

# Reliability Analysis of Safety Intervehicle Communications in a Highway Environment

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**Abstract**—Vehicular network can improve traffic safety by exchange of safety-related messages between the vehicles within the neighborhood of each other. However, many research related to vehicular network depends heavily on system-level simulation. In this paper, we use tools from stochastic geometry to analyze the reliability of the transmission of periodic message in a highway scenario. A Poission point process (PPP) based-network and slotted-aloah MAC scheme is assumed. Three-slope pass loss and rayleigh-lognormal fading are taken into consideration. Coverage probability and packet reception rate (PRR) are adopt as the reliability metrics. We then derive the closed form expressions of coverage probability and PRR. Finally, the accuracy of the analysis is validated via simulation.

**Keywords**—vehicular safety communication; highway; reliability; multi-slope path loss; rayleigh-lognormal fading; PPP

## I. INTRODUCTION

Vehicular networks can improve traffic safety by exchange of safety-related messages. Periodic message is the most fundamental one defined in vehicular networks, which can help each vehicle keep track with the adjacent vehicles. Generally speaking, the frequency of the periodic message is 10Hz, and the intended broadcast range is 300m. Existing standard for intervehicle safety communication is DSRC, which suffers from channel congestion especially in the dense scenario because of the frequent transmission of the periodic message [1].

Many research related to vehicular networks depends heavily on extensive system-level simulation, which is time consuming. In addition, limited insights can be learned from the simulation results.

Stochastic geometry, especially Poission point process, is widely used in modeling and analysis of wireless networks like ad hoc networks, cellular networks, heterogeneous networks, etc., in the last decades [2]. By averaging over all the spatial patterns, we can get the mathematical expressions of some important performance metrics, from which we can understand how these performance metrics depend on the network parameters and get some useful insights used for protocol optimization and design.

Stochastic geometry has also been applied to intervehicle safety communications. Authors in [3] conclude that the locations of the transmitters in 802.11p tend to be a PPP, and

use PPP to approximate the performance in the dense scenario. Works in [3] is extended in [4], where the authors consider both sparse, middle and dense cases, and approximate the coverage probability by a modified matern hard core point process. Reliability of the vehicles near intersections is evaluated in [5]. Multi-lane is considered in both [6] and [7]. In [6], the performance of CSMA based network in a multi-lane highway is investigated. In [7], the authors study the modeling of the multi-hop transmissions in a multi-lane highway setup.

However, both [3-7] consider the simple singular or bounded path loss model and rayleigh fading for mathematical simplicity, which is not the case in the vehicular communication enviroment. Actually, the propagation property is much more complex. From the channel model given by 3GPP in [8], we can know the path loss exponent is distance-dependent, and shadowing also exists.

There are some research focusing on the more complicated but more realistic channel models. In [9], the performance of the downlink cellular networks is investigated under multi-slope path loss model, with a focus on dual-slope model, but the shadowing is not considered. The author in [10] consider nakagami-lognormal fading in the downlink cellular network, but they adopt the simple singular path loss model. The results in both [9] and [10] can not be directly used in vehicular networks, because of the node distribution and performance metrics of the vehicular networks is different from that of the cellular networks.

In this paper, we focus on the transmission of the periodic meassage in a highway scenario. The more realistic channel model, i.e., three-slope path loss and lognormal shadowing defined in 3GPP [8], are considered. For mathematical tractability, we consider the plain slotted-aloah MAC scheme. Some more involved but efficient MAC schemes is out of the scope of this paper, which we will consider in our future research. Coverage probability and PRR are two performance metrics we are concerned about, of which the mathematical expressions are given in closed-form.

This paper is organized as follows. In section II, system model is introduced. The reliability metrics is analyzed in section III. The accuracy of the analysis results is validated by simulations in section IV. We conclude in section V.

## II. SYSTEM MODEL

### A. Distributions of the Vehicles

We consider the highway scenario with a single road, and vehicles are randomly distributed in this road, where we assume the positions of the vehicles follow a 1-D homogeneous PPP  $\Phi = \{x_i\}$  with intensity  $\lambda$  (we use the term intensity and density interchangeably). With a little abuse of notation, we often represent one vehicle with its location in the rest of the paper. Furthermore, we assume each vehicle generate and transmit a periodic message per broadcast period (i.e., 100ms).

Soltted-Aloha scheme is considered throughout this paper, where each broadcast period is divided into  $N$  data slots, and each vehicle choose one data slot randomly in each broadcast period to transmit its periodic message. Each vehicle transmit with a fixed power  $P$ . We can quickly know from the property of thinning that the locations of the transmitting vehicles and the that of the the receiving vehicles in any data slot form two independent PPP, i.e.,  $\Phi_1$  with intensity  $\lambda_x = \lambda / N$  and  $\Phi_2$  with intensity  $\lambda_{rx} = (N-1)\lambda / N$  respectively [11]. Each vehicle transmit with a fixed power  $P$ .

### B. Channel Model

#### 1) Path Loss Model

We consider the so-called three-slope path loss model which is adopted by 3GPP [8]. Given the length of the wireless link  $r$ , the path loss function is denoted as

$$l(r) = \begin{cases} l(R_1) & , 0 \leq r < R_1 \\ A_1 r^{-\alpha_1} & , R_1 \leq r < R_2 \\ A_2 r^{-\alpha_2} & , r \geq R_2 \end{cases}$$

Where  $R_1, R_2$  are two critical distances with  $R_1 = 3$ ,  $R_2 = 50.35$ , and  $\alpha_1, \alpha_2$  are two path loss exponents with  $\alpha_1 = 2.27$ ,  $\alpha_2 = 4$ .

Obviously, this function has three slopes in dB scale, i.e., 0,  $\alpha_1, \alpha_2$ . This function is continuous and bounded in its domain. Indeed, this function is a little bit different from the multi-slope model defined in [9].

#### 2) Shadowing Model

We assume shadowing follows from a lognormal distribution with mean  $\mu$  and standard deviation  $\sigma$ , and its probability density function (pdf) is given by

$$f_1(x) = \frac{e^{-(\ln y - \mu)^2 / 2\sigma^2}}{\sqrt{2\pi}\sigma y}$$

#### 3) Fading Model

Rayleigh fading is assumed, and its pdf is given by

$$f_2(x) = \rho e^{-\rho x}$$

where  $\rho$  is the mean of the pdf, which we also term as scaling parameter in this paper.

### C. Performance metrics

If the location of one transmitting vehicle is  $x_0$ , then any vehicle located in  $y$  can receive the safety message of  $x_0$  if and only if

$$\text{SINR} = \frac{P_0}{N_0 + I} = \frac{P g_0 l(|y - x_0|)}{N_0 + \sum_{x_i \in \Phi \setminus x_0} P g_i l(|y - x_i|)} > T \quad (1)$$

where  $P_0$  is the power received at  $y$ ,  $I$  is the interference,  $T$  is the threshold for successful decoding,  $g_i$  is the channel gain which equals to the product of shadowing and fading, and  $N_0$  is the power of the thermal noise. Without loss of any generality, we consider the typical transmitter located in origin, i.e.,  $x_0 = o$ .

Two performance metrics are considered in this paper. We are first interested in the probability that any vehicle  $y$  can successfully receive the periodic message of  $x_0$ , i.e.,  $\mathbb{P}(\text{SINR}^{(y)} > T)$ , and we call this coverage probability as in [12].

Sometimes it's necessary to measure the performance of all the vehicles around the transmitter as a whole, because all these vehicles are target receivers. Therefore, we also consider PRR in this paper, which is defined as the percentage of the nodes that successfully receive a packet from the target transmitter among all the intended receivers [13]. In vehicular network setting, the intended vehicles vehicles in the set  $A = \mathcal{B}(0, R)$ . Mathematically, PRR can be defined as

$$pr \triangleq \mathbb{E}_{\Phi} \left[ \frac{\sum_{x_i \in \Phi(A) \setminus x_0} \mathbf{1}(\text{SINR}^{(x_i)} > T)}{\Phi(A \setminus x_0)} \mid \Phi(A \setminus x_0) \neq 0 \right]$$

where  $\mathbf{1}(\cdot)$  is the indicator function. Actually, the concurrent transmitters in  $A$  are also intended receivers, but these vehicles can not receive any message because of the constraint of half-duplex.

## III. RELIABILITY ANALYSIS

The pdf of the channel gain  $g$  can be expressed as

$$f_g(x) = \int_0^{\infty} e^{-x/\rho y} \frac{1}{\rho y} \frac{e^{-(\ln y - \mu)^2 / 2\sigma^2}}{\sqrt{2\pi}\sigma y} dy \quad (2)$$

The form of the above pdf is complicated at first sight and seems to be intractable. Fortunately, this pdf can be approximated by the sum of several exponential pdf, as described by the following lemma.

*Lemma 1:* if fading factor follows a rayleigh-lognormal distribution, then its pdf can be approximated with a weighted sum of several exponential distributions

$$f_g(x) \approx \sum_{n=1}^N \omega_n f(e^{-\sqrt{2}\sigma t_n + \mu} / \rho; x) \quad (3)$$

where  $N$ ,  $t_n$ ,  $\omega_n$ ,  $f(\gamma; x)$  are the number of the exponential distributions,  $n$ -th root of the Hermite polynomial, normalized weight given as  $2^{N-1} N! / [N^2 H_{N-1}(t_n)]^2$ , and exponential pdf (i.e.,  $f(\gamma; x) = \gamma e^{-\gamma x}$ ), respectively.

*Proof:* see [14, Section III]. Note that Rayleigh-lognormal fading is the special case of the nakagami- lognormal fading.

For simplicity, we define  $\tau_n = \sqrt{2}\sigma t_n + \mu$  in the rest of the paper.

#### A. Coverage Probability

*Lemma 2:* The coverage probability of the vehicle located at  $y$  can be approximated as

$$\mathbb{P}(\text{SINR}^{(y)} > T) \approx \sum_{n=1}^N \omega_n \exp\left(-\frac{T\tau_n N_0}{Pl(|y|)}\right) \mathcal{L}_T\left(\frac{T\tau_n}{Pl(|y|)}\right) \quad (4)$$

where  $\mathcal{L}_T(s) = \mathbb{E}_{\Phi_1}[e^{-sI}]$  is the Laplace transform of the interference.

*Proof:* By definition and lemma 1, we can acquire

$$\begin{aligned} \mathbb{P}(\text{SINR}^{(y)} > T) &= \mathbb{P}(h_0 > \frac{T(I+N_0)}{Pl(|y|)}) \\ &= \mathbb{E}_T \left[ \int_{\frac{T(I+N_0)}{Pl(|y|)}}^{\infty} f_h(x) dx \right] \\ &\stackrel{(a)}{\approx} \sum_{n=1}^N \omega_n \mathbb{E}_T \left[ \int_{\frac{T(I+N_0)}{Pl(|y|)}}^{\infty} f(\tau_n; x) dx \right] \\ &= \sum_{n=1}^N \omega_n \exp\left(-\frac{T\tau_n N_0}{Pl(|y|)}\right) \mathbb{E}_T \left[ \exp\left(-\frac{T\tau_n I}{Pl(|y|)}\right) \right] \\ &= \sum_{n=1}^N \omega_n \exp\left(-\frac{T\tau_n N_0}{Pl(|y|)}\right) \mathcal{L}_T\left(\frac{T\tau_n}{Pl(|y|)}\right) \end{aligned}$$

where in step (a), we use the lemma 1.

To obtain the coverage probability, we give the expression of the Laplace transform of the interference in the following lemma.

*Lemma 3:* The Laplace transform of the interference is given by

$$\mathcal{L}_T(s) = \exp\left(-2\lambda_{\text{tx}} (I_1(s) + I_2(s) + I_3(s))\right) \quad (5)$$

where

$$I_1(s) = \frac{R_1}{1 + (sPA_1)^{-1} R_1^{2.27}},$$

$$v = \left(\frac{1}{sPA_1}\right)^{\frac{1}{2.27}}, \quad \theta = \left(\frac{1}{sPA_2}\right)^{\frac{1}{4}},$$

$$a = (vR_1)^{\frac{1}{4}}, \quad b = (vR_2)^{\frac{1}{4}}, \quad c = \theta R_2,$$

$$I_2(s) = \frac{\sqrt{2}}{8\theta} \left( \ln \frac{c^2 - \sqrt{2}c + 1}{c^2 + \sqrt{2}c + 1} + 2\pi - 2\arctan(\sqrt{2}c + 1) - 2\arctan(\sqrt{2}c - 1) \right)$$

$$\begin{aligned} I_3(s) &= \frac{4}{9v} \left( 2 \sum_{k=1}^4 \sin \frac{4\pi(2k-1)}{9} \left( \arctan \frac{b - \cos \frac{(2k-1)\pi}{9}}{\sin \frac{(2k-1)\pi}{9}} \right. \right. \\ &\quad \left. \left. - \arctan \frac{a - \cos \frac{(2k-1)\pi}{9}}{\sin \frac{(2k-1)\pi}{9}} \right) - \ln \frac{1+b}{1+a} \right. \\ &\quad \left. - \sum_{k=1}^4 \cos \frac{4\pi(2k-1)}{9} \ln \frac{1 - 2bc \cos \frac{(2k-1)\pi}{9} + b^2}{1 - 2ac \cos \frac{(2k-1)\pi}{9} + a^2} \right). \end{aligned}$$

*Proof:* By definition,

$$\mathcal{L}_T(s) = \mathbb{E}_{\Phi_1, g_i} \left[ \exp\left(-s \sum_{x_i \in \Phi} P g_i l(|x_i - y|)\right) \right]$$

$$\stackrel{(a)}{=} \mathbb{E}_{\Phi_1} \left[ \prod_{x_i \in \Phi_1} \mathbb{E}_{g_i} \left[ \exp(-s P g_i l(|x_i - y|)) \right] \right]$$

$$\stackrel{(b)}{=} \exp\left(-\lambda_{\text{tx}} \int_{\mathbb{R}} \left(1 - \mathbb{E}_g \left[ \exp(-s P g l(|x - y|)) \right] \right) dx \right)$$

$$= \exp\left(-2\lambda_{\text{tx}} \int_0^{\infty} \left(1 - \mathbb{E}_g \left[ \exp(-s P g l(|x|)) \right] \right) dx \right)$$

$$= \exp\left(-2\lambda_{\text{tx}} \int_0^{\infty} \left(1 - \frac{1}{1 + sPl(|x|)}\right) dx \right)$$

$$= \exp\left(-2\lambda_{\text{tx}} \int_0^{\infty} \frac{1}{1 + (sPl(|x|))^{-1}} dx \right)$$

$$= \exp\left(-2\lambda_{\text{tx}} (I_1(s) + I_2(s) + I_3(s))\right)$$

where step (a) use the fact that  $\{g_i\}$  are i.i.d. random variables, step (b) follows from Probability Generating Functional (PGFL)

of PPP [12], and

$$I_1(s) = \int_0^{R_1} \frac{1}{1 + (sPA_1)^{-1} R_1^{2.27}} dx = \frac{R_1}{1 + (sPA_1)^{-1} R_1^{2.27}},$$

$$I_2(s) = \int_{R_2}^{\infty} \frac{1}{1 + (sPA_2)^{-1} x^4} dx = \frac{1}{\theta} \int_{\mu R_2}^{\infty} \frac{1}{1 + t^4} dt.$$

$$I_3(s) = \int_{R_1}^{R_2} \frac{1}{1 + (sPA_1)^{-1} x^{2.27}} dx = \frac{1}{\nu} \int_{\nu R_1}^{\nu R_2} \frac{1}{1 + t^{2.27}} dt$$

$$\stackrel{(c)}{\approx} \frac{1}{\nu} \int_{\nu R_1}^{\nu R_2} \frac{1}{1 + t^{2.25}} dt$$

$$\stackrel{(d)}{=} \frac{4}{\nu} \int_{(\nu R_1)^{\frac{1}{4}}}^{(\nu R_2)^{\frac{1}{4}}} \frac{y^3}{1 + y^9} dy,$$

In step (d), we make a variable substitution  $y = t^{0.25}$ . From [15, eqn. 2.146(2)], we can obtain the expressions of the last two integrations. Note that it's no hard to get the exact expression of  $I_2(s)$  (one can make a variable substitution  $z = t^{0.01}$ ), but this expression has too many terms. Thus we make a approximation in step (c). We validate the effectiveness of this approximation in section IV.

From lemma 2 and lemma 3, we can derive the expression of the coverage probability.

### B. PRR

*Lemma 4:* For a homogeneous PPP-based network, if the probability that a point located is retained as the transmitter is  $p$ , then the PRR can be approximated as follows:

$$pr = \frac{1}{|A|} \int_A \mathbb{P}(\text{SINR}^{(x)} > T) dx$$

where  $|\cdot|$  is Lebesgue measure.

*Proof:* By the definition of PRR and the property of PPP, we can get

$$\begin{aligned} pr &= \mathbb{E}_{\Phi} \left[ \frac{\sum_{x_i \in \Phi(A) \setminus x_o} \mathbf{1}(\text{SINR}^{(x_i)} > T)}{\Phi(A \setminus x_o)} \mid \Phi(A \setminus x_o) \neq 0 \right] \\ &\stackrel{(a)}{\approx} \mathbb{E}_{\Phi_2} \left[ \frac{\sum_{x_i \in \Phi_2(A)} \mathbf{1}(\text{SINR}^{(x_i)} > T)}{\Phi_2(A)} \mid \Phi_2(A) \neq 0 \right] \\ &\stackrel{(b)}{=} \frac{1}{1 - \exp(-\lambda_{\text{tx}} |A|)} \sum_{n=1}^{\infty} e^{-\lambda_{\text{tx}} |A|} \frac{(\lambda_{\text{tx}} |A|)^n}{n!} \frac{1}{|A|^n} \\ &\quad \times \iiint_{A^n} \frac{\sum_{i=1}^n \mathbb{P}(\text{SINR}^{(x_i)} > T)}{n} dx_1 dx_2 \dots dx_n \\ &= \frac{1}{|A|} \int_A \mathbb{P}(\text{SINR}^{(x)} > T) dx \end{aligned}$$

where in step (a), we approximate the number of the vehicles in  $A$  with the number of the transmitting vehicles in  $A$ . This approximation is viable because the intensity of  $\Phi_1$  is much lower than that of  $\Phi_2$ . In step (b), we use the definition of PPP and Bayes formula.

To this end, we can approximate the expression of PRR, as given by the following theorem.

*Theorem 1:* The PRR can be approximated as

$$\begin{aligned} pr &\approx \frac{1}{R} \sum_{n=1}^N \omega_n \left( R_1 \exp(-a_n N_0) \mathcal{L}_1(a_n) \right. \\ &\quad + (R_2 - R_1) \exp(-b_n N_0) \mathcal{L}_1(b_n) \\ &\quad + (R_3 - R_2) \exp(-c_n N_0) \mathcal{L}_1(c_n) \\ &\quad \left. + (R - R_3) \exp(-d_n N_0) \mathcal{L}_1(d_n) \right) \end{aligned}$$

$$\text{where } a_n = \frac{T \tau_n R_1^{\alpha_1}}{PA_1}, \quad b_n = \frac{T \tau_n (R_2^{\alpha_1+1} - R_1^{\alpha_1+1})}{PA_1 (1 + \alpha_1) (R_2 - R_1)}, \quad R_3 = \frac{R}{2},$$

$$c_n = \frac{T \tau_n (R_3^{\alpha_2+1} - R_2^{\alpha_2+1})}{PA_1 (1 + \alpha_1) (R_3 - R_2)}, \quad d_n = \frac{T \tau_n (R^{\alpha_2+1} - R_3^{\alpha_2+1})}{PA_1 (1 + \alpha_1) (R - R_3)}.$$

*Proof:* One obvious way is to use the result in theorem 1 directly. However, It's difficult to obtain the closed form expressions because of the complexity of (5).

From lemma 2, we can obtain

TABLE I. SIMULATION PARAMETERS

Parameter	Value
Packet Size	300Byte
Bandwidth	10M
Broadcast Rate	6M bit/s
Mean of Shadowing $\mu$	0
Standard Deviation $\sigma$	3dB
Scaling Parameter $\rho$	1
Transmit Power $P$	20dBm
SINR Threshold $T$	7dB
Noise Power $N_0$	-110dBm
Intended Tansmission Range $R$	300m
Road Length	10Km

$$\begin{aligned} pr &= \frac{1}{R} \int_0^R \mathbb{P}(\text{SINR}^{(r)} > T) dr \\ &\approx \frac{1}{R} \sum_{n=1}^N \omega_n \int_0^R \mathbb{E}_I \left[ \exp\left(-\frac{T \tau_n (I + N_0)}{PI(r)}\right) \right] dr \\ &= \frac{1}{R} \sum_{n=1}^N \omega_n \left( R_1 \mathbb{E}_I \left[ \exp\left(-\frac{T \tau_n R_1^{\alpha_1} (I + N_0)}{PA_1}\right) \right] \right. \\ &\quad + \int_{R_1}^{R_2} \mathbb{E}_I \left[ \exp\left(-\frac{T \tau_n r^{\alpha_1} (I + N_0)}{PA_1}\right) \right] dr \\ &\quad \left. + \int_{R_2}^R \mathbb{E}_I \left[ \exp\left(-\frac{T \tau_n r^{\alpha_2} (I + N_0)}{PA_2}\right) \right] dr \right) \end{aligned} \quad (6)$$

To overcome the intractability, we adopt the following approximation

$$\frac{1}{b-a} \int_a^b e^{f(x)} dx \approx \exp\left(\frac{1}{b-a} \int_a^b f(x) dx\right) \quad (7)$$

Actually, from Jensen's inequality, we can know that the LHS in (7) is greater than the RHS.

Using (7), we can quickly obtain the result given in the theorem.

#### IV. NUMERICAL RESULTS

The parameter used in simulation is given in TABLE I. We can quickly know that the number of available data slots is 250 in each broadcast period from the first three parameters in Table I. The simulation is averaged over  $10^5$  network realizations for each density. In addition, the number of the exponential pdf  $N$  used in lemma 2 is 4.

In Fig. I, coverage probability is plotted against increasing link length for density  $\lambda = 0.1$ . It can be observed from Fig. I that the analytical results is very close to the simulation results.

Fig. II compared the analytical result and the simulation results of the PRR for different vehicle density. As shown in Fig. II, the difference between the analysis and simulation is very small, which indicate the accuracy of the approximation we used in this paper.

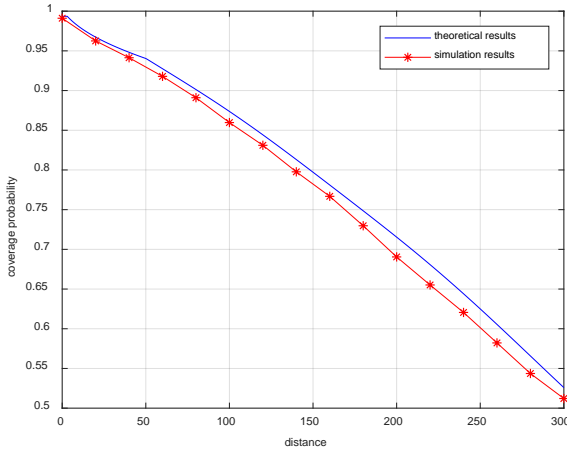


FIGURE I. COVERAGE PROBABILITY FOR DENSITY  $\lambda = 0.1$

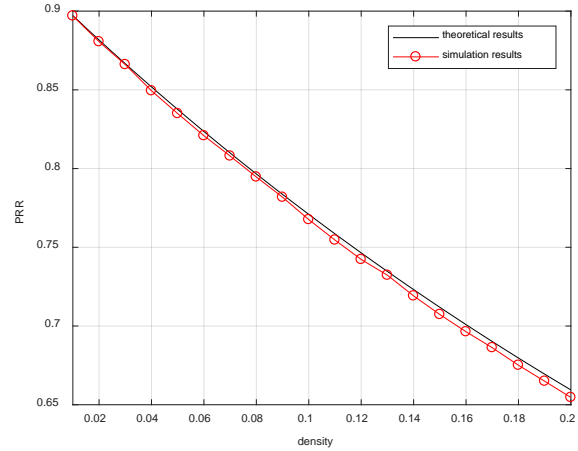


FIGURE II. PRR

#### V. CONCLUSION

In this paper, we use tools from stochastic geometry to analysis two reliability metrics of inter-vehicle safety communication under the highway scenario, with a focus on the transmission of one-hop periodic messages. The locations of the vehicles is modeled as a homogeneous PPP, and slotted-aloha is used as the MAC protocol. Three-slope path loss model and rayleigh-lognormal fading model is considered. Through some reasonable approximation, we derive the closed-form expressions of coverage probability and PRR. Finally, we validate the accuracy of the theoretical analysis by simulation.

The work of this paper can be extended in several ways. One possible idea is to consider nakagami-lognormal fading, which may be more complicated but more realistic. Another way is to employ the multi-lane model, which can depict the distribution of the vehicles more accurately. Last but not least, we can focus on the more involved but more efficient MAC protocols for vehicular networks, such as CSMA/CA, TDMA, and so on.

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