Simulating the microscopic pedestrian turning region of a subway station passageway

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Abstract. This paper investigates pedestrians' behaviours in the turning regions in the Xizhimen subway station passageway in Beijing. Through data-processing and data analysis, it can be concluded that the pedestrians tend to turn approach at the turning arc and the speed during the turn decreases initially and then increases. Based on the existing social force model, this report improves the pedestrian simulation model by introducing the gravitation and piecewise functions, thereby increasing the accuracy of the conclusion. Subsequently, in this paper, the improved model is programmed to simulate the behaviours of individual pedestrians and pedestrian flows in the turning area; the results verify the effectiveness of the improved model.

Introduction

With the expansion of urban city scale and complexity of pedestrian environment, the pedestrian safety issues become increasingly prominent. Because simulation of pedestrian flows is important to optimization of pedestrian flow organization and transport facilities, therefore many scholars in traffic area dedicated to studying pedestrian flow.

The walking speed and walking tendency of pedestrians in the turning region of a subway station passageway are different from those observed in a straight passageway. Scholars in various countries have made progress in the study of the characteristics of pedestrian walking and the social force model.

For the characteristics of pedestrian behaviour in a turning region, a heuristic one-shot method to model the influence to walk the quickest path was introduced[1]. According to the characteristics of pedestrian behaviour in the turning region, the social force model is improved. In the improved model, as a result of the resistance encountered, individual pedestrians tend to decrease the walking speed constantly in the process of turning; meanwhile, because of the influence of centrifugal force, the pedestrian walk away from the circular arc[2]. By organizing 16 pedestrians with the forward direction of the different angles and different walking speeds in the turning region, the average speed of the pedestrians decreased with the angle increasing in the forward direction [3]. Increasing the turning degree of corner has negative impact on the pedestrian queues[4]. The cost potential field model describes pedestrian flow in a domain with poor visibility or complex geometries. The proposed aggregated force field was indicated to be critical for simulating over-crowded flows in special areas, such as at the turning of a corner[5]. The pedestrian's moving behaviour of walking through a corner can be described by using a block-based floor field model[6]. Right-angled egress paths decrease the flow rate and increase the escape times significantly compared with those of straight egress paths[7]. The influence of different angles on pedestrian speed via experiments indicated that the greater the angle is, the more obvious the decrease in pedestrian speed is in the merging areas[8].

However, studies on the pedestrian walking characteristics in a turning region, the changes in pedestrian speed when turning, the characteristics of pedestrian flows in the bi-direction passageways, and the walking trajectory of a pedestrian should be explored further. In this article, by studying the turning region of the bi-direction passageways in the Beijing Xizhimen subway station and analysing the microscopic characteristics of the pedestrian crowds with opposite directions in turning region, it can be concluded that pedestrians intentionally shorten the detour in the turning region. Pedestrians

gradually approach the turning arc after the start of turning and gradually leave the turning arc after passing the arc centre until completing the turning behaviour. From the turning start point, the pedestrian speed gradually decreases; after passing the turning centre arc, each pedestrian gradually accelerates to the expected speed.

Based on the conclusion of the research, further studies were conducted regarding the pedestrian trajectory and velocity model in the turning region of bi-direction passageways, and improved the existing pedestrian microscopic simulation model in the turning region. The validity of the model was verified by programming and simulating experiments.

The remainder of this paper comprises the following sections. Based on the investigation of the turning region in the Beijing Xizhimen subway station passageway, section 2 analyses the microscopic characteristics of the bi-direction pedestrian flow in this area. In section 3, the conclusion from section 2 is used to improve the microcosmic simulation model of pedestrian behaviour in subway station passageway turning region. In section 4, we take the bi-direction passageway turning area of Xizhimen subway station as the research object and use the pedestrian social force model and MATLAB to confirm the validity of the improved model. Finally, conclusions are reported in section 5.

Analysis of the microscopic characteristics of pedestrian flow in the turning region of a subway station passageway. In this paper, we consider the turning region of a bi-direction passageway in the Xizhimen subway station to be the research area and analyse the micro-traffic characteristics of pedestrian flows in the turning region by recording video files.

Linear Segment Link Analysis in Turning Region of Subway Station Passageway. The turning shape of the research area in Xizhimen subway station is composed of a circular arc and two perpendicular lines. In this paper, the passageway corresponding to the pedestrian trajectory arc is called the turning region. The midpoint of the tangent line to the arc is defined as the centre of the turning arc, and the straight line and arc section in the turning region are defined as the turning arc, as shown in Fig. 1.
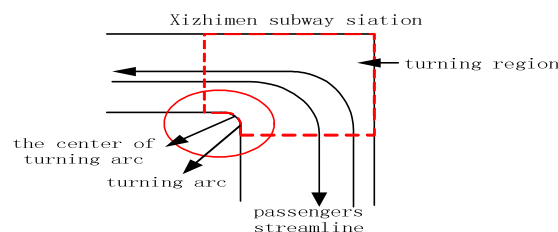

Fig.1. The parameter definition of the turning region of Xizhimen subway station passageway

Analysis of the turning trajectory. Fig.2 represents the down direction turning process of a pedestrian in the research area of Xizhimen subway station passageway. Fig.2 (a) shows that as the distance between the pedestrian and the centre of the turning arc decreases, the distance between the pedestrian and the turning arc is gradually reduced. When the pedestrian arrives at the centre of the circular arc, the distance previously mentioned is the smallest (as shown in the first picture of Fig.2 (b)). Fig. 2(b) shows that after passing the turning centre of the arc, the pedestrian gradually leaves the turning arc and automatically walk on the right side because the research area in the Xizhimen subway station is a bi-direction passageway and because Chinese people are used to walking on the right side. The down direction pedestrian crowds tend to turn approach the wall.

（a）   （b）

Fig. 2. The turning process of down direction pedestrian flow in
the turning region of Xizhimen subway station passageway

Fig. 3 shows, up direction pedestrian crowds tend to turn approach the down direction pedestrian flows in the research area of the Xizhimen subway station passageway.

Fig. 3. The turning process of up direction pedestrian flow in
the turning region of Xizhimen subway station passageway

In the turning region, as pedestrians gradually approach the centre point of theturning arc, the distance between them and the arc also decreases gradually. As shown in Fig.4, $L_1 > L_2$ and $l_1 > l_2$. After passing the centre point of the arc, the distance between the pedestrian and the arc increases gradually (as shown in Fig.4, $L_2 < L_3$ and $l_2 < l_3$) until completing the turning behaviour. The process of pedestrian turning and the walking trajectory are shown in Fig. 4. The main reason for this phenomenon is that pedestrians usually choose the shortest route. To reach the destination more quickly, pedestrians will deliberately shorten the walking distance to turn approach the turning arc in the passageway turning area.
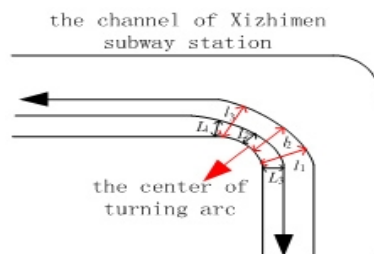

Fig. 4. Pedestrian walking trajectory in the turning region of Xizhimen subway station passageway

Analysis of passenger travelling speed in researching region. To analyse the changes in pedestrian walking speed in the turning region, we randomly extracted 20 pedestrian from the video film and analysed their turning process in the research region. Beginning from the start points in two directions and selecting five positions to record, the velocities of selected pedestrians. Specific locations used are as shown in Fig. 5, where a1 and a2 are the starting positions of pedestrian turning,c1 and c2 are the middle positions of the pedestrian turning process and are the shortest distances between the relevant pedestrian and the turn arc, and e1, e2 are the ending positions of pedestrian turning.
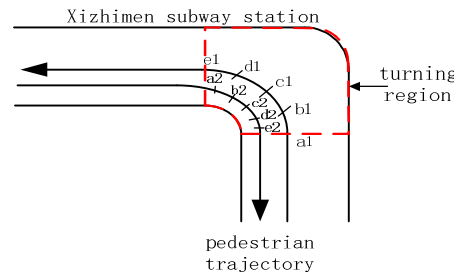
Fig. 5. The recording of velocities of turning region

Table 1 shows the average value of the velocity of 60 pedestrians, who were randomly selected from the video film with different directions at the above-described positions in the research region of Xizhimen subway station. From Table 1, the speed of the pedestrian gradually decreases from the starting position of the turning to the centre of the turning circle, and the speed gradually increases from centre of the turning circle to the ending position of the turning manoeuvre.

Table 1 Survey region Recorded velocities, Location, and Pedestrian velocities data

Up direction pedestrian flow		Down direction pedestrian flow	
location	velocity (m/s)	Location	velocity (m/s)
a1	1.34	a2	1.26
b1	1.21	b2	1.19
c1	1.19	c2	1.13
d1	1.24	d2	1.16
e1	1.28	e2	1.21

Research and improvement of pedestrians' social force model in the turning region of a subway station passageway. Based on the analysis regarding the microscopic behaviour characteristics of pedestrian flow in the passageway turning region in the section 2, this section further studies and improves the pedestrian social force model.

Existing pedestrian social force model of the passageway turning region. Charitha model (Charitha et al, 2014) added the resistance and centrifugal forces to the turning force in the social force model, as shown in Formula 1.

$$m_i \frac{d\vec{v_t}}{dt} = \vec{F}_{desired.t} + \vec{F}_{wall.t} + \vec{F}_{perdestrian.t} + j\,(\vec{F}_{turning.t} + \vec{F}_{centrifugal.t})\cdot \qquad (1)$$

In the above formula, m_i represents the pedestrian weight; $\vec{F}_{desired.t}$ denotes the pedestrian self-driving force; $\vec{F}_{wall,t}$ is the psychological force between the pedestrians and the obstacle (wall); $\vec{F}_{perdestrian.t}$ is the pedestrians interaction force (because the Charitha model simulates the behaviour of a single pedestrian, the pedestrians interaction force is ignored in the single simulation.); $\vec{F}_{turning.t}$ denotes the resistance on the pedestrians at the turning region that is in the opposite direction of the expected direction; $\vec{F}_{centrifugal.t}$ is the centrifugal force on the pedestrians at the turning region ; and j is the coefficients of the piecewise function (1 indicates that the pedestrian is at the turning region, and 0 indicates the pedestrian is on the straight line).

Charitha model indicated that there are four forces that influence a single pedestrian i in the process of turning(pedestrian self-driving force, the psychological force between pedestrians and the wall, pedestrian resistance and centrifugal force), whereas in the traditional social force model of a single pedestrian walking, the pedestrian is only affected by the first two forces. A pedestrian's

walking process contains three stages: going straight, turning, and going straight again, and only in the process of turning would pedestrians be affected by the resistance and centrifugal force. Fig. 6 shows all of the forces that a single pedestrian would experience in a passageway turning area.
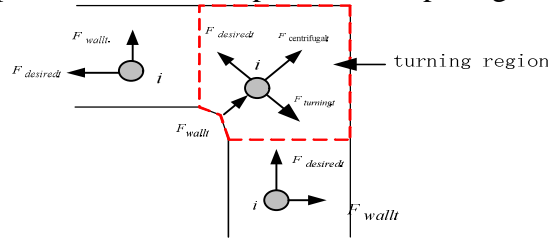

Fig. 6. The forces experienced by a pedestrian in a passageway turning area.

According to Charitha model, in the curve connecting two straight passageways, pedestrian i uses the starting position of turning as the pedestrian baseline until the end of turning, and the angle of turning for the pedestrian is 90 degrees. At this time, the direction of the intended speed of pedestrians from the initial A direction to the B direction, where directions A and B are perpendicular to each other, as shown in Fig. 7.
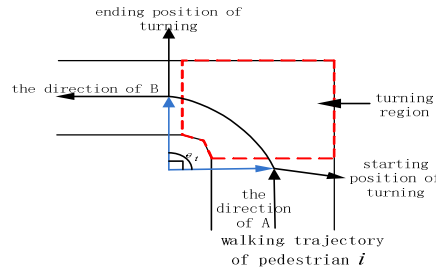

Fig. 7. The change of trajectory in the pedestrian turning process in the Charitha model

Charitha model defined that a pedestrian walks in accordance with the angular velocity w in the process of turning, as calculated in Formula 2. Because the angular velocity changes uniformly, the value of W is constant.

$$\frac{dq}{dt} = w.$$ （2）

where q is the angle at which the pedestrian turns around the starting point; w is a constant; and t represents time. The angle through which pedestrian i passed is determined by Formula 3.

$$\int_{q_t}^{q_{t+\Delta t}} dq = \int_t^{t+\Delta t} w dt .$$ （3）

where q_t is the turning angle from the time t to t+Δt, with the starting point of turning for pedestrians used as the reference line. The change of pedestrians intended direction is from q_t to $q_{t+\Delta t}$.

In Formula 3, $q_{t+\Delta t}$ is defined by Formula 4.

$$q_{t+\Delta t} = q_t + \Delta t \times w.$$ （2）

in the process of turning, the changes in the pedestrian's expected speed corresponds to the pedestrian turning angle, as shown in Formula 5, which indicates the speed from the starting position ($q_t=0$). As the angle q_t increases, the velocity in the $V_{des} \sin q_t$ direction (the beginning of the expected speed direction for pedestrian i is A) increases from 0 to V_{des}, and the $V_{des} \cos q_t$ velocity direction (the expected speed direction for pedestrian i is B after the ending of turning) decreases from V_{des} to 0.

$$v_{des,t} = \begin{pmatrix} V_{des} \sin q_t \\ V_{des} \cos q_t \end{pmatrix}.$$ （3）

where $V_{des,t}$ denotes the value of the expected speed.

The self-driving force of pedestrian i , as shown in Formula 6, is also changing in the process of turning, which drives pedestrian i to complete the turning behaviour.

$$\vec{F}_{desired.t} = m_i \bullet (\vec{v}_{des.t} - \vec{v}_t)/t .$$ （4）

Where $\vec{v}_t$ is the actual speed of the pedestrian at time t and t is the response time, reflecting the effects of acceleration and deceleration of the pedestrian.

The resistance influences pedestrians in the process of turning, as shown in Formula 7: the strength of the resistance corresponds to the actual speed at the starting point of turning and the angle from the starting point of turning. The higher the pedestrians' actual speed and the higher angle of turning, the greater is the resistance faced by pedestrians. Because of the resistance, the speed of the turning process has a downward trend.

$$\vec{F}_{turning.t} = A \bullet \exp(d \bullet q_t)\vec{n}_l .$$ （5）

where A is a parameter of the social force model that influences the pedestrian speed; d is a parameter of the social force model that influences the expected angle; and $\vec{n}_l$ is a unit vector pointing in the opposite direction of the expected velocity direction.

The whole turning process of a pedestrian is similar to the movement of Newtonian particles around the centre. Considering this situation, Charitha model added the centrifugal force into the model to describe why there is always a distance between a pedestrian and the turning arc in the process of turning; this force is given by Formula 8.

$$\vec{F}_{\text{centrifugal}.t} = k \bullet \left|\vec{V}_t\right|^2 \bullet \vec{n}_p .$$ （6）

where k is the parameter of the model; k is a constant; $\vec{n}_p$ is the unit vector pointing in the direction perpendicular to pedestrian's walking direction; and $\vec{V}_t$ denotes the speed of pedestrian at time t. Under the action of the centrifugal force, the walking trajectory of pedestrian in the process of turning is an arc shape.

Improvement of the existing passageway turning model. According to the analysis of the pedestrian turning behaviour in the second section, the existing model is improved in this part.

The forces that influence the behaviour of the single pedestrian i in the turning region can be divided into two stages. The first stage is before the pedestrian arrive at the centre of turning circle ($q_t \leq p/4$). Pedestrian i is influenced by the self-driving force, resistance, gravity and the psychological force that results from the interaction between pedestrian and wall. In the second stage, pedestrians are no longer affected by gravitation and resistance after they pass the centre of the circular arc ($q_t > p/4$), and they are only affected by the self-driving force and the psychological force between the pedestrian and the wall, as shown in Fig. 8.
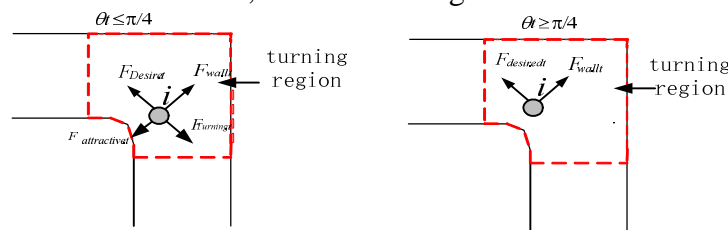

Fig. 8. The forces influence pedestrian in the turning stage

To describe the situation mentioned above, the piecewise function j_1 is introduced in the existing model, as shown in Formula 9, where j_1 is determined by the angle q_t .

$$\begin{cases} j_1=1, & q_t \leq p/4 \\ j_1=0, & q_t \geq p/4 \end{cases} .$$ （9）

After adding the piecewise function in Formula 7, the updated formula is given as follows:

$$\vec{F}_{turning.t} = j_1 A \bullet \exp(d \bullet q_t)\vec{n}_l .$$ （10）

In the process of turning, a pedestrian tends to choose the shortest path; when a pedestrian arrives at the turning centre point, the distance between the pedestrian and the turning arc is the shortest. After passing the turning arc, a pedestrian would gradually move away from it because of the self-driving force. Based on this feature, the centrifugal force (Formula 8) in the existing model was removed, and the gravitational formula was introduced in this article. As shown in formula 11, the strength of the gravity is related to the magnitude of angle q_t and the distance between pedestrian i and the turning arc; meanwhile, at the beginning of the turning, pedestrian i experience the highest level of gravity. As the angle q_t gradually increases and the distance $d_{i,wall}$ to the arc gradually decreases, the gravity felt by pedestrian i gradually decreases, and the direction of the gravity is as shown in Fig. 9. From the beginning of the turning process around the centre of the turning arc, a pedestrian is still influenced by the gravity; thus, we added the piecewise function in the gravity formula, as shown in Formula 11.

$$\vec{F}_{attractive.t} = j_1 k_{attractive} \bullet d_{i,wall} \bullet \cos(q_t) \bullet \vec{n}_i . \tag{7}$$

where $k_{attractive}$ is the gravity coefficient; $\vec{n}_i$ is the unit vector, the direction of which is perpendicular to the expected direction and points to the inside of the pedestrian trajectory; and q_t is the turning angle from the starting position of the turning process.
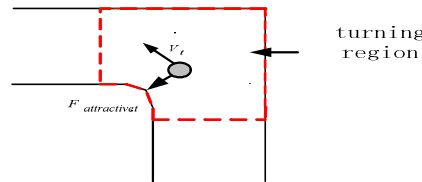

Fig. 9. The gravitational force of the pedestrian in the turning region

In the process of approaching the turning centre of the arc, pedestrians are subject to the self-driving force and constantly change their direction of walking until they arrive at the arc centre. In this process, the pedestrian's target direction can be determined by Formula 12.

$$\vec{e}_i = \begin{pmatrix} \sin q_t \\ \cos q_t \end{pmatrix}. \tag{8}$$

where $\vec{e}_i$ denotes the pedestrian's target direction.

According to the driving force of pedestrian i, as shown in Formula 13, the driving force of the pedestrian is continuously changing throughout the process of turning, thus driving pedestrian i to complete the turning behaviour.

$$\vec{F}_{desired.t} = m_i \bullet (V_{des} \bullet \vec{e}_i - \vec{v}_t) / t . \tag{9}$$

where V_{des} represents the pedestrian's expected speed; $\vec{v}_t$ is the actual speed of the pedestrian at time t; and t is the response time of the pedestrian, reflecting the effects of the acceleration and deceleration of the pedestrian.

After passing the centre of the circle, pedestrians are no longer affected by the gravity and resistance. Because of the influence of the expectation effect, pedestrians tend to accelerate their speeds; at this stage, the target point of each pedestrian is the ending position of turning, $P_{f(x,y)}$, and the direction of target point can be determined by the formula 14.

$$\vec{e}_i = (P_{f(x,y)} - P_{(x,y)}) / \left\| P_{f(x,y)} - P_{(x,y)} \right\|. \tag{14}$$

where $P_{f(x,y)}$ is the ending position of turning of the pedestrian and $P_{(x,y)}$ is the actual position of pedestrian at the moment. After the improvement, the forces that influence pedestrian behaviour in the turning regions are transferred from formula 1 to formula 15:

$$m_i \frac{d\overset{u}{v_t}}{dt} = F_{desired.t} + F_{wall.t} + F_{perdestrian.t} + j_1(F_{turning.t} + F_{attractive.t}) \; . \qquad （15）$$

Analysis of the simulation of pedestrians' social force model in a subway station passageway. This part involves the updated programs of the pedestrian social force model in a subway station passageway turning region. To verify the validity of the model, we selected reasonable simulation parameters: the research region in Xizhimen subway station was selected as the research background, and simulation programs were used to simulate the pedestrian flows in the turning region.

Parameter Setting of Pedestrian Social Force Model in passageway

(1)Setting the pedestrian attribute information

According to previous studies of pedestrian simulation parameters [9,10], in this paper, we assume that the pedestrian weights and radius are normally distributed.

(2)Selecting the parameters in social force model

According to the relevant research studies[10], the main parameters we selected for use in the improved social force model.

(3)Simulation environmental parameters

For the bi-direction passageway region of Xizhimen subway station, which we used as the simulation environment in this study, the length and width are each five meters, and the shape of the arc is circular, as shown in the Fig. 10.
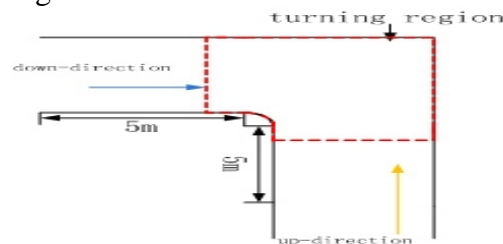

Fig. 10. The design of the simulation environment in the passageway with the turning region

Analysis of simulated pedestrian flow in the turning region of a subway station passageway. The following parameters were used in the simulation of the bi-direction pedestrian flows: the number of simulation steps is three hundred, the length of each step is 0.2s, and the arrival rate is 200 people per minute. The following is the analysis regarding the simulation of the bi-direction pedestrian flows.

(1)Analysis of the behavioural characteristics of the bi-direction pedestrian flow

Fig. 11 shows the turning process of the bi-direction pedestrian flows. Fig. 11(a) shows the turning process of pedestrian 33, who is moving in the down direction. The first Fig. shows the position at which pedestrian 33 starts to turn, and the distance between pedestrian 33 and the turn arc is 0.9m. The second Fig. shows that pedestrian 33 is at the centre of the turning arc and that the distance between pedestrian 33 and the turn arc is 0.5m. The third Fig. shows the position at which pedestrian 33 completes the turning process; the distance between the pedestrian 33 and the turn arc is 0.8m. Fig.11 (b) shows the turning process of pedestrian 35, who is moving in the up direction. The first Fig. shows the position at which pedestrian 35 starts to turn, and the distance between pedestrian 35 and the turn arc is 2.8 m. The second Fig. shows that pedestrian 35 is at the centre of the turning arc and that the distance between pedestrian 35 and the turn arc is 1.5 m. The third Fig. shows the position at which pedestrian 35 ends the turn; the distance between pedestrian 33 and the turn arc is 2.1m.

It can be concluded that the turning process of bi-direction pedestrian flows is the same as that in one-way pedestrian flow, which is consistent with the feature that the pedestrian gradually approaches the turning arc until arriving at the centre of the arc, at which point, the distance between the pedestrian and the turning arc is the smallest. After passing the arc centre, pedestrians gradually leave the turning arc. When the pedestrians reach the end of the turning process, the pedestrians would

continue moving because of the herd mentality. There is a slight deviation in the distance between the start point and the end point because a single pedestrian is influenced by the pedestrian flow.
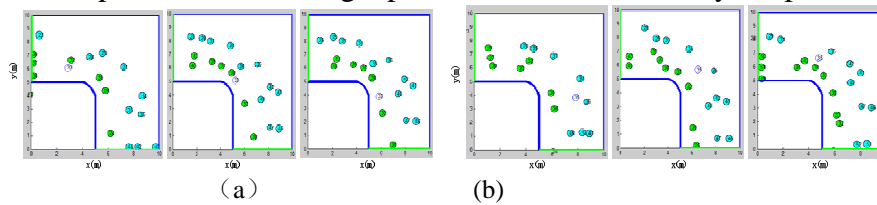

（a）　　　　　　　(b)

Fig. 11. Turning procedure of pedestrians

For pedestrian 33, who is moving in the down direction, the steps at the start point and at the ending point are 36 and 49, respectively. According to the changes in pedestrian's instantaneous velocity, the speed of pedestrian 33 at the starting position of the turning process, the turning arc centre, and the ending position of the turning process is $0.95\,m\ /\ s$, $0.59\,m\ /\ s$, and $0.94\,m/s$, respectively, as shown in Fig. 12, the steps of pedestrian 35, who is moving in the up direction, are 41 and 51 at the beginning and ending positions, respectively. According to the changes in pedestrian instantaneous velocity, the speed of pedestrian 35 at the starting position of the turning process, at the turning arc centre, and at the ending position of turning is $1.2\,m\ /\ s$, $0.87\,m\ /\ s$, and $1.49\,m/s$, respectively, as shown in Fig. 12. Fig. 12 reveals that the changes in the tendency regarding the bi-direction pedestrians are consistent with the results concluded in the second part.
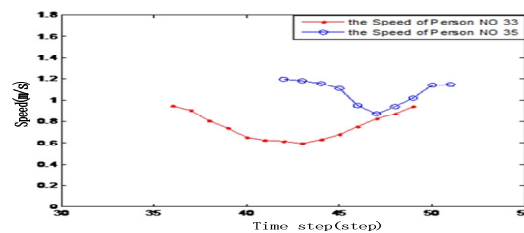

Fig. 12. Variation of the speed of pedestrian in the turning region

(2) Analysis of the conformity phenomenon of the pedestrian flow

After simulating 30 steps of the pedestrian flow, the pedestrian flow is self-organized into two directions because of the herd mentality of the pedestrians moving in the same direction. Fig. 13 shows the simulation results after the pedestrian flow is divided into two parts. The up direction flow of pedestrians walking is approach the down direction flow of pedestrians walking in the process of turning; the down direction pedestrian flow approach the turning circle in the process of turning, which reduces the possibility of collision between pedestrian flows. The pedestrians moving in the two directions are influenced by the herd mentality, which is consistent with the investigation regarding the turning region in Xizhimen subway station mentioned in the second section.
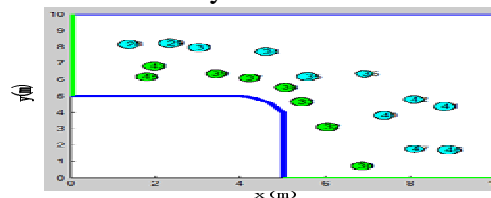

Fig. 13. Two directions of pedestrians flow have come into being

Conclusion

In this article, we analysed the microscopic traffic characteristics of pedestrians in a turning region of the Xizhimen subway station passageway in Beijing and concluded that the pedestrian usually turn approach the turning arc at the turning region, the speed during the turn decreases initially and then increases. Moreover, we improved the existing social force model by removing the centrifugal force from the original model and adding the gravitational force from the starting point of the turning to the centre line of the turning arc in the model. To describe the changes in speed and gravitation in the

process of turning of a pedestrian, the piecewise function j_1 was introduced, and the gravitation coefficient $k_{attractive}$ was calibrated. To verify the validity of the improved model, we programmed the model for existing and improving model to simulate the behaviours of the individual pedestrian and pedestrian flows in the turning region. The simulation results show that improving model is more consistent with the reality than exiting model in the research turning region.

References

[1] Kretz, T., The use of dynamic distance potential fields for pedestrian flow around corners. International Conference on Evacuation Modeling and Management. 2009: 353–365.

[2] Charitha, D., Omid, E., Majid, S., Martin, B., Exploring pedestrian walking through angled corridors. Transportation Research Procedia. 2014(2): 19-25.

[3] Charitha, D., Omid, E., Majid, S., Nirajan, S., Pedestrian Walking Characteristics Through Angled Corridors. Transportation Research Board Record: Journal of Transportation Research Board. 2014（2421): 41-50.

[4] Guo, R.Y., Tang, T.Q., A simulation model for pedestrian flow through walkways with corners. Simulation Modelling Practice and Theory. 2012(1): 103-113.

[5] Jian, X.X., Wong, S. C., Zhang, P., Choi, K., Li, H., Zhang, X., Perceived cost potential field cellular automata model with an aggregated force field for pedestrian dynamics. Transportation Research Part C: Emerging Technologies 42(2014): 200-210.

[6] Li, S., Li, X., Qu, Y., Jia, B., Block-based floor field model for pedestrian's walking through corner. Physica A 432 (2015): 337–353.

[7] Charitha, D., Majid, S, Nirajan, S.. Martin, B., Turning Angle Effect on Emergency Egress Experimental Evidence and Pedestrian Crowd Simulation. Transportation Research Board Record: Journal of Transportation Research Board. 2015（2312): 120-127.

[8] Nirajan, S., Gong, Y.S., Shi, X.M., Ye. Z.R., Examining influence of merging architectural features on pedestrian crowd movement. Safety Science, 2015(75): 15–22.

[9] Daniel, P., Gilman, M., Moldovan, H., A modification of the Social Force Model can reproduce experimental data of pedestrian flows in normal conditions. Physica A. 2009(388): 3600-3608.

[10] Yu, Q,. Improving pedestrian social force model based on pedestrians predicting theory in the rail transit station. Beijing Jiaotong University, 2015.