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A Multi-Gears Cellular Automata Model for Traffic Flow Based on Kinetics Theory

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Abstract—**Traffic flow modeling based on cellular automata (CA) has gained considerable importance as one effective tool to successfully simulate complex traffic systems and understand their behavior. However, most of the existing CA models assumes a constant acceleration rate for the vehicles, which is an over-simplification and should be avoided. In fact, most conventional vehicles have multiple gears transmissions. Thus, when one vehicle reaches its top gear, its acceleration is only a fraction of that available at lower speeds. In this paper, a simple and reliable CA model oriented to faithfully reproduce the acceleration profile of vehicles is proposed. For this purpose, a multi-regime constant acceleration model is introduced. In this way, the proposed model can have many points of discontinuity when a vehicle is accelerating based on the vehicle velocity and multiple gears. Simulation results indicate that the performance of vehicles accelerating from a stopped position is reproduced more in line with that obtained from real vehicles, when a larger number of gears is considered. Moreover, the resulting model is more in line with acceleration profile of the vehicles in the real world without seriously jeopardizing its computational efficiency.**

Keywords-microscopic traffic flow models; cellular automaton; limited uniform acceleration/deceleration capability; heterogeneity of acceleration

I. INTRODUCTION

In the last years, traffic flow modeling based on cellular automata (CA) has gained considerable importance as one effective tool to successfully simulate complex traffic systems and understand their behavior. Cellular automata models are discrete dynamical (microscopic) systems, in which the movement of individual vehicles is governed by drivers' behaviors, road topology and headway distribution. Hence, when CA models are applied in traffic research, they describe and update the position and velocity of each vehicle by using rules deduced from practical traffic experience that focus on vehicle's interaction. One of the main advantage of CA models is that CA have low computational cost and provide a possibility to conduct large-scale real simulations (see for example [1-2]).

Since the introduction of the Nagel and Schreckenberg (NaSch) in 1992 [3], various modifications and extensions have been proposed (for reviews see, e.g., [4-5]). Recently, with the aim to reproduce in a better way the deceleration process of vehicles some CA models have been introduced. One of the first efforts to introduce limited deceleration capability into CA traffic flow modeling is the Krauss and Wagner's [6] model, although the value of deceleration capability is very conservative. Later, Lee et al. [7] also introduced a traffic model that limits the vehicles' acceleration and deceleration rates to realistic values, but the model is not realistically safe, as occasionally large decelerations are also needed to avoid collisions. Bham and Benekohal [8] proposed a model (so called CELLSIM), whose formulation uses concepts of cellular automata (CA) and car-following (CF) models and has realistic acceleration and deceleration rates for vehicles, in this model a follower accelerates, decelerates or maintains speed (cruise) according to three special cases of the space gap versus the desired space gap. The underlying rules of motion in all of these models are, however, also quite complicated.

In [9] the authors proposed a CA model which focus in reproducing observed traffic phenomena by identifying the fundamental responsible mechanisms; this allows a much simpler model structure that faithfully reproduces the phenomena with high computational efficiency, still required for largescale microscopic simulations [10]. More recently, this model was extended by proposing a still simple and reliable cellular automata model oriented to faithfully reproduce reactions of drivers (hereafter referred by LAI-E). The main two new features introduced are: 1) Different limited acceleration and deceleration capabilities according to vehicle type, i.e. car or truck, and 2) Vehicles kinetics is based on uniform accelerated motion, rather than in impulsive accelerated motion as in most exiting CA models [11]. Thus, the LAI-E model overcomes the two major limitations of CA models for traffic flow that are the use of unrealistic acceleration/deceleration rates and their inability to perform smooth approach to slower or stopped vehicles. Besides, the model is minimal and very simple, executing only one state dependent action per time step per each vehicle. However, the LAI-E model assumes a constant acceleration rate for vehicles, like the most traffic flow CA models. This is a dramatic over-simplification and should be avoided.

Most real conventional vehicles are designed by using multiple gears in such a way that higher acceleration rate is provided at lower speeds and lower acceleration rate at higher speeds, and therefore, vehicles use the minimum of their acceleration capacity near their maximum desired velocity. Thus, when one vehicle reaches its top gear, the acceleration is only a small fraction of that available at lower speeds (hereafter, this traffic flow feature will be referred as the adaptive acceleration profile). Hence, it is not correct to use a continuous acceleration model, with a single gear, since it provides same constant acceleration rate at higher and lower speeds. In our knowledge, the only CA model that have considered an adaptive acceleration profile is the CELLSIM model [8]. In this model, a dual-regime constant acceleration model is addressed in a manner that two different acceleration rates overcome this problem to an extent by providing higher acceleration rate at lower speeds and lower acceleration rate at higher speeds. Therefore, acceleration rate is constant until the change in rate of speed. The CELLSIM model thus provides a more realistic acceleration behavior that the most of CA models, however, the dual-regime model does not show change in speed for every gear change. The model has only one point of discontinuity based on the increasing of the velocity of the vehicles and the rest of the discontinuity points are derived from decelerations.

In this paper, the main goal is to further extend the LAI-E model proposed in [11] by proposing a still simple and reliable cellular automata model oriented to more faithfully reproduce the acceleration profile of vehicles. For this purpose, the acceleration profile of the LAI-E model is modified by introducing a multi-regime constant acceleration model in such a way that the modified model can have many points of discontinuity when a vehicle is accelerating, i.e., when the acceleration rate changes based on the vehicle velocity and multiple gears. Besides, the acceleration profile of the proposed model is also discontinuous when a vehicle decelerates due to the two types of possible deceleration defined in the LAI-E model: the normal and emergency braking. Simulation results indicate that the performance of vehicles accelerating from a stopped position is reproduced more in line with that obtained from real vehicles, when a larger number of gears is considered. The resulting model is more in line with acceleration profile of the vehicles in the real world without seriously jeopardizing the computational efficiency of the original model.

The rest of this paper is organized as follows. In section II, a brief description of the LAI-E model is presented. A multi-gear cellular automata model for traffic flow based on kinetics theory is introduced in section III. Section IV presents simulation results from the proposed model. Finally, the concluding remarks and a summary of findings are discussed.

II. The LAI-E Model

As the model proposed in this work is an extension of the so-called LAI-E model [11], a brief description of this is given in this section.

The LAI-E model is a probabilistic cellular automaton. The main features of the LAI-E model are: 1) Different limited acceleration and deceleration capabilities according to vehicle type, i.e. car or truck, and 2) Vehicle's kinetics is based on uniform accelerated motion, rather than in impulsive accelerated motion as in most exiting CA models. Thus, at the core of the dynamics of the LAI-E model is the definition of three safe following distances to accelerate, decelerate or keep the velocity of a vehicle (d_{acc}, d_{dec} and d_{keep}, respectively) and avoid collisions at future times.

For the definition of the safe distances, it is considered that a vehicle will be able to accelerate, keep its velocity or slow down at time-step *t+1* and, in next time-step, it will begin to slow down suddenly until it stops; such that it still avoids collisions with the vehicle ahead, although the latter also slows down abruptly at any time-step.

The LAI-E model calculates the safe following distances in an analytic way, based on kinetic theory (for a detailed explanation see [11]). Thus, in the LAI-E model, two scenarios for the occurrence of collisions must be considered to define the safe following distances: (i) *stopped vehicle* (the criterion D^{stop}), where the leader vehicle stops and the follower vehicles collides with it; and (ii) *moving vehicle* (the criterion D^{mov}), where the collision takes place with both vehicles in motion.

Let sub-indexes *f* and *l* to denote the leader and follower vehicles, respectively. Let x_{f_t} and v_{f_t} respectively denoting the position and velocity of vehicle *f* at time-step *t* (it is assumed that vehicle *l* precedes vehicle *f*, which has a position x_{l_t} and velocity v_{l_t} in an analogous way). Besides, let $t_{b_l}=v_{l_{t+1}}/a_{max_l}$ and $t_{b_f}=v_{l_{t+1}}/a_{max_f}$ denoting the braking times from time *t+1*, for leader and follower, respectively. Then, the safe following distances are defined as follows, for each one of the considered cases [11] (see Table I for a summary of the parameters and variables of the LAI-E):

(i) Stopped vehicle. Let $\ddot{x}$ denoting the minimum required distance for vehicle *n* to slow down from its velocity v_t to 0. Besides, let D^{stop} denoting the minimum safe following distance that should exist between two consecutive vehicles *f* and *l*. The condition for the leader and follower to avoid collisions when stopped is. $\ddot{x}_l - l_f > \ddot{x}_f$. Then, by using basic definitions of uniformly accelerated motion (see [11]), D^{stop} is calculated in analytic way as follows:

$$D^{stop} = \frac{\left(v_{f_t}+a_{f_t}\right)^2}{2a_{max_f}} - \frac{\left(v_{l_t}-a_{max_l}\right)^2}{2a_{max_l}} + \frac{\left(a_{max_l}+a_{f_t}\right)}{2} - v_{l_t} + v_{f_t} + l_f \tag{1}$$

here a_{f_t} will take values a_f, 0, or $-a_f$ to define the safe following distances required by a follower vehicle to accelerate (D^{stop}_{acc}), keep its velocity (D^{stop}_{keep}) or slow down, (D^{stop}_{dec}), in the next time step, when at least the leader vehicle has stopped.

(ii) Moving vehicle. This case only occurs when vehicles with different deceleration capabilities are considered. In this situation, it is possible that even though the final gap between two braking vehicles satisfies the conditions established in (1), a collision may happen at some instant of time before the complete stop, if the instantaneous gap between the two vehicles takes a negative value. In this case, because both vehicles are moving, the following conditions should be met for each time $t+\tau$,

$$0 < \tau < t_{b_l}$$

$$0 < \tau < t_{b_f}$$

Based on these conditions, the condition for the leader and follower to avoid collisions, that is $\ddot{x}_l - l_f > \ddot{x}_f$, and by using basic definitions of uniformly accelerated motion, a minimum safe following distance for the moving vehicle case, D^{mov} , is determined by (2).

TABLE I. VARIABLES AND PARAMETERS OF THE LAI-E MODEL

Variables		Parameters	
x_n	position of the nth vehicle	v_{max}	maximum velocity
v_n	velocity of the nth vehicle	l_n	length of vehicle *n* (in cells)
d_{acc}	safe following distance to accelerate	R_s	slowing down probability
d_{keep}	safe following distance to maintain velocity	a_1	maximum normal acceleration
d_{dec}	safe following distance to decelerate	a_{max}	maximum deceleration
a_n	calculated normal acceleration/deceleration	R_a,R_0,R_d	acceleration probabilities
t_{b_l}, t_{b_f}	breaking times for leader and follower vehicles	*ng*	number of gears
		Δx	cell size in meters

$$D^{mov} = \frac{a_{max_l} + a_{f_t}}{2} - \frac{\left(v_{l_t} - a_{max_l} - \left(v_{f_t} + a_{f_t}\right)\right)^2}{2\left(a_{max_l} - a_{max_f}\right)} - v_{l_t} + v_{f_t} + l_f \tag{2}$$

In this way, as a function of the action taken by the follower, a_{f_t} can takes the values a_f, 0, or $-a_f$ in (2) to accelerate (D^{mov}_{acc}), keep its velocity (D^{mov}_{keep}) or slow down, (D^{mov}_{dec}), respectively.

Thus, the safe following distances d_{acc}, d_{dec} and d_{keep} are determined based on (1) and (2). For a detailed explanation of the process used for this, please see [11].

It is important to say that for a given choice of cell length, all calculations involved in (1)-(2) can be performed off-line by means of fixed look-up-tables that contain the distances for vehicles to accelerate, keep the velocity, and decelerate in a safe way. Then, the computational cost of calculating the safety distances can be considered very low.

Based on the analysis for definition of the safe distances presented in previous subsections, the updating rules of the model proposed in this work can be defined now.

The proposed model consists of *N* vehicles moving in one direction on a one-dimensional lattice of *L* cells. Each cell is either empty, or is occupied by just one vehicle (or part of one vehicle) traveling with velocity *v*, that takes discrete values ranging from 0 to v_{max}. Vehicle position is related with the cell that its rear bumper is occupying. The other vehicle's state, its integer velocity, corresponds to the number of cells that a vehicle advances in one-time step. Transitions in time are from t → t + 1, that implies the time step, *t*, is equal to 1s. This time step is on the order of human reaction time as pointed out in [12]. For an arbitrary configuration, one update of the system consists of the following five consecutive steps, which are performed in parallel for all vehicles:

S1: Safe distances. Obtain the value for the safe following distances according to (1) and (2).

S2: Slow to accelerate. Determine the stochastic noise parameter R_a, dependent on the vehicle's speed v_n (see Ref. [9]).

S3: Decision making. Let $a_{f_{t+1}}$ denoting the acceleration that the vehicle *f* will apply the next time step.

S3a: Acceleration. If $d_t \in [d_{acc}+\infty)$ then

$$a_{f_{t+1}} = \begin{cases} a \text{ if randf}() \le R_a \\ 0 \text{ otherwise} \end{cases}$$

S3b: Random slowing down.

If $d_t \in [d_{keep}, d_{acc})$ or $(v_f = v_{max}.)$ then

$$a_{f_{t+1}} = \begin{cases} -a_1 \text{ if randf}() \le R_s \\ 0 \text{ otherwise} \end{cases}$$

S3c: Braking. If $d_t \in [d_{dec}, d_{keep})$ then

$$a_{f_{t+1}} = \; -a_1$$

S3d: Emergency braking. If $d_t < d_{dec}$ then

$$a_{f_{t+1}} = \; -a_{max}$$

where $-a_{max}$ is the maximum deceleration of the vehicle n applied during one time-step.

S4: Action:

$$v_{f_{t+1}} = \; \min(\max(0, v_{f_t} + a_{f_{t+1}}), v_{max})$$

S5: Vehicle movement.

This step represents uniformly accelerated motion of vehicles as follows:

(i) If $(a_{f_{t+1}} > 0)$

$$x_{f_{t+1}} = \lfloor x_{f_t} + v_{f_t} + \frac{1}{2} a_{f_{t+1}} \rfloor$$

(ii) Otherwise,

$$x_{f_{t+1}} = \lfloor x_{f_t} + v_{f_t} t_m + \frac{1}{2} a_{f_{t+1}} t_m^2 \rfloor$$

Only in (ii) case, t_m denotes the time when the follower vehicle stops, that is

$$t_s = \min(1, \text{abs}(v_{f_t} / a_{f_{t+1}})) \text{ and } a_{f_{t+1}} \in \{-a_1, -a_{max}\}$$

here, randf()∈[0,1] is a uniform random number (specifically drawn for the vehicle *f* at time *t*).

Step S2 defines the slow to accelerate parameter R_a, which denotes the probability of vehicles to accelerate based on the velocity of vehicles (see [9] for a deeper explanation).

$$R_a=\min(R_d,R_0+v_n(R_d-R_0)/v_s) \tag{3}$$

Note that for a given value for R_0, R_d, and v_s, the calculation involved in (3) for the acceleration probability R_a can be performed off-line and stored in a fixed table, that then relates the value of R_a with the vehicle velocity v_n. In practice, the use of this table reduces the use of three parameters, R_0, R_d, and v_s, to only one, R_a.

Steps S3a to S3d are designed to determine the decision of a driver at the current time step: to accelerate, decelerate or maintain his/her velocity. Then, the velocity of vehicles is updated based on the driver decisions in step S4, while step S5 updates position. Thus, the LAI-E model builds a set of update rules close to real driver's behavior.

III. The Multi-gear LAI-E Model

The LAI-E models assumes a constant acceleration rate for the vehicles, which is an over-simplification and should be avoided. In order to faithfully reproduce the acceleration profile of the conventional vehicles which have multiple gears transmissions, in this section the LAI-E model is extended. For this purpose, a multi-regime constant acceleration model is introduced to the LAI-E model, which is referred as the *adaptive acceleration profile*. In such a way that, the resulting model can have many points of discontinuity when a vehicle is accelerating based on the vehicle velocity, the numbers of gears considered for a vehicle, and the two types of deceleration of the model.

Thus, the value for the limited acceleration of the LAI-E model used by the rule S3a, a, is determined based on the fact that a vehicle has multiple gear ratios (or simply "gears"), with the ability to switch between them as speed varies. Therefore, the dynamics of a car vary with its speed and its number of gears: at low speeds, the normal acceleration is larger; while at cruising or maximum speeds acceleration is smaller.

Let ng be the number of gears considered for a car. Let $\Delta g=v_{max}/ng$ be the size for each gear ratio where the acceleration value should be adapted (gear shifts are realized). Then, the limited acceleration value in normal conditions, a, will be determined by (4).

$$a=\left\lfloor a_1 \frac{ng-\lfloor v/\Delta g \rfloor}{ng} \right\rfloor \tag{4}$$

here v denotes the current velocity of the car and a_1 the possible maximum acceleration value in normal conditions. It can be observed from (4) that the larger the velocity is the smaller the acceleration value is, as occurs with the real vehicles.

In this way, instead to apply a constant acceleration as in the LAI-E model, a pre-calculated a value obtained by (4) is used by the step S3a of the proposed model. For a given choice of gears numbers, all calculations involved in (4) can be performed o off-line. After these off-line calculations, a fixed table can be generated that contain the acceleration value for vehicles based on its current velocity. Keeping in mind the use of table-look-up, the computational cost of calculating a is very low and therefore, the simplicity of the LAI-E model is preserved.

IV. Simulation Results

In this section, simulation results from the proposed multi-gear LAI-E model are presented. Simulations are carried out in a one lane road with a ring topology. The time-step is always taken to be 1 s, therefore, transitions are from $t \rightarrow t+1$. Velocities and positions of vehicles are updated according to the steps S3a-S3d, S4 and S5 of the new model.

With the idea for showing that the proposed model is more in line with the behavior of real vehicles, the trajectories of real vehicles accelerating from a stopped position up to reach their maximum velocity are compared with those obtained from simulation. For this purpose, simulations are carried out on a road of L_{lane} = 200,000 cells (50 km). The length of one cell is chosen to be Δx = 0.25 m. Homogeneous vehicles with a length of 5.0 m (20 cells) and v_{max} = 115.2 km/h (128 cells/s) are considered. The parameters of the model are set as R_d = 1.0, R_0 = 1.0, R_s = 0, v_s = 32 cells/s (28.8 km/h), a_{max_n}= 24 cells/s^2 (6 m/s^2), a_1 = 20 cells/s^2 (5 m/s^2). It is important to note that, in order to observe more clearly the gears behavior random effects on acceleration/deceleration are not considered.

In Figure I, trajectories obtained empirically for two models of vehicles, a Volkswagen Jetta 2013[13] and a Honda Civic 2000[14], are compared with those obtained from simulation when a different number of gears is considered. As can be observed from this figure, the larger the number of gears is the better the approximation of the results is. On the other hand, the corresponding velocities variations are showed in Figure II. It is important to note from this figure that in order to have a smoother and more realistic acceleration, at least two gears have to be taken into account. In addition, the number of discontinuity points when a vehicle is accelerating is proportional to the quantity of gears used due to the acceleration value is adapted in accordance with the current velocity and the number of gears; unlike when one gear is only used and the acceleration rate is always constant until the maximum velocity of the vehicle is reached, which is unrealistic.

Although, the results obtained of the modified model presented in this work improved that from the LAI-E model, it is necessary to calibrate the model with different scenarios, types of vehicles and drivers behavior.

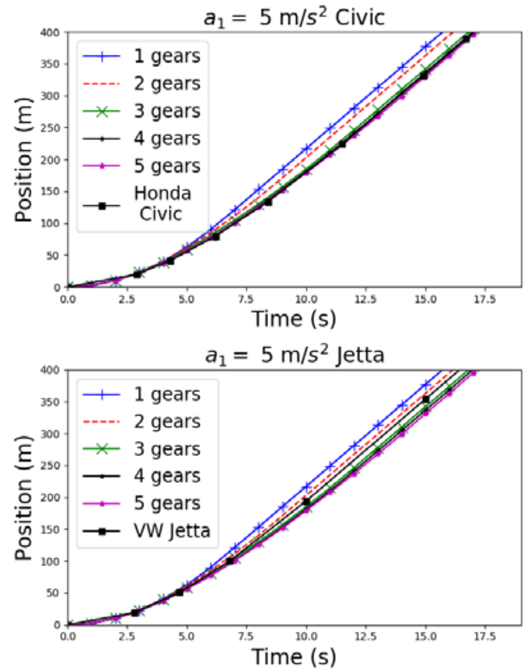

FIGURE I. VEHICULAR TRAJECTORIES WHEN VEHICLES REACH THEIR MAXIMUM VELOCITY FROM A STOPPING POSITION. COMPARISON WITH TWO MODELS OF REAL VEHICLES: A HONDA CIVIC 2000 (TOP) AND A VOLKSWAGEN JETTA 2013 (BOTTOM).

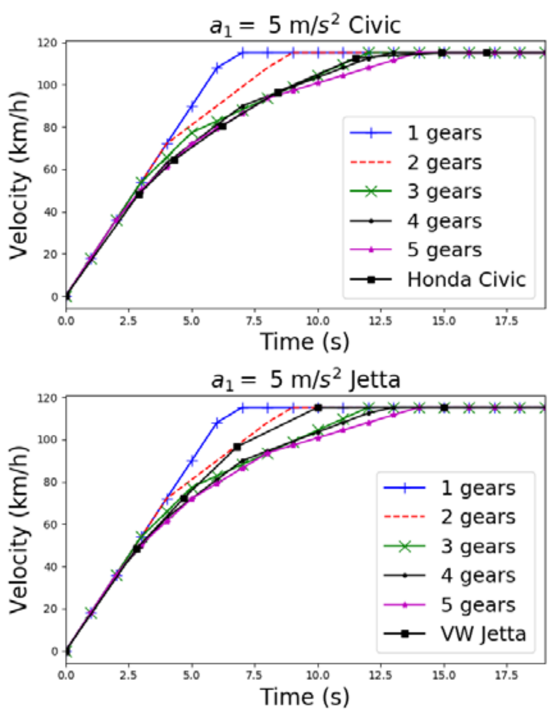

FIGURE II. SPEED VARIATIONS WHEN VEHICLES REACH THEIR MAXIMUM VELOCITY FROM A STOPPING POSITION. COMPARISON WITH TWO MODELS OF REAL VEHICLES: A HONDA CIVIC 2000 (TOP) AND A VOLKSWAGEN JETTA 2013 (BOTTOM).

V. CONCLUSIONS

In this paper, a reliable and simple cellular automata model oriented to faithfully reproduce acceleration profile of vehicle capacities and drivers reactions was presented. For this purpose, the acceleration profile of a recently presented CA model for traffic flow was modified by introducing a multi-regime constant acceleration model based on the velocity and multiple gears. In such a way that the modified model can have many points of discontinuity when a vehicle is accelerating.

A comparison of simulation results of the proposed model with that from real data for two models of vehicles indicate that the model presented in this work reproduce in a better way the acceleration profile of the vehicles. In addition, the model preserves its computational simplicity, with the perspective of look-up-tables, even considering the multi-gear extension, the computational cost is not increased and the set of rules makes it possible to use it in real-time simulation of large traffic networks. This important feature of the CA models is thus preserved.

For future work, the extension of the model for multilane systems will be considered to analyze the behavior when heterogeneous vehicles are considered. It is necessary to calibrate the model with different scenarios, types of vehicles and drivers behavior.

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