Impulsive Control of a Chaotic Financial System

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Abstract. The impulsive control of a finance system is investigated in this paper, and the finance system can be asymptotically controlled to the origin by using impulsive control. Based on the new comparison theorem of impulsive system, this paper obtains some sufficient for the stabilization of the finance system via impulsive control with varying impulsive intervals, A numerical simulation illustrates the effectiveness of the proposed result.

Introduction

The chaotic financial system is a complex nonlinear system, which has some interesting phenomena, such as the limit cycle, stable periodic orbits, quasi periodic attractor, and chaotic attractor. After the system enters the chaotic state, the stable periodic orbit turns into an unstable periodic orbit, and how to turn the chaotic motion of the system into a regular motion is the purpose of chaos control. The pulse control systems have been considered by a number of authors [1-4]. The recent, impulse control is also widely used to control chaotic systems [5-7].

Gao qin et al.[8] formulated a financial model, studied the chaos and bifurcations problems of this financial model, and got some good results. In this paper, we study the stability of this chaotic financial model by using the pulse control method, and obtain the sufficient conditions for asymptotic stability of the financial system. The results show that some beneficial adjustments can be made to the operation of the market economy.

The pulse stability of the financial model

Gao qin et al.[8] discussed a financial model, shown as following.

$$\begin{cases} \dot{x}(t) = z(t) + (y(t) - a)x(t), \\ \dot{y}(t) = 1 - by(t) - x^2(t), \\ \dot{z}(t) = -x(t) - cz(t), \end{cases} \tag{1}$$

Where $a \geq 0$ is the saving amount, $b \geq 0$ is the cost per investment, and $c \geq 0$ is the elasticity of demand of commercial markets. Chaos is found through numerical simulation (Fig. 1).
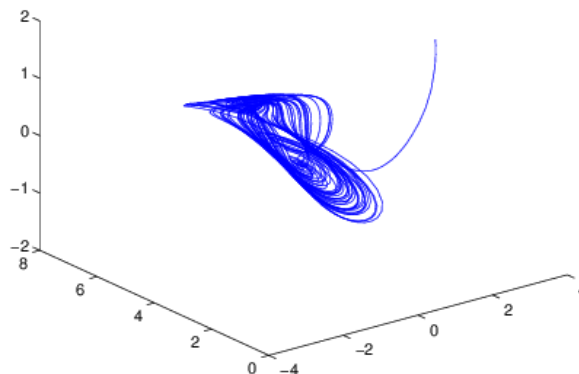

Fig.1 chaotic attractor: $a = 4, b = 0.1, c = 1$.

Let (x^*, y^*, z^*) is a steady-state of the system (1) and $X_1 = x - x^*, X_2 = y - y^*, X_3 = z - z^*$, $X^T = (X_1, X_2, X_2)$, the system (1) can become

$$\dot{X} = AX + \Phi(X). \tag{2}$$

Where $A = \begin{pmatrix} y^* - a & x^* & 1 \\ -2x^* & -b & 0 \\ -1 & 0 & -c? \end{pmatrix}$ $\Phi(X) = \begin{pmatrix} X_1 X_2 \\ -X_1^2 \\ 0? \end{pmatrix}$

We add impulsive control to the system (2)

$$\begin{cases} \dot{X} = AX + \Phi(X) & t \neq t_k \\ \triangle X \mid_{t=t_k} = I(k, X) = BX & t = t_k, k = 1, 2, 3, \cdots \end{cases} \tag{3}$$

Where $\{t_k : k = 1, 2, 3, \cdots\}$ satisfy $\Delta_1 = sup_{0 \leq j \leq \infty}\{t_{2j+1} - t_{2j}\} < \infty$, $\Delta_2 = sup_{0 \leq j \leq \infty}\{t_{2j} - t_{2j-1}\} < \infty$ and $t_{2j+1} - t_{2j} \leq \varepsilon(t_{2j} - t_{2j-1})$, ε is constant.

Theorem 1. Suppose that $n \times n$ matrix P is symmetric and positive definite, $\lambda_1 > 0, \lambda_2 > 0$ is the maximum and minimum eigenvalue, respectively. Let $Q = PA + A^T P$ where A^T is transpose of A, Q is maximum characteristic value of $P^{-1}Q$, d is maximum characteristic value of $P^1(I + B^T)P(I + B)$. And if there is a $\xi > 1$ and non-increment function $K(t) \geq m > 0$, which is differentiable at $t \neq t_k$ satisfies

$$-\frac{D^+ K(t)}{K(t)} \leq q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} \leq \frac{1}{(1+\varepsilon)\Delta_2} \ln \frac{K(t_{2k}^+)K(t_{2k-1}^+)}{K(t_{2k+1})K(t_{2k})\xi d^2} \tag{4}$$

Or

$$-\frac{D^+ K(t)}{K(t)} \leq q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} \leq \frac{1}{max\{\Delta_1, \Delta_2\}} \ln \frac{K(t_k^+)}{K(t_{k+1})\xi d^2} \tag{5}$$

Then the pulse system (3) is asymptotically stable, where $M = max|X_1|$. (Particularly, when $P = I$, $\lambda_1 = \lambda_2 = 1$)

Proof: Suppose $V(t, X) = X^T PX, t \neq t_i, P \neq I$, we have

$K(t)D^+V(t, X) + D^+K(t)V(t, X) = K(t)[X^T(A^T P + PA)X + X^T P\Phi(X) + \Phi^T(X)PX] + \dot{K}(t)V(t, X)$

$\leq K(t)[X^T QX + 2\sqrt{X^T PX}\Phi(X)\sqrt{\Phi^T(X)P\Phi(X)}] + \dot{K}(t)V(t, X)$

$\leq K(t)[qX^T PX + 2\sqrt{\lambda_2}\sqrt{X^T PX}\sqrt{\Phi^T(X)\Phi(X)}] + \dot{K}(t)V(t, X)$

$\leq K(t)[qX^T PX + 2\sqrt{\lambda_2}\sqrt{X^T PX}\sqrt{X_1^2(X_1^2 + X_2^2 + X_3^2)}] + \dot{K}(t)V(t, X)$

$\leq K(t)[qX^T PX + 2\frac{1}{\sqrt{P}}\sqrt{X_1^2}\sqrt{\lambda_2}\sqrt{X^T PX}\sqrt{X^T PX}] + \dot{K}(t)V(t, X)$

$\leq [q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} + \frac{\dot{K}(t)}{K(t)}]K(t)V(t, X)$

And

$$K(t_k + 0)V(t_k + 0, X + BX) \leq K(t_k)(X + BX)^T P(X + BX)$$

$$= K(t_k)X^T(I + B)^T P(I + B) \leq dK(t_k)V(t_k, X)$$

Suppose that $g(t, \omega) = [q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} + \frac{\dot{K}(t)}{K(t)}]\omega, \Phi_k = d\omega$, then the conditions (1-3) in the lemma 1 [14] are satisfied. By the lemma 1, the stability of pulse system (3) is contained in the following comparison system.

$$\begin{cases} \dot{\omega}(t) = [q + 2M\sqrt{\lambda_2/\lambda_1} + \dot{K}(t)/K(t)]\omega(t), \quad t \neq t_k \\ \omega(t_k^0) = d\omega(t_k) \\ \omega(t_k^0) = \omega_0 \geq 0 \end{cases} \tag{6}$$

Now let's consider the conditions in lemma 2 [9], because

$$sup_k\{dexp[q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}}(t_{k+1} - t_k) + \ln\frac{K(t_{k+1})}{K(t_k)}]\}$$

$$\leq dexp(q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(max\{\Delta_1, \Delta_2\} < \infty$$

Then, the condition (2) in the lemma 2 [9] are satisfied.

$$(q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(t_{2k+1} - t_{2k-1}) + \ln\frac{K(t_{2k+1})K(t_{2k})}{K(t_{2k}^+)K(t_{2k-1}^+)}$$

$$= (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(t_{2k+1} - t_{2k} + t_{2k} - t_{2k-1}) + \ln\frac{K(t_{2k+1})K(t_{2k})}{K(t_{2k}^+)K(t_{2k-1}^+)}$$

$$\leq (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(\Delta_1 + \Delta_2) + \ln\frac{K(t_{2k+1})K(t_{2k})}{K(t_{2k}^+)K(t_{2k-1}^+)} \tag{7}$$

$$\leq (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(1 + \varepsilon)\Delta_2 + \ln\frac{K(t_{2k+1})K(t_{2k})}{K(t_{2k}^+)K(t_{2k-1}^+)} \leq -\ln(\xi d^2)$$

Or

$$(q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(t_{k+1} - t_{2k}) + \ln\frac{K(t_{k+1})}{K(t_i^+)} \leq (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})max\{\Delta_1, \Delta_2\} + \ln\frac{K(t_{k+1})}{K(t_i^+)} \leq -\ln(\xi d) \tag{8}$$

Hence, the condition (3) in the lemma 2 [9] are also satisfied. Then the pulse system (3) is asymptotically stable.

Theorem 2. If

$$\lim_{T \to \infty} \frac{i(t, t+T)}{T} = p = \frac{1}{\tau}$$

Where $i(t, t+T)$ is the number of t_i in the $[t, t+T]$.

And

$$0 \leq q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} \leq -p\ln(\xi d), \xi > 1$$

Then the trivial solution of the system (3) is asymptotical stable.

Proof: From the previous hypothesis, we have

$$g(t, \omega) = [q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}} + \frac{\dot{K}(t)}{K(t)}]\omega(t) = \dot{\lambda}(t)$$

Hence

$$\int_{t_k}^{t_{k+1}} \dot{\lambda}(t)dt = (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(t_{k+1} - t_k) + \ln\frac{K(t_{k+1})}{K(t_k)}$$

And $K(t)$ is non-increasing function, so $\ln\frac{K(t_{k+1})}{K(t_k)} \leq 0$, hence

$$\int_{t_k}^{t_{k+1}} \dot{\lambda}(t)dt \leq (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})(t_{k+1} - t_k)$$

By using equation (2), we have

$$\int_{t_k}^{t_{k+1}} (q + 2M\sqrt{\frac{\lambda_2}{\lambda_1}})dt + \ln(\xi d) \le 0, \xi > 1 \qquad \text{for all } k.$$

Hence

$$\int_{t_k}^{t_{k+1}} \dot{\lambda}(t)dt \le \ln(\xi d)$$

By the lemma 2 [9], the trivial solution of the system (3) is asymptotical stable.

Numerical Simulation

For the system (1), when $a = 4, b = 0.1, c = 1$, chaos is found through numerical simulation (Fig. 1).

Take the equilibrium point $P_0(0, \frac{1}{b}, 0)$ of the system (1), then

$$A = \begin{pmatrix} 6 & 0 & 1 \\ 0 & -0.1 & 0 \\ -1 & 0 & -1 \end{pmatrix} \qquad A + A^T = \begin{pmatrix} 12 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

For the sake of simplicity, Assuming $P = I$, then $q = max(12, -0.2, -2) = 12$, supposing $M = 2$, taking

$$B = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix} = \begin{pmatrix} -0.58 & 0 & 0 \\ 0 & -0.65 & 0 \\ 0 & 0 & -0.75 \end{pmatrix}$$

We have $k = max(k_1, k_2, k_3) = -0.58$, and using the Theorem 2, then

$$0 \le \tau = \frac{1}{p} \le -ln(\xi(1+k)^2 / q = 0.029, \xi = 2.3$$
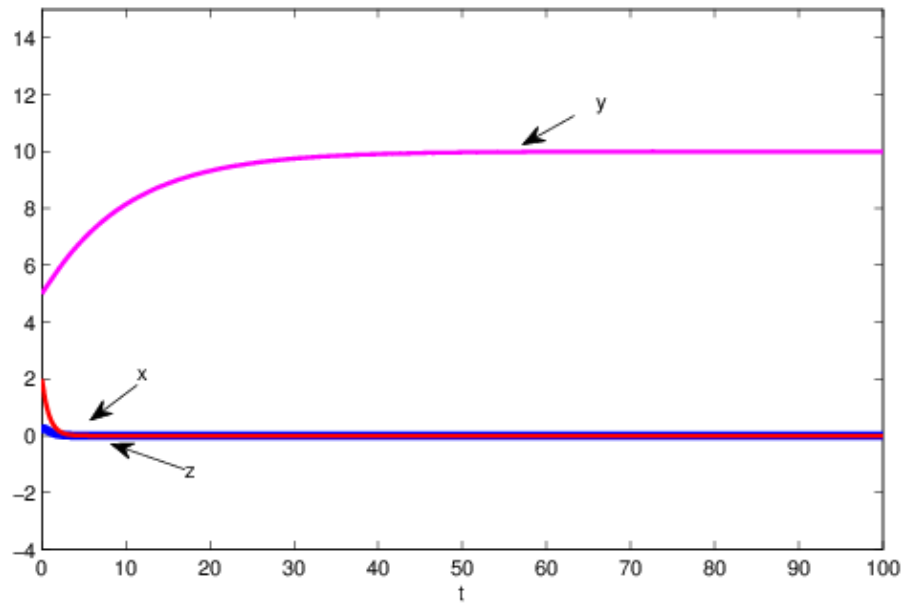
The result of numerical simulation is as shown in Fig. 2.

Fig. 2 When $a = 4, b = 0.1, c = 1$, the equilibrium $P_0(0, \frac{1}{b}, 0)$ of system (1) is asymptotically stable.

Conclusion

The impulsive control of a finance system is investigated in this paper, and the finance system can be asymptotically controlled to the equilibrium point by using impulsive control. This paper obtains

some sufficient for the stabilization of the finance system via impulsive control with varying impulsive intervals. This method can control chaotic motion effectively, can turn chaotic irregular motion into regular motion, and achieve the purpose of chaos control. In the actual commercial market, through our control, to some extent, we can effectively regulate the rapid and healthy development of economy.

Acknowledgements

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