Application of Fabry perot interferometer and error analysis

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Abstract. According to the theory of multiple-beam interference, the difference of sodium wavelength can be measured by using fabry-perot interferometer. After experiment, unary linear regression and uncertainly calculation has been down based on the raw data. At last we do qualitative and quantitative error analysis by our own sights.

Introduction.

Fabry perot interferometer（Fig.1） makes use of multiple reflections between two closely spaced partially silvered surfaces. Part of the light is transmitted each time the light reaches the second surface, resulting in multiple offset beams which can interfere with each other. The large number of interfering rays produces an interferometer with extremely high resolution, somewhat like the multiple slits of a diffraction grating increase its resolution.All manuscripts must be in English, also the table and figure texts, otherwise we cannot publish your paper.

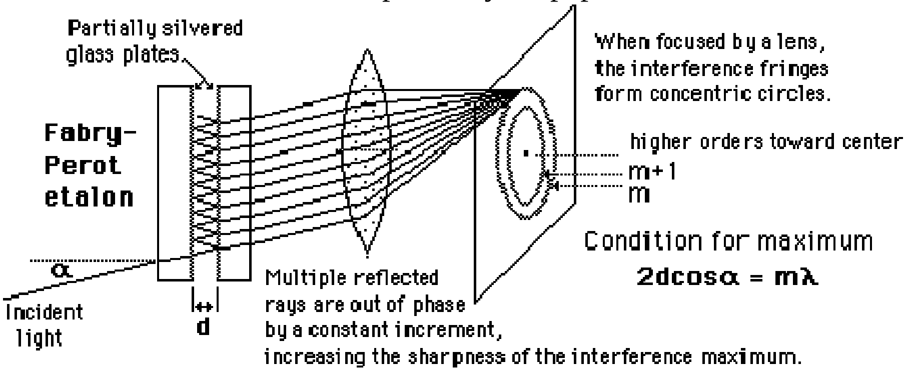

Fig. 1 Fabry perot interferometer

Measuring the differences of sodium wavelength.

We use sodium lamp as lighting source, then make sure reflectors P1 and P2 on F-P interferometer are strictly parallel. After that, fringes of equal inclination would be found on the screen. After a series of calculation, we finally get the result of wavelength differential between sodium lamp.

The experiment bases on the theory of multi-beam interference. Stripe nesting is our standard to confirm the wavelength differential between sodium lamp. We assume that wavelength of sodium lamp is λ_1 and λ_2, while $\lambda_1 > \lambda_2$. When the fringes get nested, we assume that interference fringe of λ_1 named level k_1 and interference fringe of λ_2 named level k_2+1. We have the result that

$$2d\cos\theta=k_1\lambda_1=(k_2+0.5)\lambda_2 \tag{1}$$

d means thickness of the air layer.

While $d\to d+\Delta d$, we find the nester phenomenon again. If $k_1\to k_1+\Delta k$ now, so $k_2+0.5\to k_2+0.5+\Delta k+1$.

We have the result that

$$2(d+\Delta d)\cos\theta=(k_1+\Delta k)\lambda_1=(k_2+0.5+\Delta k+1)\lambda_2 \tag{2}$$

Subtract from equation （1） and equation 2 then we get the result

$2\Delta d \cos\theta=\Delta k\, \lambda_1=(\Delta k+1)\, \lambda_2$ (3)

So

$1/\Delta k=\lambda_1/(2\Delta d \cos\theta)$，$\lambda_1-\lambda_2=\lambda_2/\Delta k$ (4)

So

$\Delta\lambda=\lambda_1-\lambda_2=(\lambda_1\lambda_2)/(2\Delta d \cos\theta)\approx\bar{\lambda}^2/(2\Delta d)$ （5）

We use F-P interferometer to accomplish the experiment. It is modified by Michael interferometer. While P2 plate position is fixed, P1 can be moved by turning the rough wheel or the handwheel to change the spacing of the plate. Recording p1's position as d1 when nested phenomenon appeared, then moving p1 and repeat the operation 10 times.

Data processing.

Table1 Position of P_1

i	1	2	3	4	5
d_i/mm	28.94101	29.22619	29.50848	29.79770	30.08108
i	6	7	8	9	10
d_i/mm	30.37569	30.70042	30.98622	31.29002	31.58570

From the introduction

$$d_i = \frac{\bar{\lambda}^2}{2\Delta\lambda} i + d_0 \quad (6)$$

let

$$i \equiv X, d_i \equiv Y, b \equiv \frac{\bar{\lambda}^2}{2\Delta\lambda}$$

based on the raw data

$$\bar{X} = 5.5, \bar{Y} = 3.0248 * 10^{-2} m$$

so

$$\overline{X^2} = 38.5, \overline{Y^2} = 9.156779 * 10^{-4} m^2, \overline{XY} = 0.1687936 m$$

so

$$b = \frac{\overline{XY} - \bar{X}\bar{Y}}{\overline{X^2} - \bar{X}^2} = 0.2943 * 10^{-3} m \quad (7)$$

The standard sodium wavelength that we use is $589.3 * 10^{-9} m$

So

$$\Delta\lambda = 5.900008325 * 10^{-10} m$$

The correlation coefficient is

$$r = \frac{\overline{XY} - \bar{X}\bar{Y}}{\sqrt{(\bar{X}^2 - \overline{X^2})(\bar{Y}^2 - \overline{Y^2})}} = 0.9998 \approx 1 \quad (8)$$

Therefore, the curve is highly linearly dependent. And then we calculated the uncertainty of $\Delta\lambda$

A type uncertainty of linear regression coefficient b is

$$u_a(b) = b\sqrt{\frac{1}{k-2}\left(\frac{1}{r^2}-1\right)} = 1.47164 * 10^{-6}m \tag{9}$$

B type uncertainty of linear regression coefficient b is

$$u_b(b) = \frac{0.00005}{\sqrt{3}}\mathrm{mm} = 2.8867 * 10^{-8}\mathrm{m} \tag{10}$$

Uncertainly of linear regression coefficient b is

$$u(b) = \sqrt{u_a^2(b) + u_b^2(b)} = 1.4717 * 10^{-6}m \tag{11}$$

Uncertainly of $\Delta\lambda$ is

$$\mathrm{u}(\Delta\lambda) = -\Delta\lambda\frac{\mathrm{u(b)}}{\mathrm{b}} = -2.9927112 * 10^{-12}\mathrm{m} \tag{12}$$

So, the result is expressed as

$$\Delta\lambda \mp u(\Delta\lambda) = (5.90 \mp 0.03) \times 10^{-10}m \tag{13}$$

Error analysis. We mainly discuss the influence of $\Delta\lambda$ from selection of di's position .It is obviously that P_1 and P_2 is not strictly parallel. We assume that the angle between P_1 and P_2 is α. When length become P_1 and P_2, named d, become larger, the force binding between P_2 and guideway become worse, that means α would become larger because the device is in poor parallelism now. According to $\Delta\mathrm{d} = \frac{\lambda^2}{2\Delta\lambda cos\alpha}$, $\Delta\mathrm{d}$ will become bigger when α gets larger.To prove the derivation above, we take two empirical data that we have done before.

Table2

i	1	2	3	4	5	6	7	8	9	10
di	28.94101	29.22619	29.50848	29.7977	30.08108	30.37569	30.68042	30.98622	31.29002	31.5857
di+1-di	0.28518	0.28229	0.28922	0.28338	0.29461	0.30473	0.3058	0.3038	0.29568	
di ‘	26.87529	27.16703	27.45817	27.74742	28.03438	28.32975	28.62105	28.91017	29.20575	29.49214
di+1'-di'	0.28374	0.29114	0.28925	0.28996	0.29537	0.2913	0.28912	0.29558	0.29639	

Then we make the figure by d_i and d_{i+1}

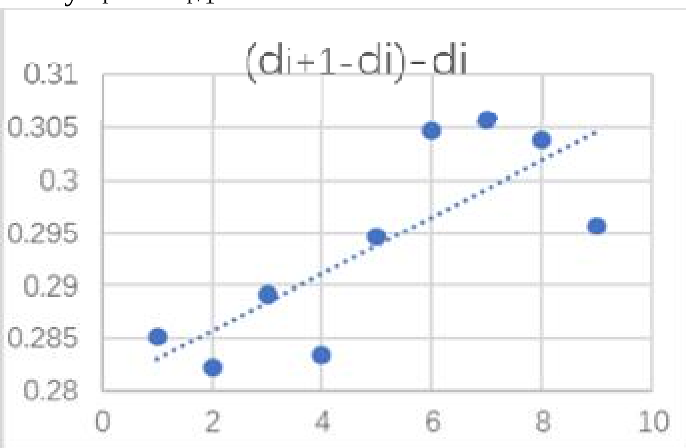

Fig 2. Relationship between d_i and d_{i+1}-d_i

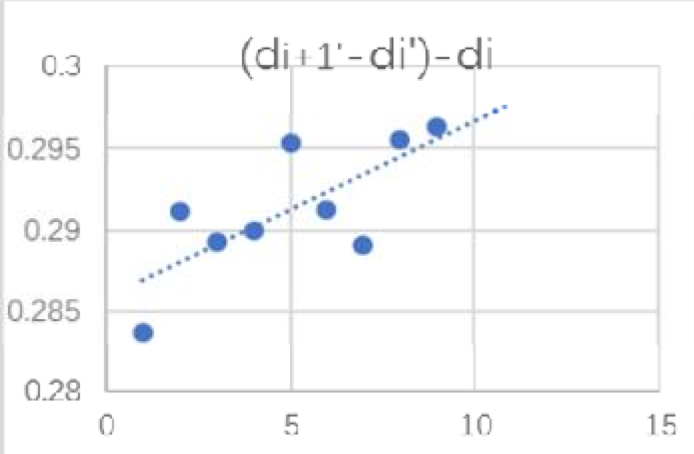

Fig 3. Relationship between d_i’ and d_{i+1}’-d_i’

Conclusions.

At first, from the figure, we can see that when d_i becomes large, Δd also become large. Secondly, comparing di from two tables, we find that Δd＞Δd’ while di＞di’. So the theoretical predictions confirmed.

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