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# Optimum Plan for Step-down-stress Accelerated Life Testing with Censoring Ⅰ and Numerical Simulation

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***Abstract*—In this paper, we obtain the optimum plan by discussing a step-down-stress accelerated life testing (SDS-ALT) satisfying some specific condition at k stresses under an exponential distribution.**

***Keywords-exponential distribution; step-down-stress accelerated life testing; accelerating life equation; numerical simulation***

## I. INTRODUCTION

Reliability evaluation for products of high-reliability and long-lifetime is one of the challenges in reliability engineering[1]. ZHANG Chun-hua[2-4] proposed a method of step-down-stress accelerated life testing (SDS-ALT). CHEN J and WANG D Y[5-6] did an experimental study on Double-crossed step-stress accelerated life testing . XU Xiao-ling, WANG Rong-hua and YU Song [7] studied step-down test based on lognormal distribution , and the finding of their study is that the efficiencies of step-down test is better than step-up test .WANG Yu-ming and SUN Yu-dong[8-9] studied double-crossed step-down-stress accelerated life test under different life distribution. LÜ Meng[10] gave an optimal design of double-crossed step-down-stress accelerated life test. Chernoff[11] initially discussed the optimum plans for simple constant-stress accelerated life tests under an exponential distribution, Mao shi song[12-13] studied constant-stress ALTs. J.F. Lawless and R. Liu [14-15] studied stress accelerated life testing . MEEKER W Q and AN Zong-wen [16-17] studied accelerated life problems based on certain distribution.

KOU Hai-xia and An Zong-wen[18] studied double synchronous-step-down-stress accelerated life testing .This paper gives the optimum plan for a step-down-stress ALT which satisfies the condition $\begin{cases} A_1 \geq A_2 \geq \cdots \geq A_K \\ \sum_{i=1}^{k} A_i = P, 0 < P < 1 \end{cases}$ under an exponential distribution at k≥3 constant stresses by minimizing the asymptotic variance of MLE in line with a linear accelerating equation .

## II. BASIC ASSUMPTION AND LEMMAS

A step-down-stress ALT with censoring I can be designed as following:

We sample n(n≥r) products at random, supposing the unit number at stress level $s_i$ to be $\tau_1$, divide them into $S_2$ groups and test the life times of the products at stress level $S_1$ respectively till $\tau_2$ products fail ,where $k-1$ is a usual stress level ,the failure data at stress level $S_k$ are denoted by $t_{i_1},\dots,t_{i_{r_i}}$ respectively (i=1,…k). Let the life times of the products satisfy the following assumptions:

A1: The life times of the products at stress $S_i$ follow an exponential distribution with $F_i(t) = 1 - e^{-\lambda_i t}$ , $T \geq 0$ , $i = 1,\dots,k$ ; $\theta_i = 1/\lambda_i$ is a mean life at stress $S_i$ .

A2: The accelerating equation between mean life $\theta$ and stress $S$ is: $\ln\theta = a + b\phi(s)$ , where $\phi(s)$ is a decreasing function of $S$ .

According to the test data and the basic assumptions, the likelihood function is

$$\delta_i = \theta_{i+1}\sum_{j=1}^{k}\frac{\tau_j - \tau_{j-1}}{\theta_j}, i = 1,2...,k-1 \quad (1)$$

Where $\tau_0 = 0$ is the total time of the test at stress $S_i$ (i=1,…,k).

By (1.1) we have:

$$L\left(\theta_1,...\theta_k;t_{11}...,t_{1_{r1}}\right) \quad (2)$$

From A2, we have : $\theta_i = e^{a+b\Phi_i}$ and put it into (1.2) , then

$$LnL(a,b) = -\sum_{i=1}^{k} r_i(a + b\Phi_i) - \sum_{i=1}^{k} T_i e^{-(a+b\Phi_i)} \quad (3)$$

**Lemma1.1:** When testing with censoring I, we have $E(U_i) = n\theta_i p_i \prod_{j=0}^{i-1}\left(1 - p_j\right), i = 1,2,\cdots,k$

Among them

$$\tau_0 = 0, p_0 = 0, p_i = 1 - \exp\left(-\frac{\tau_i - \tau_{i-1}}{\theta_i}\right) \; i = 1, \cdots, k \cdots$$

Convenient for writing, make

$$A_i = p_i \prod_{j=0}^{i-1} (1 - p_i) \tag{4}$$

## III. ASYMPTOTIC VARIANCE OF THE LOG MEAN

To compute the Fisher information matrix of the log likelihood function (1.3)

$$E\left[\frac{-\partial^2 LnL(a,b)}{\partial a^2}\right] = \sum_{i=1}^{k} E(U_i) \cdot \exp[-(a + b\Phi_i)]$$
$$= \sum_{i=1}^{k} n\theta_i p_i \prod_{j=0}^{i-1} (1 - p_i)$$
$$\exp[-(a + b\Phi_i)] = \sum_{i=1}^{k} n\theta_i p_i \prod_{j=0}^{i-1} (1 - p_i)\theta_i^{-1}$$
$$= \sum_{i=1}^{k} np_i \prod_{j=0}^{i-1} (1 - p_i) = \sum_{i=1}^{k} nA_i = A_{11} \tag{5}$$

$$E\left[\frac{-\partial^2 LnL(a,b)}{\partial b^2}\right] = \sum_{i=1}^{k} \Phi_i^2 E(U_i) \cdot \exp[-(a + b\Phi_i)]$$
$$= \sum_{i=1}^{k} \Phi_i^2 n\theta_i p_i \prod_{j=0}^{i-1} (1 - p_i)\theta_i^{-1} = \sum_{i=1}^{k} \Phi_i^2 n\theta_i p_i \prod_{j=0}^{i-1} (1 - p_i)$$
$$= \sum_{i=1}^{k} \Phi_i^2 nA_i = A_{22} \tag{6}$$

$$E\left[\frac{-\partial^2 LnL(a,b)}{\partial a \partial b}\right] = \sum_{i=1}^{k} \Phi_i E(U_i)\,\theta^{-1}$$
$$= \sum_{i=1}^{k} \Phi_i n\theta_i p_i \prod_{j=0}^{i-1} (1 - p_i)\theta^{-1}$$
$$= \sum_{i=1}^{k} \Phi_i np_i \prod_{j=0}^{i-1} (1 - p_i) = \sum_{i=1}^{k} \Phi_i nA_i = A_{12} = A_{21} \tag{7}$$

The following instructionsThe meaning of $nA_i$ : set $X$ For product life,The distribution of (1.3) so

$$p\left(\tau_{i-1} \le x < \tau_i \middle| x > \tau_{i-1}\right) = \frac{p(\tau_{i-1} \le \tau_i)}{p(x > \tau_{i-1})}$$

$$= \left\{\left[1 - \exp\left(-\frac{\delta_{i-1} + \tau_i - \tau_{i-1}}{\theta_i}\right)\right] - \left[1 - \exp\left(-\frac{\delta_{i-1}}{\theta_i}\right)\right] \Big/ \exp\left(-\frac{\delta_{i-1}}{\theta_i}\right)\right\}$$
$$= 1 - \exp\left(-\frac{\tau_i - \tau_{i-1}}{\theta_i}\right) = p_i$$

Then the Fisher information matrix of the likelihood function is: $F = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, the inverse matrix of F is:

$$F^{-1} = 1/\Delta \begin{pmatrix} A_{22} & -A_{21} \\ -A_{12} & A_{11} \end{pmatrix} \tag{8}$$

$$\Delta = A_{11}A_{22} - A_{12}^2 = (\sum_{i=1}^{k} r_i),$$

$$(\sum_{i=1}^{k} r_i \Phi_i^2) - (\sum_{i=1}^{k} r_i \Phi_i)^2 = \sum_{j=2}^{k} r_1 r_j (\Phi_1 - \Phi_j)^2$$
$$+ \sum_{j=3}^{k} r_2 r_j (\Phi_2 - \Phi_j)^2 + \cdots + r_{k-1} r_k (\Phi_{k-1} - \Phi_k)^2$$

And so the asymptotic variance of the log mean life at usual stress is :

$$AsVar(Ln\hat{\theta}_0) = (1, \Phi_0)F^{-1}(1, \Phi_0)' =$$
$$1/\Delta(A_{22} - 2A_{12}\Phi_0 + \Phi_0^2 A_{11}) = 1/\Delta(\sum_{i=1}^{k} r_i \Phi_i^2 - 2\Phi_0 \sum_{i=1}^{k} r_i \Phi_i$$
$$+ \Phi_0^2 \sum_{i=1}^{k} r_i) = \sum_{i=1}^{k} r_i (\Phi_i - \Phi_0)^2 \Big/$$
$$\left[\sum_{j=2}^{k} r_1 r_j (\Phi_1 - \Phi_j)^2 + \sum_{j=3}^{k} r_2 r_j (\Phi_2 - \Phi_j)^2 + \cdots + r_{k-1} r_k (\Phi_{k-1} - \Phi_k)^2\right] \tag{9}$$

## IV. OPTIMAL DISTRIBUTION OF THE FAILURE NUMBER IN A STEP-DOWN-STRESS ALT WITH CENSORING Ⅰ

To make computing and application easy, we take k accelerating stress levels satisfying:

$$\begin{cases} \Phi_0 - \Phi_1 = d\Delta \\ \Phi_{i-1} - \Phi_i = \Delta \end{cases} \tag{10}$$

where d is a integer, $\Delta$ is a constant, i=2, …, k.

Put (3.1) into (2.5), then:

$$AsVar(Ln\hat{\theta}_0)=\frac{\sum_{i=1}^{k} r_i(d+i-1)^2}{\sum_{\substack{i=1,\ldots,k-1\\ j=2,\ldots,k\\ r<j}} r_i r_j (j-i)^2}=\frac{R}{S} \quad (11)$$

The optimum test plan in a step-down-stress ALT ,on one hand, requires the minimum Asymptotic variance of estimator at normal stress so as to improve the preciseness of statistical analysis, on the other hand ,needs to satisfy that the failure numbers at higher stress levels are not smaller than that at the minimum stress level so as to get more failure data in a shorten time, moreover, the following conditions should also be satisfied:

$$\begin{cases} A_1 \ge A_2 \ge \cdots \ge A_K \\ \sum_{i=1}^{k} A_i = P, 0 < P < 1 \end{cases} \quad (12)$$

In the rest of the paper, we assume that (4.3) holds and give the optimal distribution plan of r failure numbers at k accelerating stress-levels by regarding the minimum asymptotic variance of the log mean as principle.

**Theorem** For a step-down-stress test with censoring I at k stress levels, if (3.3) holds, the optimum failure numbers of transformation are $A_1$ diminishing.

Proof:

$$\sum_{i=1}^{k} A_i = p \quad , \quad \sum_{i=1}^{k} nA_i = np \quad , \quad nA_k = np - nA_1 - \cdots - nA_{k-1} \quad ,$$

$$\frac{\partial nA_k}{\partial nA_1} = -1, \quad \frac{\partial R}{\partial nA_1} = \frac{\partial}{\partial nA_1}\left[\sum_{i=1}^{k}(h+i-1)^2 nA_i\right]$$

$$= \frac{\partial}{\partial nA_1}\left[h^2 nA_i + (h+1)^2 nA_2 \cdots (h+k-1)^2 nA_k\right]$$

$$= h^2 - (h-k-1) < 0 (k>1)$$

$$\frac{\partial S}{\partial nA_1} = \frac{\partial}{\partial nA_1}\begin{bmatrix} nA_1 nA_2 + 2^2 nA_1 nA_3 + \cdots + (k-1)^2 nA_1 nA_k + nA_2 nA_3 \\ +2^2 nA_2 nA_4 + \cdots + (k-2)^2 nA_2 nA_k + \cdots + nA_{k-1} nA_k \end{bmatrix}$$

$$= 1^2 nA_2 + 2^2 nA_3 + \cdots + (k-1)^2 nA_k - (k-1)^2 nA_1$$
$$- \cdots (k-2)^2 nA_2 - \cdots - 2^2 nA_{k-2} - 1^2 nA_{k-1}$$

$$= -(k-1)^2 nA_1 - \left[(k-2)^2 - 1^2\right] nA_2 - \cdots - \left[\left[1^2 - (k-2)^2\right] nA_{k-1} + (k-1)^2 nA_k\right]$$

$$\frac{\partial S}{\partial nA_1} = (k-1)^2 (nA_k - nA_1) + \left[(k-2)^2 - 1^2\right](nA_{k-1} - nA_2)$$
$$+ \cdots + \left[\left(\frac{k}{2}\right)^2 - \left(\frac{k}{2}-1\right)^2\right]\left(nA_{\frac{k}{2}+1} - nA_{\frac{k}{2}}\right)$$

$$= \sum_{i=1}^{k/2}\left[(k-i)^2 - (i-1)^2\right](nA_{K-i+1} - nA_i)$$ from (3.3) to $A_1 \le A_2 \le \cdots \le A_k$

$$i = 1, 2, \cdots, k/2, (k-i)^2 - (i-1)^2 > 0$$

$$\frac{\partial S}{\partial nA_1} = (k-1)^2 (nA_k - nA_1) + \left[(k-2)^2 - 1^2\right](nA_{k-1} - nA_2)$$
$$+ \cdots + \left[\left(\frac{k+1}{2}\right)^2 - \left(\frac{k-3}{2}\right)^2\right]\left(nA_{(k+3)/2} - nA_{(k-1)/2}\right)$$
$$+ \left[\left(\frac{k-1}{2}\right)^2 - \left(\frac{k-1}{2}\right)^2\right] nA_{(k+1)/2}$$

$$= \sum_{i=1}^{(k-1)/2}\left[(k-i)^2 - (i-1)^2\right](nA_{k-i+1} - nA_i).$$

From (3.3)to $A_1 \le A_2 \le \cdots \le A_k$. When $i \le (k-1)/2$,

$(k-i)^2 - (i-1)^2 > 0$ $nA_{k-i+1} - nA_i \ge 0$, $\frac{\partial S}{\partial A_1} \ge 0$, $\frac{\partial S}{\partial A_n} \ge 0$, $R > 0, S > 0$

$$\frac{\partial As\,\mathrm{var}\left(\ln\hat{\theta}_0\right)}{\partial nA_1} = \left(\frac{\partial R}{\partial nA_1} S - \frac{\partial R}{\partial nA_1} R\right) \Big/ S^2 < 0$$

By the type known $\partial As\,\mathrm{var}\left(\ln\hat{\theta}_0\right)$ about $A_1$ Strictly decreasing and decreasing is independent of the value of $A_k$.

## V. Optimal Distribution of the Sample Number in a Step-down-stress ALT with Censoring Ⅰ

One of the purposes to accelerate life tests is to shorten the testing time and cut down the testing cost ,therefore ,one can assume that the mean times of the step-down-stress SDS-ALT at time $\tau$ geometric sequence. $A_1 = A_2 = \cdots = A_k = \frac{P}{K}$.

$A_i = A_{i-1}, i = 2, \cdots k$ and $A_i = (1-p_1)\cdots(1-p_{i-1})\, p_i$

$\left(1-p_1\right)\cdots\left(1-p_{i-2}\right)\left(1-p_{i-1}\right)p_i=\left(1-p_1\right)\cdots\left(1-p_{i-2}\right)p_{i-1}$ so $p_i-\frac{p_{i-1}}{1-p_{i-1}}$, one ore $p_i=\exp\left(-\frac{\tau_i-\tau_{i-1}}{\theta_i}\right)$,

$$1-e-\frac{\tau_i-\tau_{i-1}}{\theta_i}=\left(1-e-\frac{\tau_{i-1}-\tau_{i-2}}{\theta_{i-1}}\right)\Big/ e-\frac{\tau_{i-1}-\tau_{i-2}}{\theta_{i-1}}$$ Taylor

$$=e-\frac{\tau_{i-1}-\tau_{i-2}}{\theta_{i-1}}-1$$

show

$$e^x\approx 1+x,1-\left(1-\frac{\tau_i-\tau_{i-1}}{\theta_i}\right)=\left(1+\frac{\tau_{i-1}-\tau_{i-2}}{\theta_{i-1}}\right)-1$$

So, $\frac{\tau_i-\tau_{i-1}}{e^{b\Phi_i}}=\frac{\tau_{i-1}-\tau_{i-2}}{e^{b(\Phi_i+d)}}$, $\frac{\tau_i-\tau_{i-1}}{\theta_i}=\frac{\tau_{i-1}-\tau_{i-2}}{\theta_{i-1}}$,

$$\frac{\tau_{i-1}-\tau_{i-2}}{\tau_i-\tau_{i-1}}=e^{bd},i-2,\cdots k.$$

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