

The Research on Characteristics of Good Mathematical Cognitive Structure Based on Flow-Map Method

—Taking Function as an Example

Zhaohua Qu*

School of Mathematics and Statistics
Shandong Normal University
Ji'nan 250358, China
zhaohua_qu@163.com

Zezhong Yang

School of Mathematics and Statistics
Shandong Normal University
Ji'nan 250358, China

Abstract—This study was conducted to probe the features of mathematics knowledge and their connections in good mathematical cognitive structure (GMCS) with the flow-map method and the help of both function as material and 72 random senior one students as participants. This paper conducted flow-map interview and flow maps constructions, and then analyzed cognitive variables and knowledge processing strategies, knowledge details, recurrent linkages and misconceptions. The results indicated GMCS had several features. (1) It contained a relatively larger amount of more inclusive and abstract knowledge and knowledge about or focused closer on core concept. (2) It contained relatively more knowledge processed by higher-order conditional inferring and comparing and contrasting, as well as the radical defining. (3) It contained a larger amount of relatively more exact knowledge. (4) It contained more connections and was more compact and likely to be activated. (5) Connections contained in it were the ones between parallel knowledge as well as between inclusive knowledge. And (6) it excluded any misconceptions about connotation of core concept. Compared with most of existed researches, this paper quantitatively analyzed features of mathematics propositions and their linkages in GMCS with a new method, namely flow-map method.

Keywords—*Good mathematical cognitive structure; Mathematics Proposition; Outstanding students in mathematics; Flow-map method*

I. INTRODUCTION

Mathematical cognitive structure (MCS) was a specific concept brought forward by mathematical education researchers in the late 19th century on light of cognitive structure in psychology [1-4]. It meant the contents and connection of mathematics knowledge in mind, and was an internal hypothetical structure [2, 5-8]. Many mathematical education researches had demonstrated that MCS was mainly constructed in the process of mathematics learning and played a vital role in individual mathematical activities [9, 10]. It could affect individual's understanding, mastering as well as applying mathematics knowledge, etc. [11-14]. So, almost all teachers expected to help their students to form a good mathematical cognitive structure (GMCS). To achieve it, many

appeared researches on GMCS obtained fruitful results in recent years.[3,14-19] Nevertheless, the results of existed researches were mainly speculative reasoning based on psychological theories and practical teaching experience. That was, they were mainly quantitative instead of qualitative. Moreover, they focused mostly on absolute mathematical concepts and scarcely on other contents in GMCS, particularly propositions. Therefore, it is significant to conduct a quantitative study on characteristics of propositions in GMCS. For one thing, it will enrich the results in this field, for another, it might set an orientation for improving teaching and learning. So the paper chose some outstanding senior high school first-grade students and did a quantitative study towards mainly propositions in their mathematical cognitive structures.

II. METHODOLOGY

A. Participants

This paper chose 24 first-grade outstanding students in mathematics coming from eight senior high schools (three in Ji'nan city, two in Qingdao city, and respective one in Zibo city, Weifang city and Taian city of Shandong province) in total as participants. The reason for choosing outstanding students as participants was that it was usually believed their mathematical cognitive structures were the best [7]. Besides, respectively another 24 first-grade middle and general level students in the same senior high schools were chosen as references. The differentiations of three levels of students were mainly based on their usual performances and daily achievements. Outstanding students generally behaved positively, efficiently and effectively, and they usually made high and steady achievements with reasonable learning methods [20-22].

B. Material

This paper chose function as the core concept of following flow-map interview. The reason was that function was the thread running through the whole of mathematics at the stage of senior high school. It was worth to mention that Klein ever advocated function should be the core content of secondary school mathematics. [23]

This work is partially supported by the Research and Innovation Fund for postgraduate students of Shandong Normal University (No. SCX201731).

C. Method

The paper adopted flow-map method. This new method was first brought forward by Anderson and Demetrius in 1993 and used for probing and quantitative analyzing of students' cognitive structures. [24] It was different from ever before methods for studying cognitive structure, such as concept map, cards sorting and words association, etc.[25] And it had several characteristics as follows.[7, 24] (1) It adopted audio-taping to collect data beginning with an interview. The interview had two steps. The first step was to let the student narrate what they had thought focused on the interview questions and the second step was to let he/she amend or supplement what he/she had narrated based on his/her audio-taping. The data from both above steps were used to draw a flow map representing the student's cognitive structure. (2) It could be used for probing the connections between complex ideas such as propositions, besides absolute concepts in a student's cognitive structure. (3) The flow map of a certain student could represent both sequential and network features of knowledge in his/her cognitive structure. (4) A flow map represented the student's natural stream of thoughts without presupposing the amount of knowledge in his/her mind or imposing a predetermined hierarchical, network or other variety of structure. (5) The student needed not any relevant training for performing flow map or to draw a flow map by his/her own so that it might easily perform and relatively improve the reliability.

D. Data Collection

Based on the requirements of flow-map method, this study began with eliciting the student's ideas in his/her mind. In order to better elicit relatively complete knowledge about function in participants' cognitive structures, the study decided to conduct the one-on-one flow-map interview at the end of the second term when they had almost finished curriculum concerning function. The interview included two parts as mentioned above. The questions of the first part were as follows. (1) Which concepts or knowledge points should be contained in function as you think? (2) Could you please elaborating on the concepts or knowledge points that you have just mentioned? (3) Could you tell me the relationship among the ideas you have mentioned?

The interview process was audio-taped by a recording pen. Then the interviewer asked the student listen to the narrative of his/her tape-recording and try his/her best to add more. This process was audio-taped by another recording pen. It was the second part of the interview, namely meta-listening period called by Tsai [7].

E. Data Analyses

For the audio data above, this paper then represented all interviewed students' mathematical cognitive structures via flow maps constructions based on the requirements of flow-map method. Each student had a unique flow map. A finished flow map was shown in Fig.1.

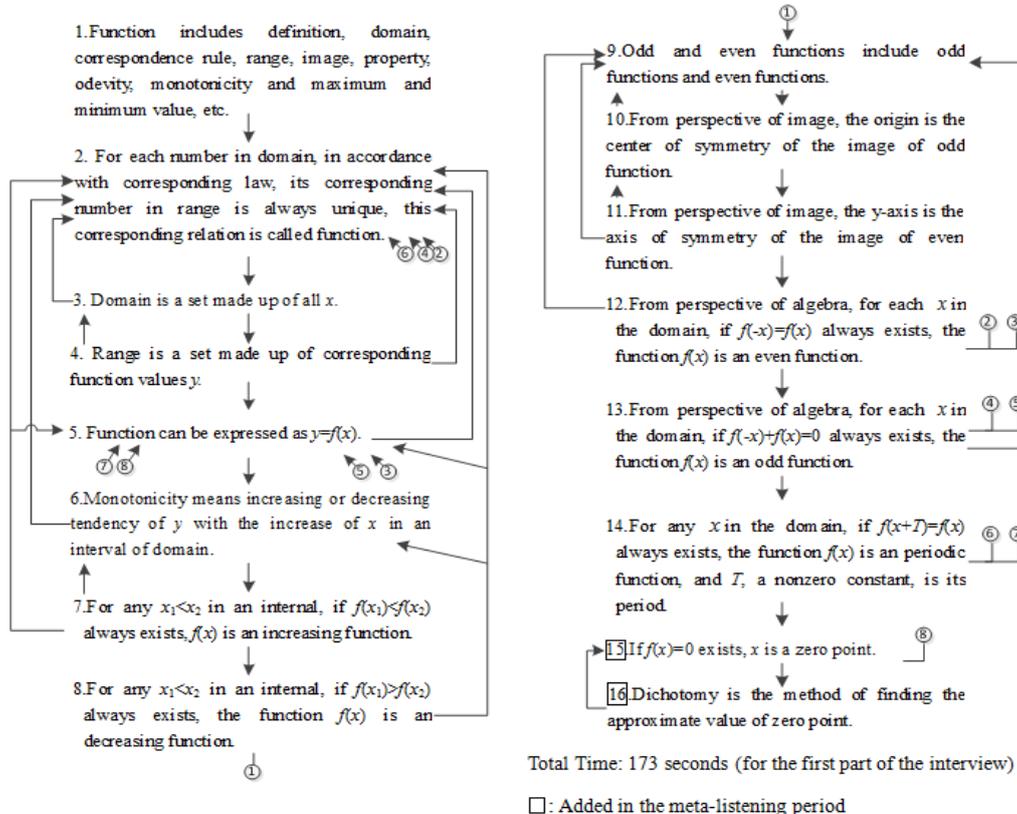


Fig. 1. A student's flow map on function

TABLE I. THE QUANTITATIVE CONTENTS OF FLOW MAPS

Dimensions	Categories	Connotations
Cognitive variables	Extent	The total number of statements shown in the flow map.
	Richness	The total number of recurrent arrows, representing the number of recurrent linkages.
	Integratedness	The proportions of recurrent linkages, equal to number of recurrent linkages/(number of statements + number of recurrent linkages).
	Misconceptions	The number of inexact or incorrect statements in the flow map.
	Knowledge retrieval rate	Statements the student narrated per second, equal to number of statements elicited in the first part of the interview/ total time used in that interview period.
	Flexibility	The number of statements the student narrated in the meta-listening period, indicating he/she idea change as a result of that period.
Knowledge processing strategies	Defining	Defining a concept or scientific term.
	Describing	Depicting a phenomenon or a real thing.
	Comparing and Contrasting	Showing the similarities or differences between different things.
	Conditional Inferring	Inferring what will happen under a supposing condition.
	Explaining	Providing two things or causal relationship between them with proving.

The followings were the procedures of drawing a flow map. Firstly, a researcher listed all complete statements on the basis of audio-taping, numbered orderly each one and linked them with linear arrows. A complex statement should be separated into individual statements. But a complex statement contained obvious comparing and contrasting should be regarded as a complete whole. Taking the flow map in Fig.1 as an example, there were 16 statements in total. Secondly, the researcher looked through each statement except the first general one to find the connections between it and the ones before it and linked them with recurrent arrows. It was mainly on the basis of the occurrences of revisited ideas. In order to avoid repetition, the recurrent linkages were drawn back to the earliest revisited statement. For example, statement 4 in the flow map in Fig.1, “range is a set made up of corresponding function values y ”, basically includes three revisited unit ideas: range, function value y and set. While both range and function value y appeared firstly in statement 2, and set in statement 3. So, two recurrent arrows were drawn back from statement 4. One was drawn back to statement 2 (about range and function value y) and one back to statement 3 (about set). Thirdly, the researcher marked each misconception with a five-pointed solid star and each statement added in meta-listening period with a box. For instance, there were two statements (i.e. 15 and 16) added in meta-listening period in the flow map in Fig.1, but no misconception. Finally, the researcher computed the total time of the flow-map interview and entered it in the flow map. It should be noted that the total time only included the time for the student thinking, organizing and replying to questions in the first part of interview, and excluded the time for the interviewer speaking and meta-listening. For instance, the total time the student mentioned in Fig.1 used was 173 seconds.

The paper then conducted quantitative analyses in light of finished flow maps. The contents basically included six quantitative variables (i.e. extent, richness, integratedness, misconceptions, knowledge retrieval rate and flexibility) representing students’ cognitive structures and five knowledge processing strategies (i.e. defining, describing, comparing and contrasting, conditional inferring and explaining) indicating their processing operations. The details were shown in TABLE I. [7, 24, 26]

Taking the flow map shown in Fig.1 as an example, its quantitative results on the basis of the above connotations were as follows. (1) cognitive variables: the total number of

statements was 16, namely the extent value was 16; the total number of recurrent arrows was 24, namely the richness value was 24; so the integratedness value equaled to $24/(16+24)$, namely 0.60; the number of misconceptions was 0; the number of statements added in meta-listening period was 2, namely the flexibility value was 2; and the value of knowledge retrieval rate equaled to $14/173$, namely 0.08. (2) knowledge processing strategies: for all 16 statements, the 2th, 3th and 4th were respectively definitions of connotation, domain and range of general function; the 1th, 5th, 6th, 9th, 10th, 11th and 16th respectively depicted inclusions, expression, monotone and odd-even functions, and method for finding a proximate zero point, etc.; the 7th, 8th, 12th, 13th, 14th and 15th were all processed by conditional inferring; and not any was obviously processed by the strategy of comparing and contrasting or explaining. Therefore, the number of statements processed by these strategies was respectively 3, 7, 0, 6 and 0.

Afterwards, the paper conducted further analyses on the basis of these flow maps data, which mainly involved knowledge details, recurrent connections and detailed misconceptions.

F. Reliability

The reliability of flow map included two aspects of sequential and recurrent linkages. The researcher draw twice all flow maps on light of existed audio-taping data after a time lapse. The inter-coder agreement for sequential statements was 0.98 and for recurrent linkages was 0.96.

III. RESULTS

A. The Quantitative Analysis of Mathematical Cognitive Structure

The quantitative dimensions of students’ cognitive structure include cognitive structure variables and knowledge processing strategies. The former included six aspects: extent, richness, integratedness, misconception, information retrieval rate and flexibility. And the latter included five types: defining, describing, comparing and contrasting, conditional inferring and explaining. [7]

This paper respectively conducted quantitative analysis of three levels of students’ mathematical cognitive structures. The details were shown in TABLE II.

TABLE II. THE RESULTS OF QUANTITATIVE ANALYSES OF COGNITIVE STRUCTURES

Dimensions	Categories	OS ^a	MLS ^b	GLS ^c	AS ^d
Cognitive Structure Variables	Extent	12.54	8.58	4.46	8.53
	Richness	14.75	7.75	1.88	8.13
	Integratedness	0.50	0.37	0.19	0.35
	Misconception	0.25	0.54	1.08	0.63
	Knowledge Retrieval Rate	0.09	0.08	0.07	0.08
	Flexibility	1.21	0.42	0.08	0.57
Knowledge Processing Strategies (Percentages of knowledge)	Defining	21.93	16.99	20.56	20.03
	Describing	56.48	66.02	67.29	61.56
	Comparing and Contrasting	6.98	4.85	4.67	5.86
	Conditional Inferring	14.29	11.65	7.48	12.21
	Explaining	0.33	0.49	0.00	0.33

^a Outstanding Students, ^b Middle Level Students, ^c General Level Students, ^d All Students. The same below.

From TABLE II, it could be seen that the average extent value of outstanding students' mathematical cognitive structures was about 1.5 times as large as that of middle level students' and about 3 times as large as that of general level students'.

The mean richness value of outstanding students' mathematical cognitive structures was 1.9 times larger than that of middle level students' and 7.8 times larger than that of general level students'.

The mean interestedness value of both outstanding and middle level students' mathematical cognitive structures were larger than that of all students'. However, the mean integratedness value of general level students' mathematical cognitive structures was smaller than that of all students'.

The average misconception of outstanding students was significantly fewer than that of all students, and the average misconception of middle level students was slightly fewer than that of all students. But the average misconception of general level students was significantly larger than that of all students.

The mean knowledge retrieval rate of outstanding students was higher than that of both middle and general level students, and the knowledge retrieval rate of middle level students was higher than that of general level students. However, there was no obvious differentiation.

The average flexibility value of outstanding students' mathematical cognitive structures was significantly larger than that of all students'. However, the flexibility value of middle level students' was slightly fewer than that of all students' and that of general level students' was obviously smaller than that of all students'.

The top three knowledge processing strategies used by all three levels of students were orderly describing, defining and conditional inferring. As for knowledge processed by describing, general level students' mathematical cognitive structures possessed them most, followed by middle level and outstanding students'. Moreover, the amount of knowledge

processed by describing in middle and general level students' mathematical cognitive structures were respectively larger than that in all students'. But the case of outstanding students was against that. As for defining, it was a kind of knowledge processing strategy used quite frequently by all three levels of students. Compared with middle and general level students, outstanding students narrated most frequently with defining strategy. As for conditional inferring, outstanding students used it most frequently, followed by middle and general level students. Moreover, the use frequency of outstanding students was higher than that of all students. But the cases of middle and general level students were all against that. The case of use of comparing and contrasting was similar. Last but significantly, whether outstanding or middle or general level students who used relatively less explaining.

B. The knowledge details about Function in Students' Mathematical Cognitive Structures

According to the flow maps representing visually students' cognitive structures, this research counted and summarized knowledge details recalled by most students. This might help with detecting which types of knowledge were focused in students' cognitive structures. The details were shown in TABLE III. It must be said that some similar statements about specific functions were just replaced by a general form (e.g. "sine function was symmetric about origin" "cosine function was symmetric about y-axis" were all replaced by "*** function was symmetric about **") because of too many of them.

From TABLE III, it could be seen that all three levels of students in the flow-map interview highlighted mostly specific elementary functions, analytical formula, image, monotonicity, odevity, domain, range, and max-minimum, etc., which are usually required in tests and examinations. However, another knowledge points about function, such as zero point and period, etc. were stated by few students. Specifically, regarding outstanding students, 70 percent of them whose mathematical cognitive structures contained elementary knowledge, such as what is function, domain and range, etc., no less than 50 percent of them whose mathematical cognitive structures contained knowledge about odevity and monotonicity of general function, and more than 50 percent of them whose mathematical cognitive structures contained propositions about domain, analytical formula, and image of various specific functions. As for middle level students, respectively more than 80, 50 and 45 percent of them whose mathematical cognitive structures contained propositions about analytical formula, monotonicity and domain of specific functions, and more than 30 percent of them whose mathematical cognitive structures contained propositions about range, image, and period of specific functions. Moreover, nearly 30 percent of middle level students whose mathematical cognitive structures contained propositions about odevity of specific functions. As for general level students, more than 60 percent of them whose cognitive structures contained knowledge about analytical formula of specific function, more than 20 percent of them whose cognitive structures contained knowledge about odevity and monotonicity of general function, and less than 20 percent of them whose cognitive structures contained any other knowledge.

TABLE III. THE KNOWLEDGE DETAILS ABOUT FUNCTION RECALLED BY STUDENTS IN THE FLOW-MAP INTERVIEW

Modules	Knowledge Details	OS(%)	MLS(%)	GLS(%)	AS (%)
General statements	Function contains analytical formula, image, and properties etc.	75.00	58.33	45.83	59.72
	Function has three elements: domain, range, and corresponding law.	20.83	12.50	0.00	11.11
	Functions include liner, quadratic function, etc.	50.00	83.33	70.83	68.06
	Trigonometric functions include cosine, sine and tangent functions, etc.	16.67	20.83	8.33	15.28
Connotation and properties of General function	Function means for each number in domain, according to the definite law, its corresponding number in range is unique.	75.00	25.00	16.67	38.89
	Function means dependent variable varies as independent variable varies.	0.00	16.67	0.00	5.56
	Function can be expressed as $y=f(x)$.	45.83	29.17	12.50	29.17
	Domain is a set made up of all x .	70.83	12.50	0.00	27.78
	Range is a set made up of all y .	70.83	12.50	0.00	27.78
	Odeivity means a certain function is an odd or even function.	58.33	29.17	20.83	36.11
	The image of even function is symmetric about y-axis, while the image of odd function is symmetric about origin.	37.50	16.67	8.33	20.83
	Even function meets $f(-x)=f(x)$, while odd function meets $f(-x)=-f(x)$.	37.50	16.67	8.33	20.83
	Monotonicity means a certain function is an increasing or a decreasing function.	54.17	33.33	20.83	36.11
	Increasing function means y increases as x increases, while decreasing function means y decreases as x increases.	41.67	12.50	8.33	20.83
	Max-minimum values include maximum value and minimum value.	8.33	4.17	0.00	4.17
	If $f(a)=0$ exists, the constant a is a zero point of $f(x)$.	4.17	0.00	0.00	1.39
	For each X in domain, if $f(x+T)=f(x)$ always exists, $f(x)$ is a periodic function and T is its period.	8.33	0.00	0.00	2.78
Connotation, image and properties of specific functions	** (The analytical formula of) function is $y=**x$.	54.17	87.50	62.50	68.06
	The domain of ** function is **.	50.00	45.83	8.33	34.72
	The range of ** function is **.	45.83	37.50	4.17	29.17
	The image of **function is **.	54.17	33.33	12.50	33.33
	**function is an odd (even) function.	25.00	29.17	4.17	19.44
	**function is symmetric about **.	4.17	20.83	8.33	11.11
	**function is increasing (decreasing) at a certain interval.	25.00	54.17	8.33	29.17
	The maximum (minimum) of ** function is**.	0.00	16.67	4.17	6.94
	The period of ** function is **.	33.33	33.33	0.00	22.22
The zero point of ** function is **.	4.17	8.33	16.67	9.72	

C. The Analyses of Recurrent Linkages between Statements Recalled by Students

According to flow maps visually representing students' cognitive structures, this study respectively counted and analyzed recurrent linkages between statements recalled by three levels of students in the flow-map interview. After analyzing, we classified these connections into five relatively large categories. The first meant the connections between some elementary knowledge about core concepts, which majorly meant connotation, domain, range, the express $y=f(x)$ ("analytical formula" called by many students), and image. The second meant the connections between statements about general properties and statements about core concepts. General properties meant odeivity, monotonicity, max-minimum, period, and zero point, etc. Although domain and range were generally

classified into properties, general properties in this case excluded them lest repetition. The third meant the connections between statements about specific functions and statements about core concepts and general properties, majorly made up of two parts, namely the connections between statements about specific functions and statements about core concepts and the connections between statements about specific functions and statements about general properties. The fourth meant the connections between statements about general properties, mainly made up of two parts: the connections between statements about odd and even functions and the connections between statements about monotone functions. The fifth meant the connections between statements about specific functions, mainly trigonometric, quadratic, and linear functions. The details were shown in TABLE IV.

TABLE IV. THE DETAILS OF RECURRENT CONNECTIONS BETWEEN STATEMENTS

Recurrent connections	OS(%)	MLS(%)	GLS(%)	AS(%)
between statements about core concepts	18.93	4.81	8.89	13.65
between statements about general properties and statements about core concepts	17.51	0	0	10.58
between statements about specific functions and statements about core concepts and general properties	10.73	3.74	0	7.68
between statements about specific functions and statements about core concepts	7.06	2.67	0	5.12
between statements about specific functions and statements about general properties	3.67	1.07	0	2.56
between statements about general properties	20.06	9.63	26.67	17.24
between statements about odd and even functions	14.41	9.09	22.22	13.31
between statements about monotone functions	5.37	0.53	4.44	3.75
between statements about specific functions	32.20	81.82	64.44	50.51

Cont. to table IV				
between statements about trigonometric functions	14.41	47.59	4.44	24.23
between statements about quadratic functions	9.60	12.83	17.78	11.26
between statements about linear functions	0.85	0.53	6.67	1.19
between statements about logarithmic and exponential functions	4.80	12.30	28.89	9.04

From TABLE IV, generally, it could be seen that connections between statements about various specific functions accounted for 50.51 percent with the largest proportion, in which the most was between statements about trigonometric functions accounting for 24.23 percent. Followed by connections between statements about general properties (17.24 percent), between statements about core concepts (13.65 percent), and between statements about general properties and core concepts (10.58 percent). The connections between statements about specific functions and statements about core concepts and general properties just accounted for 7.68 percent. Specifically, the top three recurrent connections between statements recalled by outstanding, middle and general level students were respectively the ones between specific functions, between general properties and between core concepts. But their percentages were significantly different from each other.

The connections between statements about specific functions recalled by outstanding students just accounted for 32.20 percent, while recalled by middle and general level students respectively ran to 81.82 and 64.44 percent. The connections between statements about general properties recalled by outstanding students accounted for 20.06 percent, while recalled by middle and general level students accounted respectively for 9.63 and 26.67 percent. The majority was

connections between statements about even and odd functions. The connections between statements about core concepts recalled by outstanding students accounted for 18.93 percent, while recalled by middle and general level students just accounted respectively for 4.81 and 8.89 percent. Moreover, it should be noted that the connections between statements about general properties and statements about core concepts recalled by outstanding students accounted for 17.51 percent. But, identical types of statements recalled by middle or general level students did not produce any linkage. The connections between statements about specific functions and statements about core concepts and general properties recalled by outstanding students accounted for 10.73 percent, while recalled by middle level students just accounted for 3.74 percent. By the same token, the identical types of statements recalled by general level students did not produce any connection.

D. The Analyses of Misconceptions stated by Students in the Flow-Map Interview

According to flow maps visually representing students' cognitive structures, this study conducted specifically analyses of misconceptions in the flow maps. The details were shown in TABLE V.

TABLE V. THE MISCONCEPTION DETAILS RECALLED BY STUDENTS

Modules	Misconceptions	OS(%)	MLS(%)	GLS(%)	AS(%)
Connotation of general function	Function means for any element in A, its corresponding unique element always exists in B. A and B were nonempty sets.		4.17		1.39
	Function means for each independent variable, its corresponding dependent variable always exists.		4.17		1.39
	The essence of function is dependent variable varies as independent variable varies.		4.17		1.39
	Function is a figure such as a line or curve.			4.17	1.39
	Function is a set.			4.17	1.39
	Function is a combination of figure and image, namely function not only is a figure, but also contains allure of image.			4.17	1.39
	The analytical formula of function is $y=f(x)$.	8.33	4.17		4.17
	The corresponding relation of function is its analytical formula.		4.17		1.39
Properties of general function	The corresponding relation of function means linear or quadratic function, etc.			4.17	1.39
	The extent of domain(range) is R(i.e. the set of all real numbers).			4.17	1.39
	When $f(-x)=f(x)$ exists, $f(x)$ is an even function. Or when $f(-x)=-f(x)$ exists, $f(x)$ is an odd function.	4.17	4.17	4.17	4.17
	Odd (even) functions satisfy $f(x)=-f(x)$.		4.17	8.33	4.17
Specific functions and their properties	Odd functions satisfy $f(x)=0$.	4.17		4.17	2.78
	Functions can be compared in size and applied for calculating.			4.17	1.39
	The analytical formula of quadratic function is $y=ax^2+bx+c$.			12.50	4.17
	The analytical formula of linear function is $y=kx+b$.			8.33	2.78
	The analytical formula of proportional function is $y=kx$.			4.17	1.39
	The analytical formula of inverse proportional function is $y=k/x$.			4.17	1.39
	The proportional function becomes linear function with an additional b.			4.17	1.39
	When the exponent is larger (smaller) than 0, a certain exponential function is an increasing (decreasing) function.			4.17	1.39
	When the base number is larger (smaller) than 0, a certain logarithmic function is an increasing (decreasing) function.			4.17	1.39
	The intersection point of the quadratic curve and x-axis is $-b \pm \sqrt{b^2 - 4ac} / 2a$.			4.17	1.39
The intersection point where the image of logarithmic function cuts x-axis is 1.			4.17	1.39	
The zero point of cosine function is $(\pi/2+k\pi,0)$.		4.17		1.39	

From TABLE V, it could be seen that all three levels of students had misconceptions in modules of both general and specific functions. As for misconceptions about radical connotation of general function, respectively 8.33 and 4.17 percent of outstanding and middle level students mistook the express $y=f(x)$ for analytical formula. It was seriously incorrect because $y=f(x)$ is just a symbol for function while analytical formula is one of representations of function. And respectively 4.17 percent of middle and general level students misunderstood the corresponding relation for analytical formula and specific functions such as linear and quadratic function, etc. Moreover, respectively 4.17 percent of middle level students whose cognitive structures contained inexact function relation to varying degree, overlooking the relation between function and mapping, no limiting the unique corresponding function value of each independent variable, and blurring the relation between definite function and indefinite correlation. And 1.41 percent of general level students whose cognitive structures contained functions replaced by images, sets or figures. As for misconceptions about properties of general function, respectively 4.17 percent of outstanding, middle and general level students who overlooked the limit of domain when stating propositions about even and odd functions. Moreover, respectively 4.17 percent of middle level students and 8.33 percent of general level students blurred the relational expression used for judging odd and even functions. 4.17 percent of general level students whose cognitive structures contained functions could be compared in size and calculated like concrete figures. 4.17 percent of general level students mistook domains and ranges of all functions for the set of all real numbers. As for misconceptions about specific functions and their properties, respectively 4.17 percent of middle level students and 8.34 percent of general level students who blurred zero point, root of corresponding equation and intersection point where x-axis cutting image of a certain function. More than 29 percent of general level students did not limit non-zero coefficient when stating propositions about connotations or analytical formulas of quadratic, linear or proportional function. Moreover, 4.17 percent of general level students blurred base number of exponential (logarithmic) function and exponent.

IV. DISCUSSION

Many recent studies in this field showed MCS played an important role in individual mathematics activities. So, almost all mathematics teachers expected to help their students fostering a GMCS. In order to achieve this, quite a few of existed researches had involved two significant aspects of mathematical cognitive structures, namely content and organization. However, most of these researches focused on absolute mathematical concepts and mainly adopted theoretical and qualitative methods. There still existed a gap in the field of researching on mathematical propositions in good cognitive structures with quantitative methods. So this paper chose the function as material and 72 random senior one students as participants and conducted a quantitative study on GMCS with the flop-map method. This study mainly involved knowledge variables, processing strategies, main types of proposition knowledge, recurrent connections between propositions and

misconception details in good mathematical cognitive structures.

As for cognitive structure variables, firstly, extent meant the amount of statements in the flow map of a certain student, representing the amount of ideas in his or her MCS. From date analyses above, it could be seen the extent of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', exactly almost respectively 1.5 and 3 times as large as the latter two, which indicated outstanding students' cognitive structures contained a larger number of mathematical ideas. Secondly, richness meant the number of recurrent arrows in the flow map, representing the number of recurrent connections between ideas in his or her MCS. From analyses above, it could be seen the richness of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', exactly respectively 1.9 and 7.8 times larger than the latter two, indicating more connections between ideas in outstanding students' cognitive structures. Thirdly, integratedness meant the ratio of the number of recurrent arrows to the sum of statements and recurrent arrows in the flow map, representing integration degree of his or her cognitive structure. From results above, it could be seen the integratedness of outstanding students' mathematical cognitive structures was larger than that of middle and general level students', almost respectively 1.4 and 2.6 times as large as the latter two, indicating a more compact proposition network in outstanding students' cognitive structures. Fourthly, misconception meant the number of incorrect or inexact statements in the flow map, representing the correctness of his or her MCS. From date analyses above, it could be seen the number of misconceptions in outstanding students' mathematical cognitive structures was smaller than that of middle and general level students', exactly respectively less than a half and quarter of the latter two, which indicated outstanding students' cognitive structures contained less inexact ideas, namely with a high correctness. Fifthly, knowledge retrieval rate meant the number of statements per second recalled by the student during the first part of interview. From results above, it could be seen the knowledge retrieval rate of outstanding students was higher than that of middle and general level students, which indicated outstanding students' cognitive structures were more likely to be activated. And sixthly, flexibility is defined as number of statements elicited in the second part of the interview, namely the meta-listening period. From data analyses above, the flexibility of outstanding students' cognitive structures was larger than that of middle and general level students', exactly near 3 times as large as middle level students' and 15 times larger than general level students'. This indicated outstanding students' mathematical cognitive structures were more likely to be activated with further self-prompts.

As for the amount of knowledge processed by each processing strategy, from results above, the majority of ideas in all three levels students' mathematical cognitive structures were the ones processed by describing. But, outstanding students' cognitive structures contained more knowledge processed by defining, comparing and contrasting as well as conditional inferring than middle and general level students'. This indicated outstanding students' mathematical cognitive

structures contained relatively more ideas processed by radical defining as well as relatively higher-order conditional inferring and comparing and contrasting, besides most ideas processed by describing.

As for knowledge details, from results above, ideas contained in outstanding students' mathematical cognitive structures were majorly about odd and even functions, analytical formulas, domains and images of specific functions as well as connotation, domain and range of general function. Ideas contained in middle level students' cognitive structures were mainly analytical formulas, monotonicity, domains, ranges, images, period and odevity of various specific functions such as trigonometric and quadratic functions, etc. Ideas in general level students' cognitive structures were mainly about analytical formulas of specific functions. These indicated outstanding students' cognitive structures contained relatively more ideas about connotation, domain, range, monotone and odd-even qualities of general function and ideas about analytical formulas, domains and images of specific functions that were more abstract and general and closer to the core concept "function".

As for recurrent connections between ideas in students' mathematical cognitive structures, from results above, outstanding students' cognitive structures contained connections not only majorly between specific functions, between core concepts and between general properties, but also relatively more between general properties and core concepts and between specific functions and general properties as well as core concepts. While connections in both middle and general level students' cognitive structures were majorly the ones between specific functions and between odd-even functions. That was, relatively more connections contained in outstanding students' mathematical cognitive structures from inclusive knowledge as well as parallel knowledge. While connections in both middle and general level students' cognitive structures were mainly from parallel knowledge and little inclusive knowledge.

As for misconceptions, from results above, all students had misconceptions in aspects of connotations and properties of both general and specific functions to varying degrees. It seemed to indicate unreal and unstable connections between the above and existed knowledge in their cognitive structures. Nevertheless, outstanding students had few misconceptions, particularly no misconception about the essence of function, namely corresponding relationship. While both middle and general level students had relatively more misconceptions, particularly the ones about connotation of function. Misunderstanding and even rote learning might mostly account for their frequent misconceptions.

It was universally believed that outstanding students had a GMCS [7]. So GMCS should have several characteristics as follows. (1) It should contain a relatively larger number of ideas about specific functions focused on core concepts (e.g. the ones about domains, images and analytical formulas of various specific functions) as well as more abstract and inclusive ideas (e.g. the ones about connotation, domain, range, ode-even and monotone properties of general function). (2) It should contain relatively more knowledge processed by higher-

order strategies (i.e. conditional inferring and comparing and contrasting) as well as the radical strategy (i.e. defining), besides the majority processed by describing. (3) It should contain a larger amount of knowledge with relatively higher correctness. (4) It should contain more connections and be more compact and likely to be activated. (5) Connections contained in it should be the ones not only between parallel knowledge (e.g. the ones between ideas about specific functions, the ones between ideas about core concepts, the ones between ideas about general properties), but also between inclusive knowledge (e.g. the ones between general properties and core concepts, the ones between specific functions and core concepts). (6) It should contain not any misconceptions about connotation of core concept.

V. CONCLUSION

Compared with existed researches, this study further analyzed knowledge, particularly propositions in GMCS from main perspective of knowledge details, connections between them and misconceptions. Generally, the results were shown as follows. (1) GMCS contained a relatively larger amount of more inclusive and abstract knowledge, such as propositions about connotation, domain, range, odd-even and monotone properties of general function. Moreover, it contained relatively more knowledge about core concepts, such as propositions about connotation, domain and range of function. Even specific knowledge in GMCS focused closer on core concept, such as propositions about analytical formula, domain and image of a certain function. (2) GMCS contained relatively more knowledge processed by higher-order conditional inferring and comparing and contrasting, as well as the radical defining, besides the majority processed by describing. (3) GMCS contained a larger amount of relatively more exact knowledge. (4) GMCS contained more connections and was more compact and likely to be activated. (5) Connections contained in GMCS were the ones between parallel knowledge (e.g. the ones between propositions about specific functions, the ones between propositions about core concepts and the ones between propositions about general properties) as well as between inclusive knowledge (e.g. the ones between general properties and core concepts and the ones between specific functions and core concepts). (6) GMCS contained not any misconceptions about connotation of core concept.

VI. LIMITATIONS AND FUTURE DIRECTION

Although this study found some features of knowledge type and their connection of GMCS, there are at least three limitations that should be considered. Firstly, the adopted material was only about function and excluded other more inclusive concept. Actually, an individual MCS should include the whole of mathematics knowledge containing function, even knowledge in other subjects such as linguistics and physics [6]. Secondly, some aspects of this study were superficial and incomprehensive. For instance, the paper analyzed knowledge processed by different strategies without further finding which knowledge processed by which strategy. And for the connections in students' mathematical cognitive structures, this paper only analyzed them with flow-map method and without auxiliary methods, such as concept mapping, a method by the

student performing on his/her own. It might to some degree cause incomplete and unreal representation of knowledge connection in student' MCS. Thirdly, there were only 24 participants who all came from Shandong province. A small number of participants and a narrow district to some extent decreased the accuracy and generalizations of the results. So, further enlarging participants from number and district and the range of material, as well as improving the analyses of connections in MCS with comprehensive methods should be taken into considerations in the future research.

REFERENCES

- [1] C.H. Cao, J.F. Cai, Introduction to Mathematics Education, Nanjing: Jiangsu Education Publishing House, pp.52-53, 1989.
- [2] R.B. Tu, Epistemology of Mathematics Teaching, Nanjing: Nanjing Normal University Publishing House, pp.172-175, 2003.
- [3] P. Yu, Review on Researches about Psychology of Mathematics Education for 30 Years in Mainland China. Beijing: Science Press, 2011.
- [4] A. Pease, M. Guhe, A. Smaill, Developments in Research on Mathematical Practice and Cognition. Topics in Cognitive Science. Cognitive Science Society, Inc. All rights reserved. ISSN:1756-8757 print/1756-8765 online DOI: 10.1111/tops.12021.no.5, pp.224-230, 2013.
- [5] Bruner, A selection of Bruner's Education Works, R. Zh. Shao, Trans, Beijing: People's Education Press, pp. 42-57, 1989.
- [6] Sh.Q. Li, PME: Psychology of Mathematics Education, Shanghai: East China Normal University Press, pp.24- 25, 2001.
- [7] C.C. Tsai, "Probing Students' Cognitive Structures in Science: the Use of a Flow Map Method Coupled with a Meta-Listening Technique", Studies in Educational Evaluation, pp.257-268, 2001.
- [8] X.Y. He, "Instructional Strategies on Constructing well Mathematical Cognitive Structure", Journal of Mathematics Education, vol.11, no.1, pp. 24-27, 2002.
- [9] Ch.W. Zhang, "The Psychological Meaning of Students' Mathematical Cognitive Structure in Mathematics Teaching", Research of Mathematics Teaching-Learning, no.12, pp.2-4, 2003.
- [10] P. Yu, Psychology of Mathematics Education, Nanning: Guangxi Education Publishing House, pp.55-77, 2004.
- [11] Skemp, The psychology of learning mathematics, Published by Penguin Books, Limited, 1971.
- [12] Q. Yang, "On the Influence of Cognitive Structure on Mathematics Learning -- One of the Explores about Psychological Factors that Have Influence on Mathematics Learning", Journal of Mathematics Education, no.1, pp.66-70, 1993.
- [13] M.L. Zhang, "How the Knowledge and Cognitive Structure Promote Each Other?", China Education Daily, no.5, pp.11-23, 2007.
- [14] Y. Zhang, "The Functions of Good Mathematical Cognitive structure", Proceedings of Conference on Psychology and Social Harmony (CPSH2012), Wuhan University Scientific Research Publishing, Apl.4, 2002.
- [15] G.M. Wang, Y. Wang, "The Comparison of Top and Ordinary Students' Mathematical Cognitive Structures in Senior High School, the Possible Reasons for difference, and Teaching Suggestions", Reference for Middle School Mathematical Education, no.12, pp. 1-4, 2004.
- [16] B. Han, G.M. Wang, "Role of Cognitive Structure and Reflections on It in the Process of Problem Solving", Educational Department of Journal of Junior Mathematic, no.6, pp.5-7,2005
- [17] X.F. Jin, "Again on the Construction of Students' Good Mathematical Cognitive Structure", New Curriculum Research, no.209, pp.179-181,2011.
- [18] Ch.X. Zhao, "Optimizing and Perfecting Students' Mathematical Cognitive Structure in Inquiry Learning", Educational Practice and Research, no.5, pp.53-55, 2013.
- [19] D.D. Sun, "The Research on Organization of Senior High School Students' Mathematical Cognitive Structures Based on Network Analysis", Ji'nan: Shandong Normal University, 2017.
- [20] C.J. Maker, "Curriculum Development for the Gifted (Hakluyt Society third series)", Aspen Publications, pp. 1-5, 1981.
- [21] NCTM, Principles and Standard for School Mathematics, Reston, NJ, 2000.
- [22] D.T. Johnson, "Teaching Mathematics to Gifted Students in a Mixed-Ability Classroom", Eric DIGEST E594, ERIC Publications,pp.1-2, 2000.
- [23] F. Klein, Elementary Mathematics Under the High Viewpoint, X. Q. Shu, Trans, Hubei Education Publishing House, vol.1, 1989.
- [24] O.R. Anderson, O.J. Demetrius, "A flow-map method of representing cognitive structure based on respondents' narrative using science content", Journal of Research in Science Teaching, pp.30, 953-969, 1993.
- [25] J.W. Zhang, Q. Chen, "Test Method of Cognitive Structure", Journal of Psychological Science, no.6, pp.750-751, 2000.
- [26] C.C. Tsai, "Content analysis of Taiwanese 14 year olds' information processing operations shown in cognitive structures following physics instruction, with relations to science attainment and scientific epistemological beliefs", Research in Science & Technological Education, no.17, 125-138, 1999.