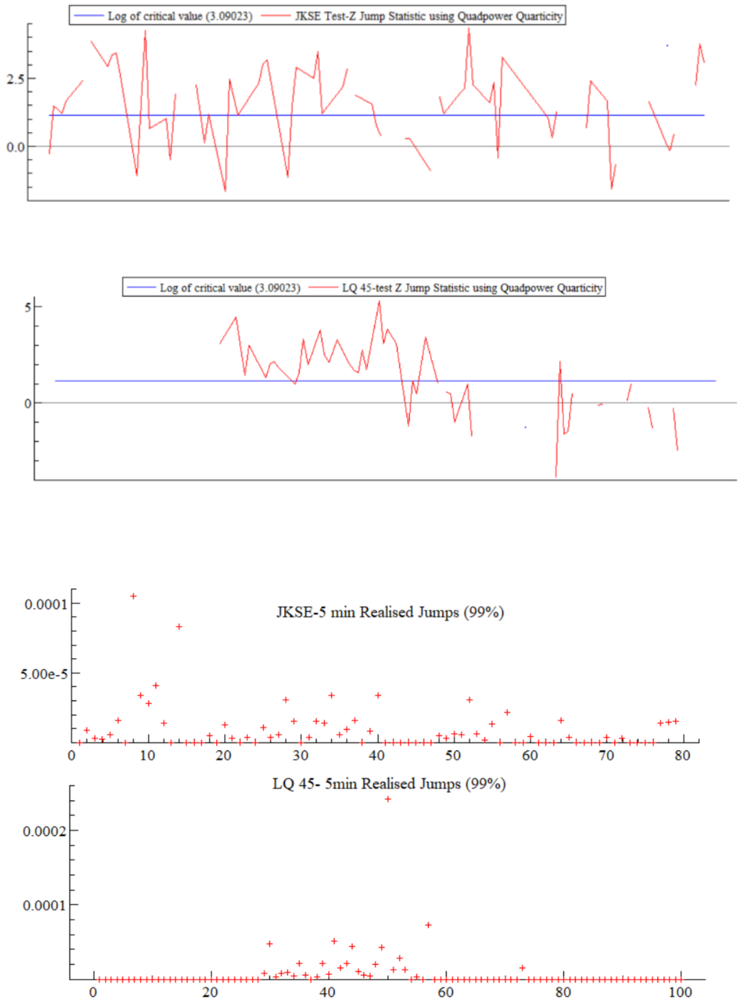

Figure 1. JKSE and LQ45 Comparasion Test Result

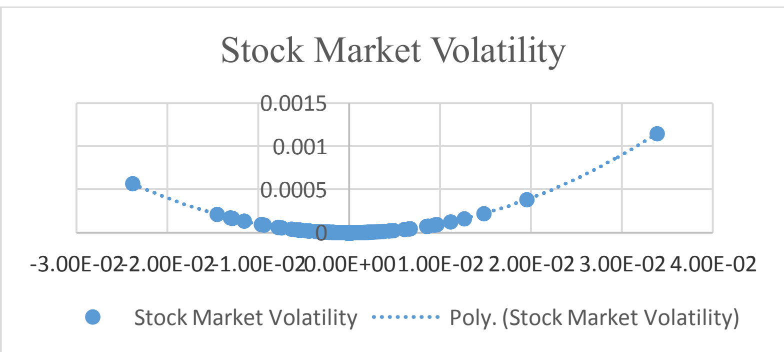

Figure 2. JKSE Volatility

Firm characteristics can have strong impacts on firms' sensitivity to systemic risk. For example, we would expect the larger firms are less vulnerable to market risks, and hence have a lower beta compared with small. To explore the roles of firms' characteristics in the estimated beta jump and continuous, we rank stocks in terms of size, and level of idiosyncratic volatility. Firm size is measured by market capitalization and the level of idiosyncratic volatility is measured using a daily return regression:

$$\bar{r}_{i,d} - \bar{r}_{f,d} = \alpha_i + \beta_i(\bar{r}_{0,d} - \bar{r}_{f,d}) + \phi SMB_d + \psi HML_d + \epsilon_{i,d}, \qquad (11)$$

Where ri,d and r0,d are return of the i-th stock and market portfolio on day d, and $r_{f,d}$ is the risk free rate on day d. SMB and HML are the daily Fama-French factors. Standard deviation of the regression residual in each month is taken as idiosyncratic volatility of each stock.

Figure 3. Jump Return in Stock Market

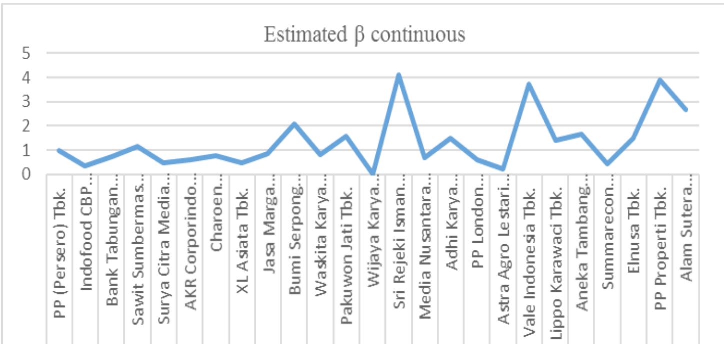

Figure 4. Small Size

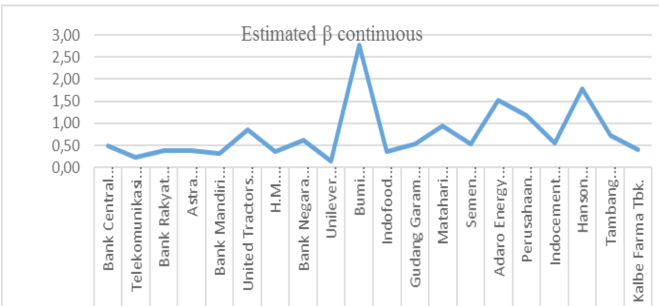

Figure 5. Large Size

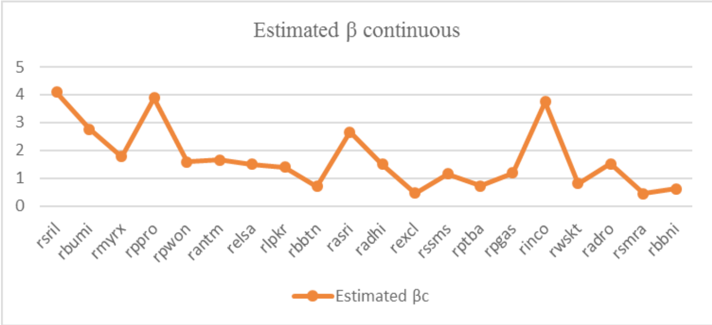

Figure 6. Low Idiosyncratic Volatility

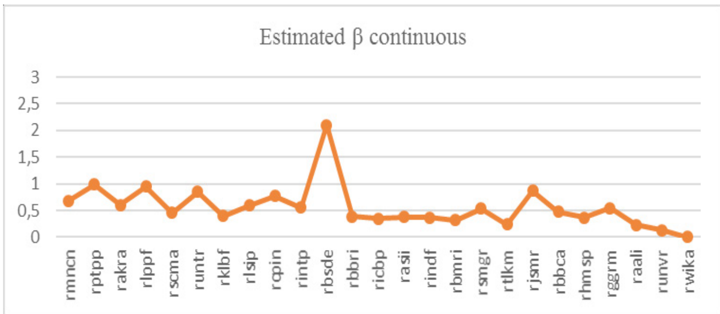

Figure 7. High Idiosyncratic Volatility

Figure 3 and 4 portray that small size companies have larger beta than large size companies with the highest beta is 4% and the average beta is 1,32%. Furthermore, low volatility firms have smaller beta with the average 0,56% as opposed to high volatility firms with the average 1,71%.

4.2 Portfolio Diversification with Betas

This study decomposed beta using equally weighted total realized volatility. For example, Christensen *et al.* (2014) suggest pre-averaging method based higher frequency data. However, one thing to be noted is that this method has not been widely adopted for small samples. To our knowledge, there is no comparable study for a broad range of assets in a portfolio context as undertake here. Concern about the appropriate choice of sampling frequency also suggest that one should be wary of the apps effect; the reason that both (Patton, 2012) and (Todorov V. B., 2010) use lower frequency sampling than is employed in this study.

Even though we did not find any significance jumps in individual stock, we find jumps in level 90% in our portfolio. Figure 8 depict diversification effect can be employed to decrease the total realized volatility by simply adding the number of stocks. As a result, the lowest volatility is the portfolio that consist of 45 stocks.

This study also constructed the same equally weighted portfolio and estimate systematic continuous and discontinues risk. (Vitaly Alexeev M. D., 2016) used **βj x ki,t** as a proxy discontinuous systematic risks where **βc** is discontinuous beta and *ki,t* is jump volatility. Furthermore, continuous systematic risk can be constructed as the same way as discontinuous.

$\beta c \times Bv_{i,t}$, where βc is continuous beta and **Bvit**, is the bipower variation as a proxy of continuous volatility.

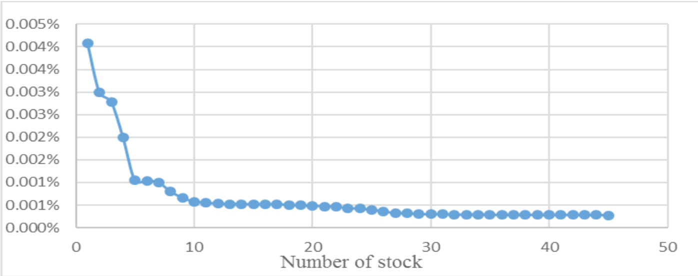

Figure 8. Porfolio and RV

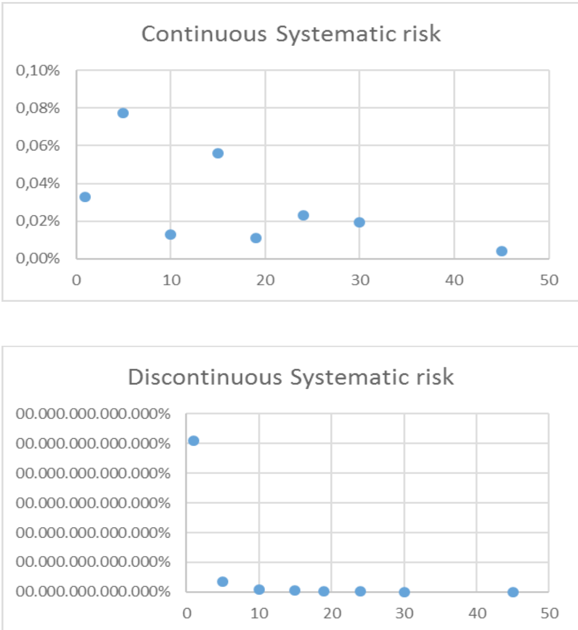

Figure 9. Effect of diversification on systematic risk

Figure 8 shows that there is no decrease pattern in continuous systematic risk as the number of stocks in portfolio n increase. It means that the portfolio consists of persistent continuous systematic risk even when the number of n stocks in the portfolio is being increased. On the other hand, it was found that there is a very small percentage of discontinuous systematic risk and this risk can be overlooked when there are at least 10 stocks inthe portfolio.

5. Conclusion

This paper estimates CAPM beta for both continuous and jump components for constituent stocks market using six-month sample from March to August 2017. This study utilized 5 min interval data set from Thompson Reuters, and the findings show that there is significance jumps components in Indonesia stock market (JKSE). However, we do not find any significance jumps component in individual stock market in LQ 45.

It was discovered that for individual small size companies have larger beta than large size companies with the highest beta is 0,00004% and the average beta is 0,0000132%. Furthermore, low volatility firms have smaller beta with the average 0,000056% as opposed to high volatility firms with the average 0,0000171%.

This research reveal that continuous volatility in stock market is 0,0000123% and jump volatility is 0,0000434%. It means that in there are short run or dynamic volatility instead of persistent volatility in stock market. We investigate that diversification effect can be employed to decrease the total realized volatility by simply adding the number of stocks. The implication for investor is they can reduce both continuous and jump volatility by simply increase the number of n portfolio with equally weighted. This method is effortlessly to reduce total risk in our portfolio. The findings show no similar pattern for continuous and jump systematic risk. However, the results show that investor can omitted jump systematic risk when they have at least 10 stocks in their portfolio.

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