The Product and Covering of Lattice –Valued Tree Automata

Zhonggang Hu[1,a]

College of Ethnic Education, Nanchang Institute of Science& Technology, Nanchang, Jiangxi, 330108, China

[a]huzhonggang@126.com

Keywords: Lattic-valued tree automata; Homomorphism; Full direct product; Restricted direct product; Cascade product; Wreath product; Coverings

Abstract. In this paper, the concept of full direct product, restricted direct product, cascade product, wreath product and covering is given. The relation between the products of the lattice-valued tree automata, the covering relation between the lattice-value tree automata, and the covering relation between the product of lattice-valued tree automata and the product of the lattice-valued tree automata covering them are discussed.

Introduction

The tree automata can be regarded as the generalization of the classical word automata[1]. The systematic exposition of the classical tree automata can be found in literature[2]. Li Y M and Pedrycz W put forward the theory of lattice-valued automata, which constructed the fuzzy automata in a broader framework. Literature[3] is the generalization of the key concepts and conclusions of the literature[4,5]. In this paper, literature[6] studies the congruence and homomorphism of lattice-valued tree automata. In automata theory, product is one of the basic operations, and the product and covering relation of different forms play a very important role in the decomposition of automata. Literature [7] studies the product of lattice-valued finite automata, and literature [8] studies the product and covering relation of fuzzy finite-state machines. In this paper, the concept of product and covering of the lattice-valued tree automata is given, the covering relation between the products of the lattice-valued tree automata is studied, and the covering relation between the product of lattice-valued automata and the product of the lattice-valued tree automata covering them is also discussed.

Basic Concepts and Symbols

Definition 2.1[9]. Assuming that $\sum$ is a nonempty set, $rk:\sum\to \mathrm{N}$ is a mapping, and N is the set of natural numbers, then $(\sum, rk)$ is called an order character set. $\forall k\ge 0$, $\sum_{(k)}=\{\sigma\in\sum \mid rk(\sigma)=k\}$. For simplicity, $(\sum, rk)$ is recorded as $\sum$.

Definition 2.2[10]. Assuming that X is a set of variables that does not cross with $\sum$. The minimum set that meets the condition (1) and (2) is called the item set on the $\sum$ marked by X, and it is recorded as $T_\sum(X)$. Among them:

$X\cup\sum_{(0)}\subseteq T_\sum(X)$

If $k\ge 1$, $\sigma\in\sum_{(k)}, s_1,\cdots,s_k\in T_\sum(X)$, then $\sigma(s_1,\cdots,s_k)\in T_\sum(X)$.

Note: The set $T_\sum(\varnothing)$ is recorded as $T_\sum$. Obviously, $T_\sum(X)=T_{\sum\cup X}$.

Definition 2.3[4]. we give a lattice $(L,\wedge,\vee,0,1)$, and let $\bullet$ be a binary operation in L, and make $(L,\bullet,e)$ be a monoid with a unit element e. Any $a,b,c,x\in L$, and meets the following conditions:

$a\bullet 0=0\bullet a=0$

$a\le b\Rightarrow a\bullet x\le b\bullet x$ 且 $x\bullet a\le x\bullet b$

$a\bullet(b\vee c)=(a\bullet b)\vee(a\bullet c);(b\vee c)\bullet a=(b\bullet a)\vee(c\bullet a)$

Then L is called a lattice-ordered semigroup, and is called a lattice semigroup for short. For

simplicity, it is generally recorded as L.

Definition 2.4[11]. A lattice-valued finite tree automation $M=(Q,\Sigma,L,\alpha,\nu)$. Among them, Q is the finite nonempty state set, Σ is the order input character set, L is a lattice semigroup, $\nu:Q\to L$ is the fuzzy termination state, α is a lattice-valued tree representation, which is a family mapping $\alpha=(\alpha_k)_{k\geq 0},\alpha_k:\Sigma_{(k)}\to L^{Q^k\times Q}$. For simplicity, the multiplicative operation $\bullet$ of lattice semigroup L is expressed as $\otimes$.

Let's define the function $\alpha_t:Q^k\times Q\to L, k\in \mathrm{N}, \forall t=\sigma(t_1,\cdots,t_k)\in T_\Sigma$,

$$\alpha_t(p_1p_2\cdots p_k,p)=\alpha_{\sigma(t_1,\cdots,t_k)}(p_1p_2\cdots p_k,p)$$
$$=\vee_{p_i'\in Q,i=1,\cdots,k}\otimes_{i=1}^{k}\alpha_{t_i}(p_1p_2\cdots p_k,p_i')\otimes\alpha_k(\sigma)_{(p_1',\cdots,p_k')p}.$$

Definition 2.5[6]. Assuming that $M_1=(Q_1,\Sigma,L,\alpha_1,\nu_1)$ and $M_2=(Q_2,\Sigma,L,\alpha_2,\nu_2)$ are two lattice-valued tree automata. Mapping $\varphi:Q_1\to Q_2$ is called the homomorphic mapping from M_1 to M_1, which is recorded as $\varphi:M_1\to M_2$. If it meets:

$$\alpha_{k1}(\sigma)_{(p_1,\cdots,p_k),p}\leq\alpha_{k2}(\sigma)_{(\varphi(p_1),\cdots,\varphi(p_k)),\varphi(p)},$$
$$\nu_1(p)\leq\nu_2(\varphi(p)),\quad \forall p,p_i\in Q_1,i\in[1,k],\sigma\in\Sigma_{(k)},k\geq 0$$

φ is called a strong homomorphism, if:

$$\alpha_{k2}(\sigma)_{(\varphi(p_1),\cdots,\varphi(p_k)),\varphi(p)}=\vee_{\varphi(p)=\varphi(x)}\alpha_{k1}(\sigma)_{(p_1,\cdots,p_k),x},$$
$$\nu_2(\varphi(p))=\vee_{\varphi(p)=\varphi(x)}\nu_1(x),\quad \forall p,p_i\in Q_1,i\in[1,k],\sigma\in\Sigma_{(k)},k\geq 0$$

A strong homomorphism $\varphi:M_1\to M_2$ is called an isomorphism. If φ is the bijection; If φ is the injection(surjection), then we say φ is the monomorphism mapping(epimorphism mapping).

The Covering of the Lattice-Valued Tree Automata

Definition 3.1. Assuming that $M_i=(Q_i,\Sigma_i,L,\alpha_i,\nu_i)(i=1,2)$ is the lattice-valued tree automaton. If there is the surjection $\varphi:Q_2\to Q_1$, and preserving rank $\psi:\Sigma_1\to\Sigma_2$, for $\forall q_1,q_2,\cdots,q_k,q\in Q_2,\sigma_1\in\Sigma_{k_1}$, it satisfies:

$$\alpha_{k_1}(\sigma_1)_{(\varphi(q_1),\cdots,\varphi(q_k)),\varphi(q)}\leq\alpha_{k_2}(\psi(\sigma_1))_{(q_1,\cdots,q_k),q},\quad \nu_1(\varphi(q))\leq\nu_2(q),$$

Then we say M_2 covers the M_1, we record it as $M_2\leq M_1$.

The following can be got easily:

Proposition 3.1. Assuming that $M_i=(Q_i,\Sigma_i,L,\alpha_i,\nu_i)$ is the lattice-valued tree automaton, $i=1,2,3$. If $M_1\leq M_2,M_2\leq M_3$, then $M_1\leq M_3$.

Theorem 3.1. Assuming that: $M_i=(Q_i,\Sigma,L,\alpha_i,\nu_i)(i=1,2)$ is the lattice-valued tree automation, If $\varphi:M_1\to M_2$ is the homomorphism, then there will be: (1) If the homomorphism is epimorphism, then $M_2\leq M_1$; (2) If φ is the interjection, then $M_1\leq M_2$.

Proof. $\varphi:M_1\to M_2$ is the epimorphism, so there's a full function $\varphi:Q_1\to Q_2$. Let $\eta=\varphi:Q_1\to Q_2,1_\Sigma:\Sigma\to\Sigma$. Obviously, η is the surjection. For $\forall p,p_i\in Q_1,i\in[1,k],\sigma\in\Sigma_{(k)},k\geq 0$,

$$\alpha_1(\sigma)_{(\eta(p_1),\cdots,\eta(p_k)),\eta(p)}=\alpha_1(\sigma)_{(\varphi(p_1),\cdots,\varphi(p_k)),\varphi(p)}\leq\alpha_2(\sigma)_{(p_1,\cdots,p_k),p}=\alpha_2(1_\Sigma(\sigma))_{(p_1,\cdots,p_k),p},$$
$$\nu_1(\eta(p))=\nu_1(\varphi(p))\leq\nu_2(p),$$

therefore, $M_2\leq M_1$.

$\varphi:M_1\to M_2$ is the homomorphism, so there is the mapping $\varphi:Q_1\to Q_2$. For $\forall p,p_i\in Q_1,i\in[1,k],\sigma\in\Sigma_{(k)},k\geq 0$, there is

$$\alpha_1(\sigma)_{(p_1,\cdots,p_n),p}\leq\alpha_2(\sigma)_{(\varphi(p_1),\cdots,\varphi(p_n)),\varphi(p)},\quad \nu_1(p)\leq\nu_2(\varphi(p)).$$

Let $\eta: Q_2 \to Q_1, \eta(q) = p$, $p \in Q_1, q \in Q_2$, if $\varphi(p) = q$, because φ is the interjection, so p is the only definite, so η is a full partial function. Let $1_\psi: \Sigma \to \Sigma$, for $\forall p, p_i \in Q_1, i \in [1,k], \sigma \in \Sigma_{(k)}, k \geq 0$, then

$$\alpha_{k_1}(\sigma)_{(\varphi(q_1),\cdots,\varphi(q_k)),\varphi(q)} \leq \alpha_{k_2}(\sigma)_{(q_1,\cdots,q_k),q} = \alpha_{k_2}(1_\Sigma(\sigma))_{(q_1,\cdots,q_k),q},$$
$$\nu_1(\eta(q)) = \nu_1(p) \leq \nu_2(\varphi(p)) = \nu_2(q),$$

then $M_1 \leq M_2$.

Definition 3.2. Assuming that $M_i = (Q_i, \Sigma, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued tree automaton, if φ is a full partial function from Q_2 to Q_1, and ψ is the partial function from Σ_1 to Σ_2, then we say (φ, ψ) is a weak covering from M_2 to M_1, which is recorded as $M_2 \leq_\omega M_1$. For $\forall q_1, q_2, \cdots, q_k, q \in Q_2, \sigma_1 \in \Sigma_{k_1}$, it meets:

$$\alpha_{k_1}(\sigma_1)_{(\varphi(q_1),\cdots,\varphi(q_k)),\varphi(q)} \leq \alpha_{k_2}(\psi(\sigma_1))_{(q_1,\cdots,q_k),q}, \quad \nu_1(\varphi(q)) \leq \nu_2(q),$$

the following can be got easily:

Proposition3.2. Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)$ is the lattice-valued tree automaton, $i = 1,2,3$. If $M_1 \leq_\omega M_2, M_2 \leq_\omega M_3$, then $M_1 \leq_\omega M_3$.

Theorem3.2 Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)$ is the lattice-valued tree automaton, $i = 1,2$. If $(\alpha, \beta): M_1 \to M_2$ is a homomorphism, then there are:

If this homomorphism is a strong epimorphism, and α is the injection, then $M_2 \leq_\omega M_1$;

If α is the injection, then $M_1 \leq_\omega M_2$.

The Product of the Lattice-Valued Tree Automata

Definition 4.1. Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued automaton, then the full direct product of lattice-valued automata M_1 and M_2 is $M_1 \times M_2 = (Q_1 \times Q_2, \Sigma_1 \times \Sigma_2, L, \alpha_1 \times \alpha_2, \nu_1 \times \nu_2)$, among them

$$(\alpha_{k_1} \times \alpha_{k_2})(\sigma_1 \times \sigma_2)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} = \alpha_{k_1}(\sigma_1)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}(\sigma_2)_{(q_1,q_2,\cdots,q_k),q},$$
$$(\nu_1 \times \nu_2)(p,q) = \nu_1(p) \wedge \nu_2(q),$$
$$\forall (p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k),(p,q) \in Q_1 \times Q_2, \sigma_1 \times \sigma_2 \in \Sigma_{k_1} \times \Sigma_{k_2}, k \geq 0.$$

Definition 4.2. Assuming that $M_i = (Q_i, \Sigma, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued tree automaton, then the restricted direct product of the lattice-valued tree automata M_1 and M_2 is $M_1 \wedge M_2 = (Q_1 \times Q_2, \Sigma, L, \alpha_1 \wedge \alpha_2, \nu_1 \wedge \nu_2)$, among them

$$(\alpha_{k_1} \wedge \alpha_{k_2})(\sigma)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} = \alpha_{k_1}(\sigma)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}(\sigma)_{(q_1,q_2,\cdots,q_k),q},$$
$$(\nu_1 \wedge \nu_2)(p,q) = \nu_1(p) \wedge \nu_2(q),$$
$$\forall (p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k),(p,q) \in Q_1 \times Q_2, \sigma \in \Sigma_k, k \geq 0.$$

Definition 4.3 Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued tree automaton, then the cascade product of the lattice-valued tree automata M_1 and M_2 is $M_1 \omega M_2 = (Q_1 \times Q_2, \Sigma_2, L, \alpha_1 \omega \alpha_2, \nu_1 \omega \nu_2)$, among them $\omega: Q_2 \times X_2 \to X_1$,

$$(\alpha_{k_1} \omega \alpha_{k_2})(\sigma_2)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} = \alpha_{k_1}(\omega(p_2,\sigma_2))_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}(\sigma_2)_{(q_1,q_2,\cdots,q_k),q},$$
$$(\nu_1 \omega \nu_2)(p,q) = \nu_1(p) \wedge \nu_2(q),$$
$$\forall (p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k),(p,q) \in Q_1 \times Q_2, \sigma_2 \in \Sigma_{k_2}, k \geq 0.$$

Definition 4.4. Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued tree automaton, then the wreath product of lattice-valued tree automata M_1 and M_2 is

$M_1 \circ M_2 = \left(Q_1 \times Q_2, \Sigma_1^{Q_2} \times \Sigma_2, L, \alpha_1 \circ \alpha_2, \nu_1 \circ \nu_2\right)$, among them

$$\left(\alpha_{k_1} \circ \alpha_{k_2}\right)\left(f, \sigma_2\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} = \alpha_{k_1}\left(f\left(q\right)\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma_2\right)_{(q_1,q_2,\cdots,q_k),q},$$

$$\left(\nu_1 \circ \nu_2\right)\left(p,q\right) = \nu_1\left(p\right) \wedge \nu_2\left(q\right),$$

$$\forall\left(p_1,q_1\right),\left(p_2,q_2\right),\cdots,\left(p_k,q_k\right),\left(p,q\right) \in Q_1 \times Q_2, \sigma_2 \in \Sigma_{k_2}, k \geq 0, \forall\left(f,\sigma_2\right) \in \Sigma_1^{Q_2} \times \Sigma_2,\ f: Q_2 \to \Sigma_1$$

Theorem4.1. Assuming that $M_i = \left(Q_i, \Sigma_i, L, \alpha_i, \nu_i\right)\left(i = 1,2\right)$ is the lattice-valued tree automaton, then:

$M_1 \wedge M_2 \leq M_1 \times M_2$, among them $\Sigma_1 = \Sigma_2 = \Sigma$,

$M_1 \omega M_2 \leq M_1 \circ M_2$,

$M_1 \circ M_2 \leq M_1 \times M_2$,

$M_1 \omega M_2 \leq M_1 \times M_2$.

Proof. Definition $\varphi: Q_1 \times Q_2 \to Q_1 \times Q_2$ is the identity mapping on $Q_1 \times Q_2$. Obviously, φ is the full partial function. Definition $\psi: \Sigma \to \Sigma \times \Sigma$, For $\forall \sigma \in \Sigma$, there is $\psi\left(\sigma\right) = \left(\sigma, \sigma\right)$. ξ is a function, and

$$\begin{aligned}\left(\alpha_{k_1} \wedge \alpha_{k_2}\right)\left(\sigma\right)_{(\varphi(p_1,q_1),\varphi(p_2,q_2),\cdots,\varphi(p_k,q_k)),\eta(p,q)} &= \left(\alpha_{k_1} \wedge \alpha_{k_2}\right)\left(\sigma\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} \\ &= \alpha_{k_1}\left(\sigma\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma\right)_{(q_1,q_2,\cdots,q_k),q} \\ &= \left(\alpha_{k_1} \times \alpha_{k_2}\right)\left(\sigma \times \sigma\right)_{((p_1,q_1),\cdots,(p_k,q_k)),(p,q)} \\ &= \left(\alpha_{k_1} \times \alpha_{k_2}\right)\left(\psi\left(\sigma \times \sigma\right)\right)_{((p_1,q_1),\cdots,(p_k,q_k)),(p,q)},\end{aligned}$$

$\left(\nu_1 \wedge \nu_2\right)\left(\varphi\left(p,q\right)\right) = \left(\nu_1 \wedge \nu_2\right)\left(p,q\right) = \nu_1\left(p\right) \wedge \nu_2\left(q\right) = \left(\nu_1 \times \nu_2\right)\left(p,q\right)$, among them, $\forall p, p_i \in Q_1, q, q_i \in Q_2, i = 1,2,\cdots,n$, therefore, $M_1 \wedge M_2 \leq M_1 \times M_2$.

Definition. $\varphi: Q_1 \times Q_2 \to Q_1 \times Q_2$ is the identity mapping on $Q_1 \times Q_2$. Obviously, φ is the full partial function. Let definition $\psi: \Sigma_2 \to \Sigma_1^{Q_2} \times \Sigma_2$, $\psi\left(\sigma_2\right) = \left(f, \sigma_2\right), \forall \sigma_2 \in \Sigma_2$, among them, $f: Q_2 \to \Sigma_1, f\left(q_0\right) = \sigma_1 = \omega\left(q_0, \sigma_2\right)$,

$\forall q_0 \in Q_2, \sigma_1 \in \Sigma_1$, ψ is a function, and

$$\begin{aligned}\left(\alpha_{k_1} \omega \alpha_{k_2}\right)\left(\sigma_2\right)_{(\varphi(p_1,q_1),\varphi(p_2,q_2),\cdots,\varphi(p_k,q_k)),\varphi(p,q)} &= \left(\alpha_{k_1} \omega \alpha_{k_2}\right)\left(\sigma_2\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} \\ &= \alpha_{k_1}\left(\omega\left(p_0, \sigma_2\right)\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma_2\right)_{(q_1,q_2,\cdots,q_k),q} \\ &= \alpha_{k_1}\left(f\left(q_0\right)\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma_2\right)_{(q_1,q_2,\cdots,q_k),q} \\ &= \left(\alpha_{k_1} \circ \alpha_{k_2}\right)\left(f, \sigma_2\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} \\ &= \left(\alpha_{k_1} \circ \alpha_{k_2}\right)\left(\psi\left(\sigma_2\right)\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)},\end{aligned}$$

$\left(\nu_1 \omega \nu_2\right)\left(\varphi\left(p,q\right)\right) = \left(\nu_1 \omega \nu_2\right)\left(p,q\right) = \nu_1\left(p\right) \wedge \nu_2\left(q\right) = \left(\nu_1 \circ \nu_2\right)\left(p,q\right)$, among them, $\forall p, p_i \in Q_1, q, q_i \in Q_2, i = 1,2,\cdots,n$, therefore, $M_1 \omega M_2 \leq M_1 \circ M_2$.

Definition $\psi: \Sigma_1^{Q_2} \times \Sigma_2 \to \Sigma_1 \times \Sigma_2$ is $\psi\left(f, \sigma_2\right) = \left(f\left(q_0\right), \sigma_2\right)$, among them, $f: Q_2 \to \Sigma_1, f\left(q_0\right) = \sigma_1$, $\forall \sigma_2 \in \Sigma_2, q_0 \in Q_2$, and the definition φ is the identity mapping on $Q_1 \times Q_2$, it can be easily proved that $M_1 \circ M_2 \leq M_1 \times M_2$.

$$\begin{aligned}\left(\alpha_{k_1} \circ \alpha_{k_2}\right)\left(f, \sigma_2\right)_{(\varphi(p_1,q_1),\varphi(p_2,q_2),\cdots,\varphi(p_k,q_k)),\varphi(p,q)} &= \left(\alpha_{k_1} \circ \alpha_{k_2}\right)\left(f, \sigma_2\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)} \\ &= \alpha_{k_2}\left(f\left(q_0\right)\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma_2\right)_{(q_1,q_2,\cdots,q_k),q} \\ &= \alpha_{k_2}\left(\sigma_1\right)_{(p_1,p_2,\cdots,p_k),p} \wedge \alpha_{k_2}\left(\sigma_2\right)_{(q_1,q_2,\cdots,q_k),q} \\ &= \left(\alpha_{k_1} \times \alpha_{k_2}\right)\left(\sigma_1 \times \sigma_2\right)_{((p_1,q_1),(p_2,q_2),\cdots,(p_k,q_k)),(p,q)},\end{aligned}$$

$\left(\nu_1 \circ \nu_2\right)\left(\varphi\left(p,q\right)\right) = \left(\nu_1 \circ \nu_2\right)\left(p,q\right) = \nu_1\left(p\right) \wedge \nu_2\left(q\right) = \left(\nu_1 \times \nu_2\right)\left(p,q\right)$, among them ,

$\forall p, p_i \in Q_1, q, q_i \in Q_2, i = 1,2,\cdots,n$, so $M_1 \circ M_2 \le M_1 \times M_2$.

From (2) and (3), we can know that $M_1 \omega M_2 \le M_1 \times M_2$.

The following can be easily got:

Theorem 4.2. Assuming that $M_i = (Q_i, \Sigma_i, L, \alpha_i, \nu_i)(i = 1,2)$ is the lattice-valued tree automaton, $i = 1,2,3$. If $M_1 \le M_2$, then:

$M_1 \times M_3 \le M_2 \times M_3, M_3 \times M_1 \le M_3 \times M_2$; If $\Sigma_1 = \Sigma_2 = \Sigma_3 = \Sigma$, then $M_1 \wedge M_3 \le M_2 \wedge M_3, M_3 \wedge M_1 \le M_3 \wedge M_2$;

For any $\omega_1 : Q_3 \times \Sigma_3 \to \Sigma_1$, if there is $\omega_2 : Q_3 \times \Sigma_3 \to \Sigma_2$, let $M_1 \omega_1 M_3 \le M_2 \omega_2 M_3$; If (φ, ψ) is the covering of M_2 for M_1, and ψ is a surjection, then for any $\omega_1 : Q_1 \times \Sigma_1 \to \Sigma_3$, there is $\omega_2 : Q_2 \times \Sigma_2 \to \Sigma_3$, let $M_3 \omega_1 M_1 \le M_3 \omega_2 M_2$;

$M_1 \circ M_3 \le M_2 \circ M_3, M_3 \circ M_1 \le M_3 \circ M_2$.

Acknowledgements

Science and technology research project of Jiangxi Provincial Education Department(GJJ161223).

References

[1] G e' cseg F and Steinby M: Tree Automata[M], Akad e' miai Kiad o', Budapest,1984.
[2] Esik Z and Liu G: Fuzzy Tree Automata[J], Fuzzy Sets and Systems, 2007,158:1450-1460.
[3] Mordeson J N and Malik D S: Fuzzy Automata and Languages: Theory and Applications[M], London:Chapman ＆Hall, 2002.
[4] Y.M.Li and W.Pedrycz: Fuzzy Finite Automata and Fuzzy Regular Expressions with Membership Values in Lattice-Orderd Monoids[J], Fuzzy Sets and System, 2005:156(1):68-92.
[5] Sheng L and Li Y M: Regular Grammars with Truth Values in Lattice-Ordered Monoids and Their Languages[J], Soft Computing, 2006,10(2):79-86.
[6] Z.G.Hu and Q.F.Lou: Congruence and Homomorphism of Lattice-Valued Tree Automata[J], Fuzzy Systems and Mathematics, 2011,29(4): 135-138.(In Chinese)
[7] J.Liu and Z.W.Mo: The Product of the Lattice-Valued Finite Automata[J], Applied Mathematics: A Journal of Chinese Universities, 2009, 24(1):121-126.(In Chinese)
[8] J.Liu, S.Q.Sun and X.H.Ou: On Covering of Fuzzy Product Finite Machines[J], Applied Mathematics: A Journal of Chinese Universities, 2011,26(2):239-246.(In Chinese)
[9] M.Droste,W.Kuich and H.Vogler: Handbook of Weighted Automata[M], EATCS Monograph,in Theoretical Computer Science, Springer, 2009.
[10] B.Borchardt.The Myhill-Nerode theorem for recognizable tree series[C].In Proc.7th Int Lonf. Developments in Language Theory, volume 2710 of LNCS, p.146-158.Springer verlag, 2003.
[11] R.M.Zhang: Minimization of Lattice-Valued Tree Automata[D], Shanxi Normal University, 2011.(In Chinese)