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H_∞ Consensus Control of Multi-Agent Systems: a Static Output Feedback Controller with Time-Delay

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***Abstract*—This paper addresses the H_∞ consensus problem for networks of multi-agent systems subject to external disturbances. In particular, a distributed static output feedback controller with time-varying delays is provided, which based on the relative out-puts of neighboring agents. By using Schur complement lemma, Newton Leibniz formula and Jensen's inequality, sufficient conditions are derived to guarantee that consensus is achieved under an undirected communication topology, which are in terms of linear matrix inequalities (LMIs). And meanwhile the parameter of controller is also determined. Finally, a numerical example is presented to illustrate the effectiveness of the proposed method.**

Keywords- consensus; multi-agent systems; static output control; time-delay; external disturbances

I. INTRODUCTION

During the past decades, the decentralized cooperative control of multi-agent systems has attracted increasing attention from physicists, biologists, social scientists and control scientists, such as unmanned air vehicles, flocking, swarming and formation control [1]. Consensus and stability are widely accepted to investigate the agents' dynamical behaviors, and meanwhile which are also premise and fundamental problem of the cooperative control for complex systems [9,10,12]. Consensus means that all the agents' states converge to the same value, based on their interaction or feedback protocol. There have been many related results regarding the consensus problem from different viewpoints including various agents' dynamics, constraints on network topologies, control strategies and design methods. Many aspects of consensus algorithms are considered for multi-agent networked systems [4]. A consensus protocol for a linear multi-agent system is designed [6]. The consensus problem for second-order multi-agent systems with a dynamical leader is analyzed [13].

Recently the H_∞ performance index has been considered for the consensus problem of multi-agent systems. Wang et al investigated the problem of distributed reliable H_∞ consensus control for high-order networked agent systems with actuator faults [15]. Han et al presented an event-triggering protocol with random parameter for H_∞ consensus filtering problem [14]. Whereas, the method generally adopted state feedback protocols or controllers. It's well known that in many control problems not all state information can be obtained. Thus, the output feedback controller is more attractive in engineering. Qiu et al investigated the problem of robust H_∞ output-feedback control for a class of uncertain nonlinear systems based on a Takagi-Sugeno (T-S) fuzzy model [8]. The mixed H_∞ and passive control problem was studied for systems with time delays [17]. Zhang et al examined the consensus problem of static output feedback control for high-order dynamical systems [11]. A distributed static output feedback was proposed for the synchronization of multi-agent systems subject to external disturbances [18]. Liu et al proposed a dynamic output feedback protocol to deal with the H_∞ consensus problem [7]. The problem of non-fragile reduced-order dynamic output feedback H_∞ control for both continuous and discrete-time switched systems was concerned [16]. Nevertheless, neither [8] nor [17] investigated the consensus algorithms of multi-agent network systems. Reference [11] and [18] hadn't taken H_∞ problem into consideration. Reference [7] and [16] didn't consider the interference caused by time delays. To the best of our knowledge, there has been seldom research on using static output feedback controllers or protocols with time delays for H_∞ consensus problems of multi-agent network systems.

Motivated by the above discussion, we aim to investigate H_∞ consensus control problem for undirected networks subject to external disturbances. A distributed static output feedback controller with time-varying delays and interconnection of agents' outputs is adopted. Meanwhile, a measurable output is chosen. Then, by model transformation, the consensus problem is converted into the stability problem. By choosing an appropriate Lyapunov-Krasovskii function, sufficient conditions are given, and a controller parameter matrix is obtained simultaneously. Finally, a numerical example is presented to illustrate the efficiency and applicability of the proposed methods.

II. PROBLEM STATEMENT

Consider a multi-agent system consisting of n identical agents, which is characterized by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1\omega_i(t) + B_2u_i(t), \\ y_i(t) &= Cx_i(t), \end{aligned} \tag{1}$$

where $x_i(t) \in \Re^m$ is the state, $\omega_i(t) \in \Re^{m1}$ is the external disturbances that belongs to $\mathcal{L}_2[0,\infty)$, $u_i(t) \in \mathbb{R}^{m_2}$ is the control input or protocol, and $y_i(t) \in \mathbb{R}^{m_3}$ is the output. $A \in \mathbb{R}^{m\times m}$, $B_1 \in \mathbb{R}^{m\times m_1}$, $B_2 \in \mathbb{R}^{m\times m_2}$, and $C \in \mathbb{R}^{m_3\times m}$ are given matrices.

Definition 2.1 For the given initial state $x_i(0)$, if and only if $x_i(t), i = 1,2,\dots,n$ satisfies

$$\lim_{t\to\infty} \| x_i(t) - x_j(t) \| = 0, \forall i, j = 1,2,\dots,n, \tag{2}$$

then the system (1) reaches consensus.

Under the influence of external disturbances, it is hard to achieve the accurate consensus defined in (2). For the sake of analyzing the effect of disturbance to the consensus problem quantitatively, we attempt to design a controlled output function $z_i(t) \in \mathbb{R}^m$ to measure the interference of $x_i(t)$ to the average state of all agents. It is as follows

$$z_i(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t), \tag{3}$$

and with the network system of n agents, $z_i(t), i = 1,2,\dots,n$ can be written as

$$Z(t) = (L_c \otimes I_m) X(t), \tag{4}$$

where $X(t) = [x_1^T(t) \quad x_2^T(t) \quad \cdots \quad x_n^T(t)]^T$, $Z(t) = [z_1^T(t) \quad z_2^T(t) \quad \cdots \quad z_n^T(t)]^T$ and

$$L_c = \begin{cases} \frac{n-1}{n}, i = j, \\ -\frac{1}{n}, i \neq j. \end{cases} \tag{5}$$

Note that if $Z(t) = 0$, then $x_i(t) = \frac{1}{n}\sum_{j=1}^{n} x_j(t)$. That is, the network system has reached consensus as (2).

Thus, the attenuating ability of the multi-agent system against external disturbances can be quantitatively measured by

$$\| T_{z\omega} \|_\infty = \sup_{0 \neq \omega(t) \in L_2[0,\infty)} \frac{\| Z(t) \|_2}{\| \omega(t) \|_2} < \gamma, \tag{6}$$

where $\omega(t) = [\omega_1^T(t) \quad \omega_2^T(t) \quad \cdots \quad \omega_n^T(t)]^T$, $T_{Z\omega}(s)$ represents the closed-loop transfer function matrix from external disturbance $\omega(t)$ to controlled output $Z(t)$. $\gamma > 0$ is a given H_∞ performance index.

For this purpose, a distributed static output feedback controller is proposed based on the relative output of neighboring agents. Hence, there may be a delay in the procedure of information interactions and the applied controllers usually encounter difficulties regarding inaccuracies. Therefore, the following distributed static output feedback controller with time-varying delays is presented to guarantee consensus among agents in multi-agent network systems.

$$u_i(t) = K\Sigma_{j\in\mathbb{N}_i} a_{ij}(y_i(t-\tau(t)) - y_j(t-\tau(t))), i = 1,2,\dots,n, \tag{7}$$

where $K \in \mathbb{R}^{m_2 \times m_3}$ is undetermined parameter matrix, a_{ij} is the adjacency elements of interconnection graph , $\tau(t)$ satisfies $0 < \tau(t) < d$ and $\dot{\tau}(t) < h < 1$.

By Kronecker product, the system (1) with controller (7) can be rewritten as the following

$$\begin{aligned} \dot{X}(t) &= (I_n \otimes A)X(t) + (I_n \otimes B_1)\omega(t) + (L \otimes B_2 KC)X(t-\tau(t)), \\ Z(t) &= (Lc \otimes I_m)X(t), \end{aligned} \tag{8}$$

where L is the Laplacian matrix of graph, L_c is defined as (5).

Since the graph is undirected and connected, the number of zero eigenvalues of L is only one. Refer to [7], using model transformation $\hat{X}(t) = (U_L^T U^T L_C \otimes I_m)X(t)$, where U_L, U are orthogonal matrixs satisfy $[U_1 \quad U_2] \in \mathbb{R}^n$ and $U_2 = \frac{1_n}{\sqrt{n}}$ such that $U^T L_c U = \begin{pmatrix} I_{n-1} & 0_{n-1} \\ * & 0 \end{pmatrix}$. For the dimension reduction and decoupling, the consensus problem of multi-agent system is transforming into a stability problem of the following system which consists of $n-1$ independent agents.

$$\begin{aligned} \dot{\hat{x}}_i(t) &= E_i \hat{x}_i(t) + F_i \dot{\hat{\zeta}}(t) + B_i \hat{\omega}_i(t), \\ \dot{\hat{z}}(t) &= \hat{x}_i(t), \end{aligned} \tag{9}$$

where $= 1,2,\dots,n-1$, $E_i = A + \lambda_i B_2 KC$, $F_i = \lambda_i B_2 KC$ and $\hat{\zeta}_i(t) = \hat{x}_i(t-\tau(t)) - \hat{x}_i(t)$.

Remark 2.1 As a result, the system (8) is H_∞ consensus if systems (9) are H_∞ asymptotically stable. In other words, we transform the H_∞ consensus problem of networks system with external disturbance to the H_∞ stability problem of $n-1$ systems.

Remark 2.2 According to the definition of H_∞ performance (6), if $\|T_{\hat{z}\hat{\omega}}\|_\infty < \gamma$, then $\|T_{z\omega}\|_\infty < \gamma$. Through orthogonal transformation, it is readily verified that the system (8) is H_∞ consensus if the $n-1$ systems (9) are all asymptotically stable and with the H_∞ index .

One lemma will be introduced which are needed in the next section.

Lemma 2.1 [2] For arbitrary real differentiable vector functions $(t) \in \mathbb{R}^n$, time delay $\tau(t)$ with $0 < \tau(t) < t$ and constant matrix $W = W^T$, the following inequality is established

$$\frac{1}{d}[x(t) - x(t-\tau(t))]^T W [x(t) - x(t-\tau(t))]$$

$$\leq \int_{t-\tau(t)}^{t} \dot{x}^T(s) W \dot{x}(s) ds,$$

where $t \geq 0$.

III. CONSENSUS FOR LINEAR NETWORK SYSTEMS

In this section, the consensus conditions of linear network systems are derived.

Theorem 3.1 For positive constants d, h, if there exist the positive definite symmetric matrix P, Q, R, an appropriate matrix K and

$$\Theta = \begin{pmatrix} E_i^T P + PE_i + hQ + I & PF_i - (1-h)Q & PB_1 & \sqrt{d}E_i^T R \\ * & -(1-h)Q - \frac{1}{d}R & 0 & \sqrt{d}F_i^T R \\ * & * & -\gamma^2 & \sqrt{d}G_i^T R \\ * & * & * & -R \end{pmatrix} < 0, \tag{10}$$

are satisfied for $i, i = 1,2,\dots,n-1$, then systems (9) are H_∞ asymptotically stable and system (8) achieves H_∞ consensus.

Proof For system (9), we choose the following Lyapunov-Krasovskii function

$$V(t, \hat{x}_i(t), \hat{\varsigma}_i(t), \hat{\omega}_i(t)) = \sum_1^3 V_j(t, \hat{x}_i(t), \hat{\varsigma}_i(t), \hat{\omega}_i(t)), \tag{11}$$

where $V_1\left(t, \hat{x}_i(t), \hat{\zeta}_i(t), \hat{\omega}_i(t)\right) = \hat{x}_i^T(t)P\hat{x}_i(t)$,

$$V_2\left(t, \hat{x}_i(t), \hat{\zeta}_i(t), \hat{\omega}_i(t)\right) = \int_{t-\tau(t)}^{t} \hat{x}_i^T(s)Q\hat{x}_i(s)ds,$$

$$V_3\left(t, \hat{x}_i(t), \hat{\zeta}_i(t), \hat{\omega}_i(t)\right) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\hat{x}}_i^T(s)R\dot{\hat{x}}_i(s)dsd\theta,$$

P, Q and R are to stay with the proper dimensions for symmetric positive definite matrixs.

Obviously, $V\left(t, \hat{x}_i(t), \hat{\zeta}_i(t), \hat{\omega}_i(t)\right) > 0$. Then take the derivative of (11) along the solution of system (9) yields

$$\begin{aligned} \dot{V}_1 &= \dot{\hat{x}}_i^T(t)P\hat{x}_i(t) + \hat{x}_i^T(t)P\dot{\hat{x}}_i(t) \\ &= \hat{x}_i^T(t)(E_i^T P + PE_i)\hat{x}_i(t) + \hat{\zeta}_i^T(t)F_i^T P\hat{x}_i(t) \\ &\quad + \hat{x}_i^T(t)PF_i\hat{\zeta}_i(t) + \hat{\omega}_i^T(t)B_1^T P\hat{x}_i(t)PB_1\hat{\omega}_i(t), \\ \dot{V}_2 &= \hat{x}_i^T(t)Q\hat{x}_i(t) - \left(1-\dot{\tau}(t)\right)\hat{x}_i\left(t-\tau(t)\right)^T Q\hat{x}_i\left(t-\tau(t)\right) \\ &\le h\hat{x}_i^T(t)Q\hat{x}_i(t) - (1-h)\hat{\zeta}_i^T(t)Q\hat{\zeta}_i(t) \\ &\quad -(1-h)\hat{x}_i^T(t)Q\hat{\zeta}_i(t) - (1-h)\hat{\zeta}_i^T(t)Q\hat{x}_i(t), \\ \dot{V}_3 &= d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t) - \int_{t-\tau(t)}^{t} d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t)ds \end{aligned}$$

Using Lemma 2.1 and Newton-Leibniz formula, we have $\dot{V}_3 \le d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t) - \frac{1}{d}\hat{\zeta}_i^T(t)R\hat{\zeta}_i(t)$, so

$$\dot{V} = \xi_i^T(t)\Phi\xi_i(t) + d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t),$$

where $\xi_i(t) = [\hat{x}^T(t) \quad \hat{\zeta}_i^T(t) \quad \hat{\omega}_i^T(t)]$, $\Phi = \begin{bmatrix} E_i^T P + PE_i + hQ & PF_i - (1-h)Q & PB_1 \\ * & -(1-h)Q - \frac{1}{d}R & 0 \\ * & * & 0 \end{bmatrix}$.

As to $d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t)$,

$$\begin{aligned} d\dot{\hat{x}}_i^T(t)R\dot{\hat{x}}_i(t) &= d\xi_i^T(t)[E_i \quad F_i \quad B_1]^T R[E_i \quad F_i \quad B_1]\xi_i(t) \\ &= d\xi_i^T(t)\Lambda\xi_i(t). \end{aligned}$$

Considering $\dot{V} < 0$ if $\Phi + d\Lambda < 0$, using Schur complement [3], $\Phi + d\Lambda < 0$ if

$$\Phi_1 = \Phi + d\Lambda$$

$$= \begin{bmatrix} E_i^T P + PE_i + hQ & PF_i - (1-h)Q & PB_1 & \sqrt{d}E_i^T R \\ * & -(1-h)Q - \frac{1}{d}R & 0 & \sqrt{d}F_i^T R \\ * & * & 0 & \sqrt{d}F_i^T R \\ * & * & * & -R \end{bmatrix} < 0.$$

For any $T > 0$, consider the following cost function

$$J_T = \int_0^T \hat{z}_i^T(t)\hat{z}_i(t)dt - \gamma^2\int_0^T \hat{\omega}_i^T(t)\hat{\omega}_i(t)dt.$$

Under the zero initial condition $V(0) = 0$, we have

$$\begin{aligned} J_T &= \int_0^T [\hat{x}_i^T(t)\hat{x}_i(t) - \gamma^2\hat{\omega}_i^T(t)\hat{\omega}_i(t) + \dot{V}]dt - V(T) \\ &\le \int_0^T [\xi_i^T(t)(\Phi_1 + diag\{I, 0, -\gamma^2 I\})\xi_i(t)]dt - V(T) \\ &\triangleq \int_0^T \xi_i^T(t)\,\Theta\xi_i(t)dt - V(T). \end{aligned}$$

Therefore, if $\Theta < 0, J_T < 0$, *i.e.,*

$$\int_0^\infty \|\hat{z}(t)\|^2\,dt < \gamma^2\int_0^\infty \|\hat{\omega}\|^2 dt,$$

which completes the proof.

Noticing that the inequality (10) is bilinear according to matrix P, Q, R and K, the method of obtaining the undetermined feedback matrix K is presented in following by linear matrix inequalities. Without loss of generality, the matrix B_2 is with full column rank.

Theorem 3.2 For a given index $\alpha > 0$, and an H_∞ performance index γ if there exist positive definite symmetric matrix $\bar{P}$, $\bar{Q}$ and matrix $\bar{R}$ such that the following inequalities for $i = 1,2,\dots,n-1$

$$\begin{pmatrix} \Psi_1 & \lambda_i\bar{R}CH^{-1} - (1-h)\bar{Q} & \bar{P}HB_1 & \sqrt{d}\alpha A_0\bar{P} + \sqrt{d}\alpha\lambda_i C^T\bar{R}^T \\ * & -\frac{\alpha}{d}\bar{P} - (1-h)\bar{Q} & 0 & \sqrt{d}\alpha\lambda_i H^{-T}C^T\bar{R}^T \\ * & * & -\gamma^2 & \sqrt{d}\alpha B_1^{-T}H^T\bar{P} \\ * & * & * & -\alpha\bar{P} \end{pmatrix} < 0, \tag{12}$$

are simultaneously satisfied for all d and h, then systems (9) are H_∞ asymptotically stable and the system (8) achieves H_∞ consensus, where $\bar{P} = \begin{bmatrix} \bar{P}_1 & 0_{(m-m_2)\times(m-m_2)} \\ * & \bar{P}_2 \end{bmatrix} \in \mathbb{R}^m$, $\bar{Q} = H^{-T}QH^{-1}$, $\bar{R} = \begin{bmatrix} \bar{R}_1 \\ 0_{(m-m_2)\times m_3} \end{bmatrix} \in \mathbb{R}^{m\times m_3}$, $\psi_1 = A_0\bar{P} + \bar{P}A_0^T + \lambda_i\bar{R}CH^{-1} + \lambda_i H^{-T}C^T\bar{R}^T + h\bar{Q}$, $A_0 = H^{-T}A^T H^T$, $H \in \mathbb{R}^m$ is a nonsingular matrix with $H = \begin{bmatrix} (B_2^T B_2)^{-1}B_2^T \\ B_2^{T\perp T} \end{bmatrix}$, and controller parameter is given by $K = \bar{P}_1^{-1}\bar{R}_1$.

Proof Pre-multiplying and post-multiplying the inequality (10) with $\bar{V} = diag\{H^{-T}, H^{-T}, I_{m_1}, H^{-T}\}$ and $\bar{V}^T$ separately, we have

$$\begin{pmatrix} \Theta_1 & \Theta_2 & H^{-T}PB_1 & \Theta_3 \\ * & \Theta_4 & 0 & \alpha\lambda_i\sqrt{d}H^{-T}C^TK^TB_2^TPH^{-1} \\ * & * & -\gamma^2 I & \alpha\sqrt{d}B_1^TPH^{-1} \\ * & * & * & -\alpha H^{-T}PH^{-1} \end{pmatrix} < 0, \tag{13}$$

where $R = \alpha P$, $\alpha \in \mathbb{R}$, $\Theta_1 = H^{-T}(A^TP + PA)H^{-1} + \lambda_i H^{-T}(C^TK^TB_2^TP + PB_2KC)H^{-1} + hQ$, $\Theta_2 = \lambda_i H^{-T}PB_2KCH^{-1} - (1-h)H^{-T}QH^{-1}$, $\Theta_3 = \alpha\sqrt{d}H^{-T}A^TPH^{-1} + \alpha\lambda_i\sqrt{d}H^{-T}C^TK^TB_2^TPH^{-1}$, $\Theta_4 = -(1-h)H^{-T}QH^{-1} - \frac{\alpha}{d}H^{-T}PH^{-1}$.

The inequalities (12) are got with

$$\bar{P} = H^{-T}PH^{-1},$$

$$\bar{R} = \bar{P}HB_2K,$$

and this proof is completed.

IV. Numerical Example

This section presents a numerical example which demonstrates the validity of the method described above.

Example Consider the linear multi-agent system (1) with

$$A = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.4 & -1 & 0 \\ 5 & 1 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0.5 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix},$$

and the time-delay is $\tau(t) = 0.1\,sin(t)$, so $0 < d < 1$ and $h < 0.1$. All the information transmission relationships among agents are given by Fig.1. And the corresponding Laplacian matrix L is

$$L = \begin{bmatrix} 0.2 & -0.1 & -0.1 & 0 \\ -0.1 & 0.2 & 0 & -0.1 \\ -0.1 & 0 & 0.2 & -0.1 \\ 0 & -0.1 & -0.1 & 0.2 \end{bmatrix}$$

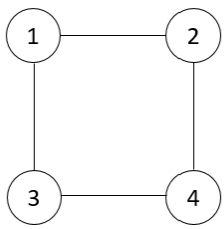

FIGURE I. UNDIRECTED INTERACTION GRAPH

The initial values of states are $x_1(0) = [10; -4; -2]$, $x_2(0) = [4; 1; 5]$, $x_3(0) = [7; 2; 9]$, $x_4(0) = [-3; 5; 3]$, $\alpha = 0.1$ and the H_∞ index γ is chosen as 1. The external disturbance is the unit sine wave. Using the method of Theorem 3.2, we can obtain the consensus feedback controller gain

$$K = \begin{bmatrix} -15.9793 & -3.9048 \\ -7.4387 & -1.5030 \end{bmatrix},$$

and under the static output feedback control, the states' curves of the closed-loop multi-agent system are shown in Fig.2, and the corresponding controlled outputs' curves are shown in Fig.3. Fig.2 and Fig.3 show that the multi-agent network system with the external disturbance is asymptotically consistent.

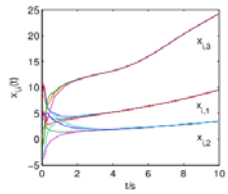

FIGURE II. THE CURVES OF LINEAR MULTI-AGENT SYSTEMS STATES $x_{i,i}$

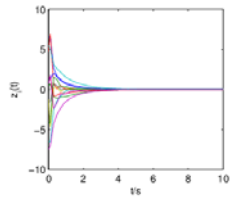

FIGURE III. THE CURVES OF CONTROLLED OUTPUTS z_i

V. Conclusion

The H_∞ consensus problem of multi-agent network systems subject to external disturbance has been studied in this paper. By adopting a distributed output feedback controller based on the relative outputs of neighboring agents with time-varying delays, and using model transformation, the H_∞ consensus problem for a linear multi-agent system is turned to be the asymptotic stability problem. Then, through Lyapunov-Krasovskii function and several lemmas we derive consensus conditions in terms of linear matrix inequalities and meanwhile obtain the controller parameters. A numerical example has shown the effectiveness of the proposed theoretical algorithms. Extension to a directed graph or more complex dynamic output feedback controller is under the future investigation.

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