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Mathematical Model and Analytical Solution for Groundwater Seepage in Confined Aquifer Subjected to Well Pumping without Penetrating Overlying Aquiclude

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***Abstract*—Groundwater in confined aquifer was pumped at constant rate through base plate of a vertical well without penetrating overlying aquiclude, a mathematical model for seepage in confined aquifer was developed, and by technique of integral transforms of Hankel Transformation and Laplace Transformation respectively, the analytical solution was attained. By a numerical example, the seepage of groundwaAter was analyzed, and some conclusions were derived: (1) The drawdown of hydraulic head in the confined aquifer decreases with the depth and with the distance away from pumping well; (2) The effect of base plate size of pumping well on the drawdown of hydraulic head is finite and mainly converges on the vicinity of base plate of pumping well; (3) The resistance coefficient of leakage B remarkably affects the distribution of drawdown, and the smaller B leads to more water leaking from aquiclude into the confined aquifer to reduce amplitude of drawdown in confined aquifer due to pumping**

Keywords-groundwater; confined aquifer; anlytical solution; nonpenetrating well; integral transfomation

I. INTRODUCTION

To analyze vertical well flow in confined aquifer, there are two kinds of mathematical models for simulation of groundwater: one model is fully penetrating well[1-6], in which pumping well fully penetrates the confined aquifer, and the other is partially penetrating well[7-9], in which the pumping well partially penetrates the confined aquifer. As it is well known, there is a overlying impermeable layer or aquiclude above the confined aquifer, and the pumping well should penetrates the overlying aquiclude in the above mentioned models[10-14]. However, in geotechnical engineering or hydraulic engineering, some vertical wells don't penetrating the overlying aquiclude, and these wells pump the water that comes from confined aquifers. For example, in foundation excavation engineering, pumping wells are applied to lower the level of water below the base plate of foundation pit, and pumping wells sometimes will not penetrate the overlying aquiclude to dewater, when confined aquifer is far below the base plate of foundation pit but the hydraulic head is above base plate of foundation pit and threaten the stability during excavation. The existing mathematical models are not developed to study the seepage of groundwater for such a kind of problem. Therefore, it is necessary to establish a new mathematical model to solve the problem.

In this paper, the mathematical model of seepage flow for confined aquifer subjected to a constant pumping rate through vertical well without penetrating overlying aquiclude was presented, and by using Hankel Transformation and Laplace Transformation, analytical solution was attained. According to the derived solution, a numerical example was used to analyze the groundwater seepage in confined aquifer and the relative influence factors.

II. MATHEMATICAL MODEL

A. Basic Assumption

Figure Ⅰ illustrates the conceptual model of seepage flow of groundwater, in which the pumping well doesn't penetrate the overlying aquiclude above the confined aquifer and the groundwater was pumped with a constant pumping rate Q. To derive the corresponding mathematical model, some basic assumptions are proposed: (1) the material in confined aquifer is anisotropic and the flux from base plate of pumping well is uniform; (2) M.s.Hantush assumptions are introduced here, that is vertical leakage from overlying aquiclude is in proportion to hydraulic head in the confined aquifer; (3) Leakage recharge from overlying aquiclude is deemed as source and sink in the mathematical model and the top of confined aquifer is regarded as impermeable boundary; (4) The lateral boundary condition is a contant hydraulic head.

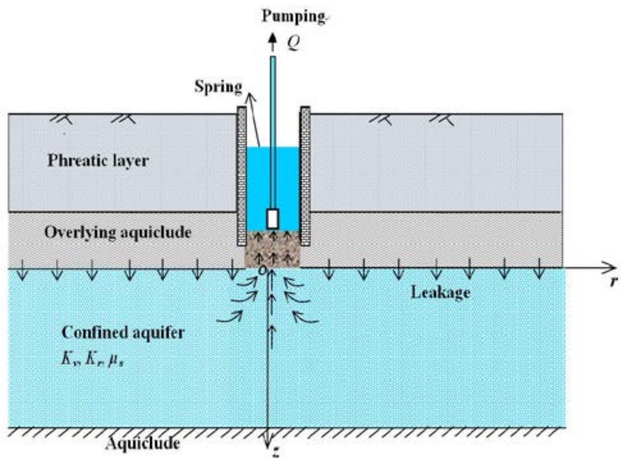

FIGURE I. CONCEPTUAL MODEL FOR SEEPAGE OF GROUNDWATER OF CONFINED AQUIFER DUE TO CONSTANT RATE PUMPING: PUMPING WELL DOESN'T PENETRATE THE OVERLYING AQUICLUDE.

B. *Governing Equations and Conditions for Seepage Field of Groundwater*

The cylindral coordinates is established as shown in the conceptual model in Figure Ⅰ. According to the conceptual model and basic assumptions, the governing equation for three dimensional seepage flow is derived as below:

$$K_r\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)+K_v\frac{\partial^2 s}{\partial z^2}-\frac{K' s}{bb'}=\mu_s\frac{\partial s}{\partial t} \tag{1}$$

The corresponding boundary conditions and initial condition is listed as:

(a) The top boundary condition for seepage flow of confined aquifer: $\begin{cases}\left.\dfrac{\partial s}{\partial z}\right|_{z=0}=0,\quad r>r_0\\ \left.\dfrac{\partial s}{\partial z}\right|_{z=0}=\dfrac{Q}{K_v'\pi r_0^2}, r\le r_0\end{cases}$; (b) Base plate boundary condition: $\left.\dfrac{\partial s}{\partial z}\right|_{z=b}=0$; (c) Initial condition: $\left.s\right|_{t=0}=0$.

Where s is drawdown of hydraulic head in confined aquifer; r and z are radial coordinate variable and vertical coordinate variable; t is time variable; Q is pumping rate of groundwater; K_r and K_v are radial permeability and vertical permeability of confined aquifer respectively; K' and K_v' are radial permeability and vertical permeability of overlying aquiclude respectively; μ_s is specific elastic storage coefficient of confined aquifer; b and b' are thickness of confined aquifer and overlying aquiclude respectively; r_0 is radius of permeable base plate of well.

C. *Laplace - Hankel Integral Transform Form of Mathematical Model*

To transfer the nonhomogeneous equation to be a homogeneous equation, let $z^*=\vartheta z$ and $\vartheta=\sqrt{K_r/K_v}$ in (1), and at the same time Laplace Transformation and Hankel transformation were applied to (1). In consequence, equation (1) in physical domain was changed into the equation in Hankel-Laplace domain as below:

$$\frac{d^2\tilde{\bar{s}}}{dz^{*2}}-\left(\xi^2+\frac{1}{B^2}+\frac{p}{a_r}\right)\tilde{\bar{s}}=0 \tag{2}$$

Where $\tilde{\bar{s}}$ is drawdown of hydraulic head in Laplace-Hankel domain; $B=\sqrt{K_r b' b/K'}$ is resistance coefficient of Leakage; $a_r=T_r/\mu_e$ is radial conductivity coefficient of confined aquifer; $T_r=K_r b$ is radial transmissivity of confined aquifer; μ_e is elastic storage coefficient of confined aquifer; ξ and p are variables in Hankel domain and Laplace domain respectively.

After Laplace transformation and Hankel transformation in order of priority, both of the upper boundary condition and lower boundary condition were changed to Laplace-Hankel domain: (Ⅰ) upper boundary condition in Laplace-Hankel domain: $\frac{d\tilde{\bar{s}}}{dz^*}=\frac{Q}{\vartheta K_v'\pi r_0 p\xi}J_1(\xi r_0)$; (Ⅱ) lower boundary condition in Laplace and Hankel domain: $\left.\frac{d\tilde{\bar{s}}}{dz^*}\right|_{z^*=\vartheta z}=0$.

Where $J_1(\xi r_0)$ is first order Bessel Function.

III. ANALYTICAL SOLUTION

A. *Solution of Hankel-Laplace Domain for Hydraulic Drawdown*

Equation (2) is ordinary differential equation, and the general solution is attained as:

$$\tilde{\bar{s}}=Ce^{-\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}z^*}+De^{\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}z^*} \tag{3}$$

Where C and D are undetermined constants.

By using boundary conditions (Ⅰ)and (Ⅱ), constants C and D could be attained as following:

$$\begin{cases}C=-\dfrac{Q\gamma_w J_1(\xi r_0)}{K_v'\pi r_0\vartheta\xi p\sqrt{\xi^2+\dfrac{1}{B^2}+\dfrac{p}{a_r}}}\dfrac{e^{2\vartheta b\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}}}{\left(e^{2\vartheta b\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}}-1\right)}\\ D=-\dfrac{Q\gamma_w J_1(\xi r_0)}{K_v'\pi r_0\vartheta\xi p\sqrt{\xi^2+\dfrac{1}{B^2}+\dfrac{p}{a_r}}}\dfrac{1}{\left(e^{2\vartheta b\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}}-1\right)}\end{cases} \tag{4}$$

By substituting determined constants C and D into (3), the solution in Laplace-Hankel domain was derived:

$$\tilde{\tilde{s}} = -\frac{QJ_1(\xi r_0)}{K_v^{'}\pi r_0\vartheta\xi}\frac{1}{p}\frac{Ch\left[\vartheta(b-z)\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}\right]}{\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}\,Sh\left(\vartheta b\sqrt{\xi^2+\frac{1}{B^2}+\frac{p}{a_r}}\right)} \tag{5}$$

B. Solution of Physical Domain

Inverse transformation of Hankel transformation and Laplace transformation were applied to (5) respectively, and solution in physical domain was attained as below:

$$s = -\frac{2Q}{K_v^{'}\pi r_0 L^2 b\vartheta^2}\sum_{m=1}^{\infty}\frac{J_1(\xi_m r_0)J_0(\xi_m r)}{\xi_m J_1^{\,2}(\xi_m L)}\left\{\frac{1}{\xi_m^{\,2}+\frac{1}{B^2}}\left(1-e^{-\left(\xi_m^{\,2}+\frac{1}{B^2}\right)a_r t}\right)\right.$$
$$\left.+2\sum_{n=1}^{\infty}(-1)^n\frac{1}{\xi_m^{\,2}+\frac{1}{B^2}+\frac{n^2\pi^2}{\vartheta^2 b^2}}\cos\left[\frac{n\pi(b-z)}{b}\right]\cdot\left[1-e^{-\left(\frac{n^2\pi^2}{\vartheta^2 b^2}+\xi_m^{\,2}+\frac{1}{B^2}\right)a_r t}\right]\right\} \tag{6}$$

IV. ANAYLSIS OF HYDRAULIC DRAWDOWN OF CONFINED AQUIFER

A. Vertical Distribution of Drawdown of Hydraulic Head

Figure Ⅱ shows the distribution of drawdown of hydraulic head along depth of confined aquifer. It can be seen that the drawdown of hydraulic head decreases with the depth in the confined aquifer and the vertical distribution of drawdown of hydraulic head is greatly affected by the distance away from pumping well. When r=50m, the drawdown is 15m at upper boundary and 11m at lower boundary, while as for r=100m, the distribution is much more uniform and the drawdown is 9.8m at upper boundary and 9.5m at lower boundary.

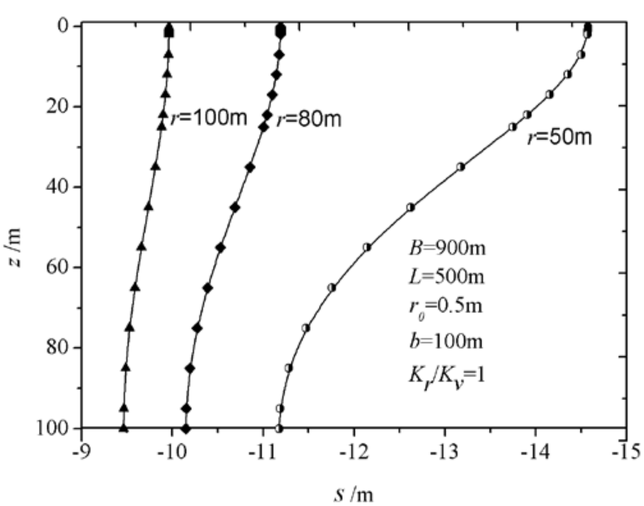

FIGURE II. DISTRIBUTION OF DRAWDOWN OF HYDRAULIC HEAD ALONG DEPTH OF CONFINED AQUIFER

B. Effect of Base Plate Size of Pumping Well on Groundwater Seepage

Figure Ⅲ shows the radial distribution of drawdown of hydraulic head along the distance from pumping well with different base plate sizes of pumping well. It indicates that the drawdown of hydraulic head increases with decreasing distance from pumping well, which shape looks like "s", and the drawdown in the vicinity of the pumping well will be greatly affected by base plate size of pumping well but the effect attenuates with increasing distance away from pumping well. As the numerical example shown, the drawdown are 185m, 217m, 229m and 233m for different base plate size of pumping well r_0=2.0m, 1.0, 0.5 and 0.1m respectively, at location r=0.1m. The difference affected by the base plate size becomes trivial for the distance r greater than 2m. It means that the effect of base plate size of pumping well on the drawdown of hydraulic head is finite and mainly focuses on the vicinity of base plate of pumping well.

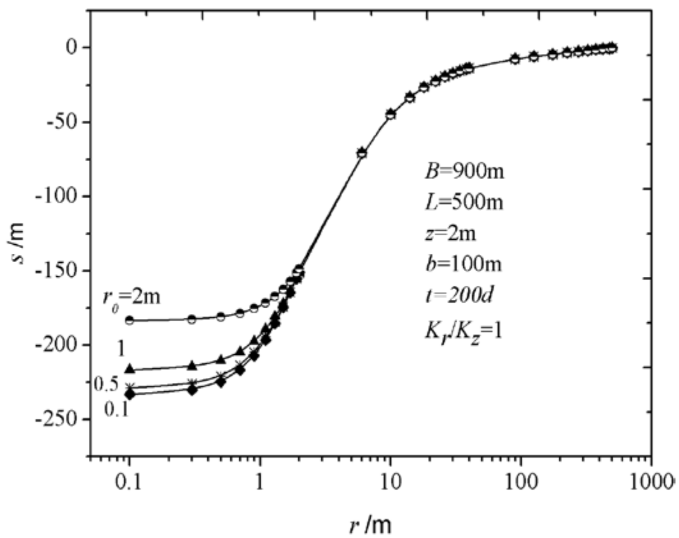

FIGURE III. EFFECT OF BASE PLATE SIZE OF PUMPING WELL ON THE DRAWDOWN OF HYDRAULIC HEAD

C. Effect of Leakage from Aquiclude on Seepage of Groundwater

Figure Ⅳ shows the effect of leakage of water from aquiclude on the seepage in confined aquifer. By comparing the curves for four B=200m，300m, 600m and 1500min Fig.4, it can be see that: (1) all curves have the same trend that hydraulic drawdown decreases with distance increase and is zero at the location L=500m for constant hydraulic head of boundary condition; (2) the resistance coefficient of leakage B remarkably affects the radial distribution of drawdown, and the smaller B will make the drawdown smaller, because smaller B means more water leakage from aquiclude into confined aquifer to reduce the water lost in confined aquifer due to pumping.

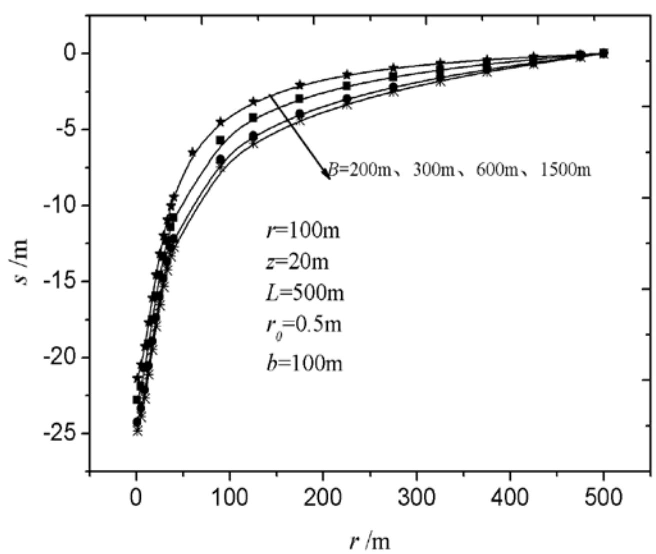

FIGURE IV. EFFECT OF LEAKAGE FROM AQUICLUDE ON THE DRAWDOWN OF HYDRAULIC HEAD IN CONFINED AQUIFER

V. CONCLUSIONS

The mathematical model of seepage flow for confined aquifer subjected to pumping well without penetrating into overlying aquiclude was developed, and by technique of integral transforms of Hankel Transformation and Laplace Transformation, analytical solution was derived. By analysis of a numerical example, conclusions were attained as following: (1) The drawdown of hydraulic head in the confined aquifer decreases with the depth and with the distance away from pumping well; (2) The effect of base plate size of pumping well on the drawdown of hydraulic head is finite and mainly converges on the vicinity of base plate of pumping well; (3) The resistance coefficient of leakage B remarkably affects the distribution of drawdown, and the smaller B will make the hydraulic drawdown smaller for smaller B leads to more water leaking from aquiclude into the confined aquifer to reduce the water lost in confined aquifer due to pumping.

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