

A Tracking Differentiator Based Backstepping Control of Missile System

Wenguang Zhang^a, Lili Yang, Junwei Lei

Department of Control, Naval aeronautical and astronautical university, Yantai, 264001, China

^aemail: zhangwenguang1024@126.com

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Abstract. To avoid the complex calculation of derivative in the backstepping design procedure, a novel kind of tracking differentiator was adopted to solve the derivative of ideal angle speed of the pitch channel missile system. Also, the ideal angle speed should be bounded or the stability of the system can not be guaranteed. And a Lyapunov function was chosen to prove the stability of the whole system. At last, the detailed numerical simulation was done to show the rightness and effectiveness of the proposed method.

Introduction

Pitch channel of missile system can be reduced and simplified to be a second order system[1-4], which is a classical method to design and analysis of the stability of missile system. And backstepping method is a widely used method because the stability of the whole system can be guaranteed by choosing a Lyapunov energy function. But the only disadvantage is that the derivative of desired value is very complex[5-8]. So a kind of tracking differentiator method was widely studied to solve derivative of some signals. But there is no difference between tracking differentiator method and traditional filter method that both need the input signal is bound or its derivative is bound. So we design a kind of tracking differentiator based backstepping controller for missile pitch channel system, and simulation shows that strategy is very effective.

Problem Description

The simplified linear model of supersonic missile pitch channel can be written as following second order system:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z + f_1 \quad (1)$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 \quad (2)$$

Where a_{ij} is air dynamic coefficient of missile, α is attack angle of missile, ω_z is the rotate speed of pitch angle, f_1 and f_2 are outer disturbance.

The control objective is to design a backstepping control law δ_z based on tracking differentiator such that the attack angle α can track the desired angle α^d . Without loss of generality, assume $\alpha^d = 1$, and the disturbance f_1 and f_2 can be observed and compensated by the designed control law δ_z .

Backstepping Controller Design

Define a new error variable as $e_\alpha = \alpha - \alpha^d$, then the first subsystem can be described as

$$\dot{e}_\alpha = \omega_z - a_{34}\alpha - a_{35}\delta_z + f_1 - \dot{\alpha}^d \quad (3)$$

Then the idea value of ω_z can be designed as ω_z^d as follows:

$$\omega_z^d = a_{34}\alpha + a_{35}\delta_z - f_1 + \dot{\alpha}^d - k_{\alpha 1}e_\alpha - k_{\alpha 2} \frac{e_\alpha}{|e_\alpha| + \varepsilon_b} \quad (4)$$

Since f_1 is unknown, we define a new variable as

$$\omega_{za}^d = a_{34}\alpha + a_{35}\delta_z + \dot{\alpha}_d - k_{\alpha 1}e_\alpha - k_{\alpha 2} \frac{e_\alpha}{|e_\alpha| + \varepsilon_b} \quad (5)$$

Then

$$\omega_z^d = \omega_{za}^d - f_1, \dot{\omega}_z^d = \dot{\omega}_{za}^d - \dot{f}_1 \quad (6)$$

And Define a new variable as $e_\omega = \omega - \omega_z^d$, then the second subsystem can be rewritten as

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 - \dot{\omega}_z^d \quad (7)$$

It can also be rewritten as

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 + \dot{f}_1 - \dot{\omega}_{za}^d \quad (8)$$

And we design a tracking differentiator to solve $\dot{\omega}_{za}^d$ as

$$\begin{aligned} z_1 &= x_1 - \omega_{za}^d \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= R^2(-z_1 - x_2/R) \end{aligned} \quad (9)$$

Then the idea control law can be designed as

$$\delta_{za} = \frac{1}{a_{25}} \left\{ -a_{24}\alpha - a_{22}\omega_z - k_{\alpha 1}e_\omega - k_{\alpha 2} \frac{e_\omega}{|e_\omega| + \varepsilon_a} - f_2 - \dot{f}_1 + \dot{\omega}_{za}^d \right\} \quad (10)$$

Here we use the value \dot{x}_1 as the value of $\dot{\omega}_{za}^d$ since $x_1 \rightarrow \omega_{za}^d$. And assume $-f_2 - \dot{f}_1$ is bounded, then there exists k_{a2} such that $|-f_2 - \dot{f}_1| \leq k_{a2}$. Also consider that $k_{\alpha 2} \frac{e_\omega}{|e_\omega| + \varepsilon_a} \approx k_{a2} \text{sgn}(e_\omega)$, so we design the real control law as

$$\delta_{za} = \frac{1}{a_{25}} \left\{ -a_{24}\alpha - a_{22}\omega_z - k_{\alpha 1}e_\omega - k_{a2} \frac{e_\omega}{|e_\omega| + \varepsilon_a} + \dot{x}_1 \right\} \quad (11)$$

Then if we choose a Lyapunov function as

$$V = \frac{1}{2}e_\alpha^2 + \frac{1}{2}e_\omega^2 \quad (12)$$

And its derivative can be calculated as

$$\dot{V} \leq -k_{\alpha 1}e_\alpha^2 - k_{a1}e_\omega^2 \quad (13)$$

So according to the Lyapunov stability theorem, the whole system is stable.

Numerical Simulations

The air coefficient of a type of missiles can be set as

$$a_{25} = -167.87; a_{35} = 0.243; a_{22} = -2.876; a_{24} = -193.65; a_{34} = 1.584$$

It is a kind of non-mini-phase system. Set control parameters as

$$R = 10; k_{\alpha 1} = 8; k_{a2} = 15; k_{\alpha 1} = 20; k_{\alpha 2} = 10$$

Then simulation results can see below figures 1-5. Figure 1 to Figure 3 shows the missile system is stable and attack angle can trace to the desired value 5 degree. And figure 4 and figure 5 shows that the estimation of angle speed and its derivative. So it can provide derivative information for the backstepping design method.

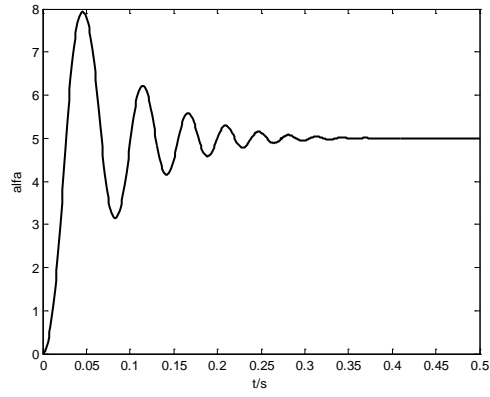


Figure 1 The curve of attack angle

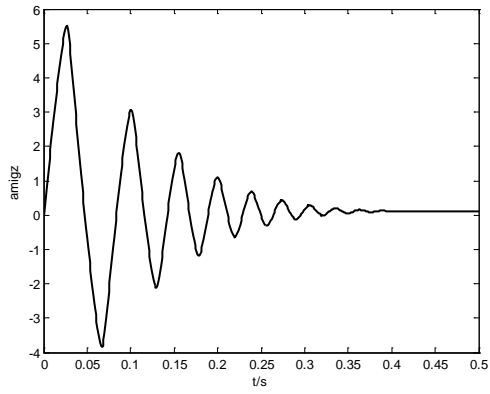


Figure 2 The figure of angle speed

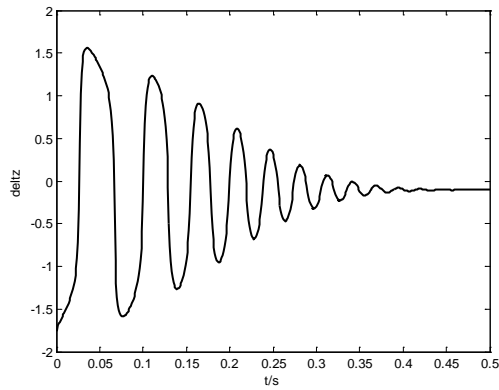


Figure 3 The curve of actuator

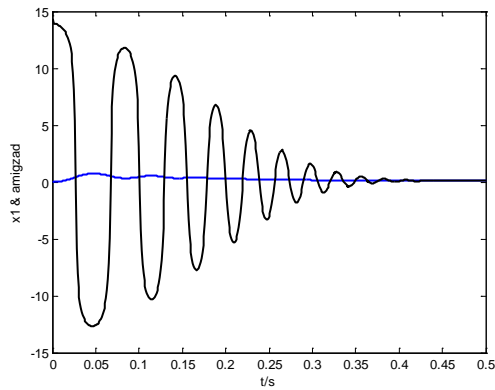


Figure 4 The estimation of ω_{za}^d

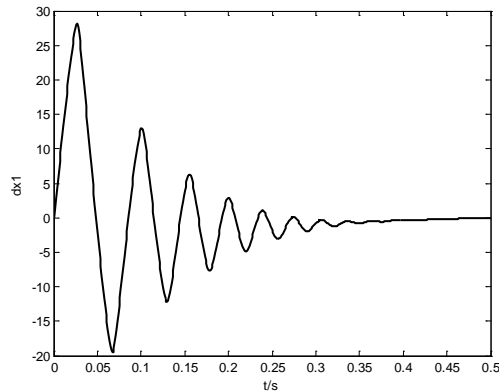


Figure 5 The state of $\dot{\omega}_{za}^d$

Conclusions

A novel tracking differentiator based backstepping method was proposed and applied in the pitch channel missile system to make the attack angle tracking to the desired value. Also a Lyapunov function method was used to prove the stability of the whole system. And detailed simulation were done to test and check the stability of proposed control strategy.

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