The Realization of the Parametric Excitation on MEMS Gyroscope

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Abstract—In this paper, parametric excitation is applied to the excitation of a micro-ring gyroscope. The primary objective of this work is to improve the dynamic gain. It is show that, by adjusting the parametric excitation parameters, the harmonic forcing can be reduced whilst the gain of frequency response can be amplified. Because of the different frequencies of the excitation and harmonic forcing, the unwanted effect of electrical feed-through can keep in a minimum value.

Keywords-MEMS gyroscope; parametric excitation; closed-loop system

I. INTRODUCTION

MEMS (Micro-Electro-Mechanical System) gyroscopes have many advantages such as low cost, small size and negligible weight compared to the conventional mechanical gyroscopes. Furthermore, they have a wide range of applications including navigation and guidance systems, automobiles and consumer electronics, so that many researchers have focused on them for the last decades. However, to amplify the effective signal and reduce the noise remains the same.

Parametric resonance techniques have been used to amplify the mechanical signal in MEMS resonators, cantilevers and in MEMS gyroscopes. Parametric excitation refers to a method of excitation whereby a resonant system may be set into oscillation by periodically varying one of the parameters of the homogeneous system. Parametric amplification of approximately two orders of magnitude has been demonstrated in a forced mode of a micro-ring gyroscope [1]. When applied to the primary mode of the gyroscope, parametric amplification enables reduced forcing amplitudes and hence reduces electrical feed-through due to parasitic capacitances.

In this paper the experimental results from a MEMS gyroscope under high gain parametric amplification are described. The amplitude of the actuating signal may then be reduced so that the noise produced by electrical feed-through will be decreased. This will significantly improve the performance of the ring gyroscope.

II. THEORY OF PARAMETRIC AMPLIFICATION

The ring gyroscope mainly has two operating modes of drive and detection, and its dynamic model can be regarded as two order system of spring mass block with two degree of freedom. The system block diagram is shown in Figure 1.

As shown in figure 1, X coordinate is drive modal movement direction, Y coordinate is sensitive modal movement direction. By applying voltage on the drive electrode, the mass block is driven by electrostatic force to do simple harmonic vibration in the X direction. When there is an angular rate input, due to the effect of the Coriolis force, mass produce relative displacement in the Y direction. The dynamic equation of two modes can be written [2, 3]:

$$\begin{cases} I_x \ddot{q}_x + \lambda_x \dot{q}_x + k_x q_x = F \\ I_y \ddot{q}_y + \lambda_y \dot{q}_y + k_y q_y = -2 I_y \Omega \dot{q}_x \end{cases} \tag{1}$$

In the formula, I_x and I_y are the rotational inertia of the drive mode and the sensitive mode respectively, q_x and q_y are displacement of mass blocks in driving modes and sensitive modes respectively, λ_x and λ_y are damping coefficients of drive modes and sensitive modes, respectively, k_x and k_y are the stiffness coefficients of the driving modes and the sensitive modes respectively.

The steady-state solutions of the drive mode are obtained by solving the upper modes:

$$q_x = A \cos(\omega t - \varphi_x)$$

$$A = \frac{M}{I_x \sqrt{(\omega_x^2 - \omega^2)^2 + (\omega \omega_x / Q_x)^2}} \tag{2}$$

$$\varphi_x = \arctan\left(\frac{\omega_x \omega}{Q_x(\omega_x^2 - \omega)} \right)$$

Here, $\omega_x = \sqrt{k_x / I_x}$ is natural angular frequency of the drive mode. $Q_x = 1 / 2\delta_x$ is quality coefficient of drive mode. $\delta_x = \lambda_x / (2 I_x \omega_x)$ is damping ratio of drive mode.
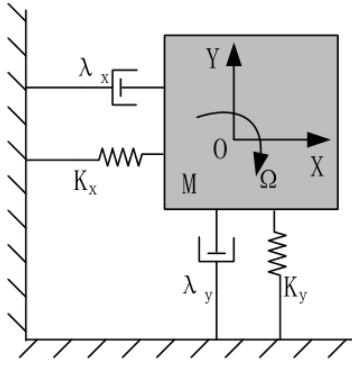
FIGURE I. MECHANICAL MODEL OF GYROSCOPE

A briefly description of parametric excitation and amplification for the MEMS gyroscope is presented in this section since a detailed description is available in [4, 5].

In order to enable parametric amplification of the MEMS gyroscope's response to harmonic forcing, the drive voltage $V_d(t)$ must be of the form:

$$V_d(t) = V_p \sin(\omega_p t + \varphi) + V_f \sin(\omega_f t) \tag{3}$$

Where: ω_f is the forcing frequency and $\omega_p = 2\omega_f$, φ is the phase advance of parametric excitation with respect of the forcing in addition.

The linearized equation of the primary motion under combined harmonic forcing and parametric excitation has the form [6]:

$$\ddot{q} + \frac{\omega_\alpha}{Q}\dot{q} + (\omega_\alpha^2 - \eta_0 + \eta_1 \sin(\omega_p t + \varphi))q = -\bar{\eta}_0 \sin(\omega_f t) \tag{4}$$

With

$$\eta_0 = \frac{\varepsilon a d}{2h_0^3}\left(\alpha + \frac{\sin(4\alpha)}{4}\right)(V_p^2 + V_f^2) + 32\alpha \frac{\varepsilon a d}{2h_0^3}V_{dc}^2$$

$$\eta_1 = 4\frac{\varepsilon a d}{2h_0^3}\left(\alpha + \frac{\sin(4\alpha)}{4}\right)V_p V_{dc}$$

$$\bar{\eta}_0 = 2\frac{\varepsilon a d}{2h_0^2}\sin 2\alpha V_{dc} V_f$$

The solution of Eq. (4) can be written by

$$q_p(t) = A\sin(\omega_f t + \psi) \tag{5}$$

Where the steady state amplitude A and phase angle Ψ of the primary mode with parametric excitation are describe

$$A(V_p, \varpi, \varphi) = \bar{\eta}_0 \frac{\sqrt{\varsigma^2 + \eta_1\cos(\varphi) + \varpi^4 + \eta_1\varpi^2\sin(\varphi) + \frac{1}{4}\eta_1^2}}{\varpi^4 + \varsigma^2 - \frac{1}{4}\eta_1^2}$$

$$\psi(V_p, \varpi, \varphi) = \tan^{-1}\left(-\frac{\varsigma + \frac{1}{2}\eta_1\cos(\varphi)}{\varpi^2 + \frac{1}{2}\eta_1\sin(\varphi)}\right)$$

$$\varpi^2 = \varpi_\alpha^2 - \omega_f^2$$

$$\varpi_\alpha^2 = \omega_\alpha^2 - \eta_0$$

$$\varsigma = \frac{\omega_\alpha \omega_f}{Q} \tag{6}$$

Here, ω_α represents the natural frequency of the ring gyroscope. $\bar{\omega}_\alpha$ represents the softened natural frequency because of the negative electrostatic stiffness caused by applied voltage so that the softened natural frequency is slightly less than the natural frequency.

The frequency response data could be represented in terms of the mobility $M(V_p, \varpi, \varphi)$, where [7]:

$$|M(V_P, \varpi, \varphi)| = \frac{\omega_f A(V_P, \varpi, \varphi)}{\bar{\eta}_0}$$

$$\theta = \psi(V_P, \varpi, \varphi) + \frac{\pi}{2} \tag{7}$$

Gain, which can quantify the effect of parametric amplification, is defined as the ratio of the Mobility magnitude with parametric excitation 'ON' to the Mobility magnitude with parametric excitation 'OFF'.

$$Gain(V_P, \varpi, \varphi) = \frac{|A(V_P, \varpi, \varphi)|_{on}}{|A(\varpi)|_{off}} = \frac{|M(V_P, \varpi, \varphi)|_{on}}{|M(\varpi)|_{off}} \tag{8}$$

III. DEVICE DESCRIPTION

The basic form of MEMS gyroscope is shown in figure 2. Compared with the traditional MEMS ring gyro referenced in [8, 9, 10], the gyro made some improvements. Two electrodes are placed between rings, rather than a single electrode, to increase the capacitance area. The positive and negative electrode formed the difference form, in order to achieve the goal of increasing the driving force and reducing noise. The dimensions are described in table 1.

The gyroscope is actuated and sensed electrostatically. Each of the electrode encompasses an angle 2α. The gyroscope vibrates in an in-plane flexural mode.

A dc voltage is applied to the ring structure. The driving signal is comprised by harmonic forcing and parametric excitation. The frequency of parametric excitation is twice as large as harmonic forcing, controlled by FPGA.

TABLE I. GYROSCOPE DIMENSIONS

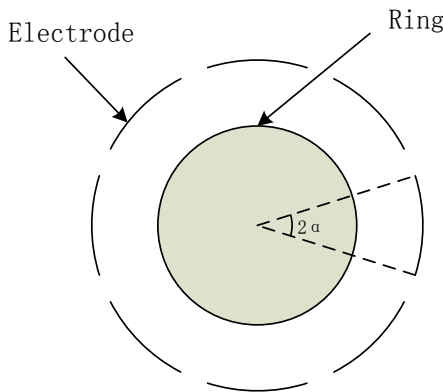| Parameter	Value
Outer/Inner radius a1/a2 (μm)	1000
Thickness d (μm)	75
Width b (μm)	5.8
Capacitive gap ho (μm)	7.2
Q-factor	25000
Natural frequency (Hz)	14870
Mass m(kg)	1.46e-7
2α (rad)	$\pi/4$

FIGURE II. BASIC FORM OF MEMS GUROSCOPE

IV. EXPERIMENTAL ARRANGEMENT

Figure 3 shows the schematic layout of the experimental system used to assess the effects of parametric excitation on the MEMS gyroscope. This paper propose a MEMS resonant gyro closed-loop driving system based on FPGA. The closed-loop driving system of MEMS ring gyroscope consists of interface circuit, mixed analog/digital circuit and signal processing circuit as shown in figure 4.

The function of the interface circuit is to translate the capacitance change caused by the mechanical vibration of the gyro drive mode into the test voltage, so as to realize the detection of the driving signal by the circuit. The mixed analog/digital circuit can be regarded as the connection device between the interface circuit and the FPGA digital system. The mechanical vibration of MEMS resonant gyroscope is converted into voltage and then sampled and quantized by ADC to become digital signal. The digital signal is processed by the algorithm designed in FPGA, and then converted into analog voltage through DAC to feedback on the gyro driving exciting electrode.

According to simulation analysis, the phase advance φ of parametric excitation to harmonic forcing is a pronounced effect to the result of the experiment. In the simulation, it is obvious that the gain is most positive affected when the phase advance is zero. However, parametric excitation have side effect to gain when phase advance is close to π. Because of this, the phase advance φ of parametric excitation to harmonic forcing in this experiment is controlled to become 0.

Similarly, dc voltage applied to the gyroscope V_{dc} is also related to the experimental result. In this experiment, V_{dc} is determined to be 5V.
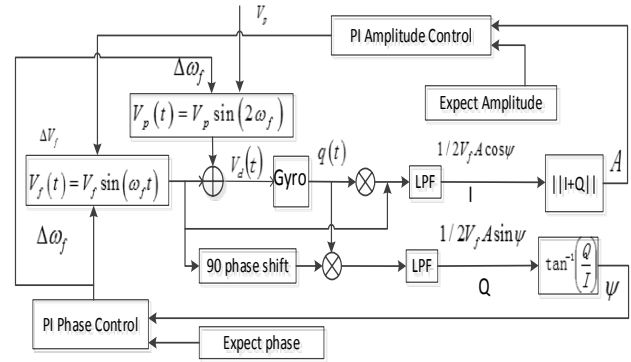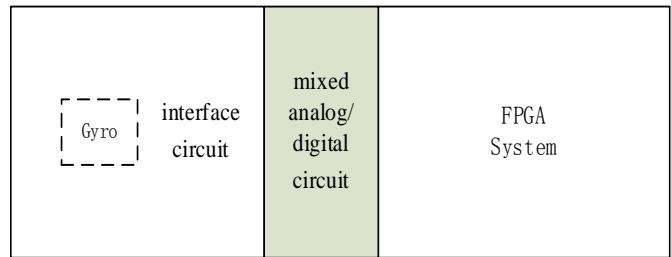

FIGURE III. CONTROL SCHEME

FIGURE IV. PCB MODEL OF THE TEST

V. EXPERIMENTAL RESULTS FOR PARAMETRIC EXCITATION

The closed loop experimental test results for the drive mode are shown in Figs. 5-6. Figure 5 shows the results of the test either with or without parametric excitation. As shown in this figure, the amplitude of the parametric is higher than drive signal to ensure the effect on the gain.

This is shown in Figure 5, the amplitude of harmonic forcing is decreased after adding the parametric excitation just like theory.
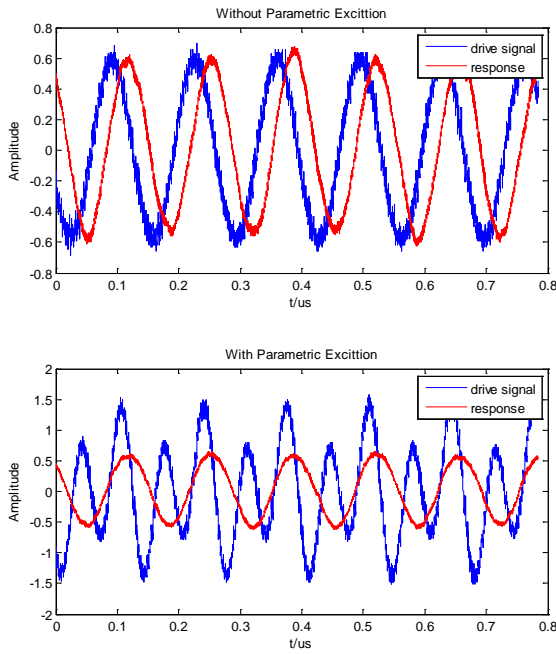
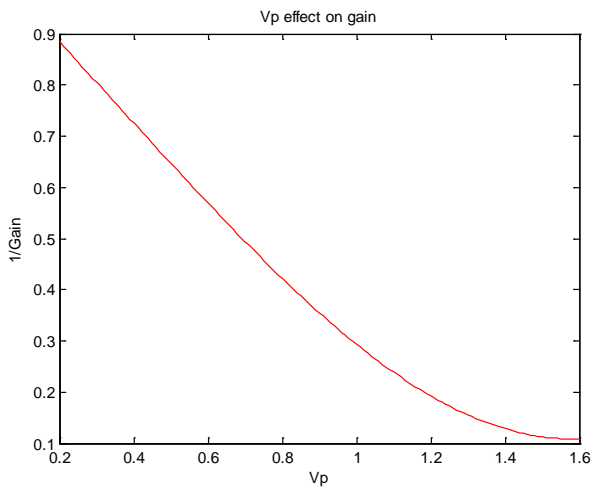FIGURE V. THE RESULTS OF ADDING PARAMETRIC EXCITATION

FIGURE VI. GAIN

Figure 6 shows the gain increase with V_p. From the figure, it can be found there is an obvious threshold about V_p and gain. When V_p <1.3V, there is a linear relationship between gain and V_p. However, when V_p >1.3V, gain tends to a constant value. Hence, the amplitude of V_p is limited to the linear range. In the linear range, with the rise of V_p, the ratio of the mobility magnitude with parametric excitation to the mobility magnitude without parametric excitation is rising correspondingly.

VI. CONCLUSION

This paper research the influence of the parametric excitation signal on the gain. As shown in the above experimental results, parametric excitation is able to reduce the amplitude of harmonic forcing because it has positive effect on the gain.

It is clear that the amplitude of the substrate voltage and parametric excitation is related to the gain. It is desirable to improve the voltage limit by improving the equipment for further experiments. After that, the experimental results are expected to be further improved

Hence, the principal benefits of amplifying the drive mode is to enable reduced harmonic forcing level. Because of this, it is able to reduce the electrical feed-through of the drive signal to the sense electrodes. Then, it could improve the signal-to-noise ratio of the MEMS gyroscope.

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