Analysis of Water Quality Variation Trend of Guanting Reservoir Based on Markov Model

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Abstract. Base on Markov model, the quality state of Guanting Reservoir is divided, the initial matrix is formed, the index weight by entropy weight is obtained, and the probability transition matrix of water quality variation trend is constructed. Then this model calculates the degree of absolute and comparative advancement, which can help us to analysis water quality variation trend of Guanting Reservoir. The comparative advancement of Guanting Reservoir is 0.0046 during 2006～2016, which indicates that the variation trend is generally good, but the trend is very slow.

Introduction

Many large-scale reservoirs were built after the founding of New China, and Guanting Reservoir is the first one. It is an important source of surface water in Peking and an ecological conservation area in Peking-Tianjin-Hebei area. Guanting Reservoir located in Zhangjiakou of Hebei Province and Yanqing of Peking, which is in upstream of Yongding River. There are three rivers, Yang River, Sanggan River and Guishui River, flowing into Guanting Reservoir. The area of Guanting Reservoir is about 46,000 km^2, accounting for about 90% of the total area of Yongding River Basin. The water quality of Guanting Reservoir has been good during the 50~60s of twentieth Century. Since the early 70s, due to the pollution of upstream industry and agriculture, the warehousing rivers were polluted. With the long-term low water level operation, the water quality of the reservoir gradually became worse and the trend of eutrophication became increasingly prominent. In May 1997, Guanting Reservoir was forced out of the source of drinking water in Peking.

Since the implementation of "the capital water resources sustainable utilization plan in the early twenty-first Century", a series of comprehensive ecological water resources and environment management have been carried out in Guanting Reservoir basin in Peking and Zhangjiakou City. "The Peking-Tianjin-Hebei collaborative development program" was passed in 2015, which defined the mountainous area of Peking and Zhangjiakou and Chengde of Hebei province as the northwest ecological conservation area, and the ecological restoration and pollution control of Guanting Reservoir are definitely required.

The change of water quality is influenced by many uncertain factors in river, lake and reservoir. And the water quality is transferred from one state to another with randomness. It is difficult to consider these factors in water quality prediction model based on mathematical expression. And it is also difficult to consider all factors in qualitative model of water quality evaluation[1]. Markov process is a kind of special stochastic process, which has the good property of describing the random change of things and no aftereffects[2,3]. The process of water quality change is a stochastic process with Markov characteristics. And there are many applications in water quality assessment and prediction based on Markov model. Zhong Zhenglin[4], who predicted the river comprehensive water quality using Markov process, He Bin[5],who calculated the probability of water quality transfer between different grades, using the principle of Markov process, and evaluated the dynamic change of water quality.Feng Wei[2], who proposed the concept of the **comparative advancement degree** on the basis of Markov model, and calculated the degree of Songhua River, Haihe River, Yellow River and Yangtze River in 2005. Ma Huiqun[3],who obtained the multi-index entropy weight Markov water quality

dynamic evaluation model.Zou Zhihong[6], who applied the gray markov model to the water quality eutrophication trend prediction in Taihu lake, Dianchi lake and Chaohu lake, and provided a new short-term trend forecasting method for water quality.

Markov Model

Markov process is a random process, which means the future state of the system is closely related to the present state, but independent of the previous state. And it has the characteristics of no aftereffects[7]. It is a stochastic process that studies the theory of transfer rules between states of an event. In the Markov model, the probability 错误!未找到引用源。 of a one-step transition from state i to a state j is called a one-step transition probability[8]. which form is:

$$P=错误!未找到引用源。$$

It is called the transfer matrix . Its i-row vectors 错误!未找到引用源。 respectively represent the probability of reaching all states in one step from the i - state . It is clear that 错误!未找到引用源。 =1. The variation trend of water quality can be predicted by probability transfer matrix . The establishment of Markov evaluation model is as follows.

Determine of State. According to the characteristics of the evaluation of water environment, the water environment quality state is divided into N states, and it is expressed as E={1，2，…，N}. The most common method is to classify water quality into six types, which is I, II, III, IV, V and bad V by adopting the "Surface Water Environmental quality Standard" (GB3838-2002), that is, E={1，2，3，4，5，6}. In the actual situation, the monitoring value of a certain index is often concentrated in the same category. If the monitoring value is classified simply according to the water quality category, the purpose of analyzing and judging the changing trend of water quality will not be achieved [1]. Considering the situation that the water quality category difference of the index monitoring value is not obvious, the method of classifying the water quality according to the national standard is not suitable, so it is necessary to set the state classification standard with smaller subordinate range.After the quality of water quality is divided, the monitoring value of each indicator can be attached to a certain state, and the indicator membership matrix is obtained,错误!未找到引用源。.

Determine the Index Weight. In the multi-index evaluation, the weight determination is the key and difficult point. At present, there are many methods to determine weight, which can be divided into two categories [3]: subjective and objective weighting methods.The subjective weighting method is that the experts determine the weight according to the importance of each index, which is easily influenced by the subjective consciousness of the experts, and can not reflect the interrelationship of the statistical data of each indicator. The entropy method in the objective weighting method is a method to determine the weight by the judgment matrix composed of the evaluation index value under the objective condition. It can eliminate the subjectivity of the weight of each factor as far as possible and make the evaluation result more realistic. The calculation steps are as follows[9,10].

1) Construct a judgment matrix 错误!未找到引用源。of n times monitoring m evaluation indexes.

2) The judgment matrix is normalized to obtain the normalized judgment matrix B, and the expression of the elements in B is:

$$B_{ij}=错误!未找到引用源。. \tag{1}$$

In the formula, 错误!未找到引用源。 are the most satisfied or dissatisfied ones in each monitoring under the same indicator.

3) According to the definition of entropy, the entropy of the evaluation index can be determined to be as followed：

$H_j=$错误!未找到引用源。. (2)

错误!未找到引用源。. (3)

4) Calculate the entropy of evaluation index

错误!未找到引用源。. (4)

5)Make 错误!未找到引用源。1, and the rest of the weights are scaled accordingly, that is,

错误!未找到引用源。，j=1,2,错误!未找到引用源。,m.
(5)

Determine the Probability Transfer Matrix. The initial matrix is as followed, in which the number of N is integers using the four-house five-entry principle,

错误!未找到引用源。. (6)

And the probability transfer matrix is calculated according to the initial matrix.

Let m indicators are monitored for n times. The total number of two adjacent monitoring indicators from i to j is 错误!未找到引用源。（i，j=1,2,……，12）, and 0≤错误!未找到引用源。≤m. The number of original samples in i-level is错误!未找到引用源。, then the transfer probability from i-level to j-level is:

错误!未找到引用源。. (7)

In the formula, 0≤错误!未找到引用源。≤1,错误!未找到引用源。.

The probability transfer matrix is 错误!未找到引用源。 (N is the number of divided states).

Calculate Absolute Advancement Degree. The calculated probability transfer matrix P can predict the future development of water quality, and have a general understanding of the trend of water quality change during the monitoring period. But in order to show the change of water quality more clearly, further calculations are needed, so the degree of advancement is necessary[11].It is assumed that if the water quality is improved, the degree of advancement is positive, and if the water quality deteriorates, the degree is negative. The more obvious the water quality is improved or deteriorated, the greater the degree of advancement.

On the basis of the above assumptions, we define the degree of advancement 错误!未找到引用源。 as:

错误!未找到引用源。. (8)

In the formula, i,j=1,2,···,12，and 错误!未找到引用源。 is called the advancement matrix of probability transfer matrix, 错误!未找到引用源。 is called the degree of absolute advancement.

Calculate Relative Advancement Degree. In order to compare the dynamic development of each index more easily, it is necessary to normalize d(s) so that the maximum relative advancement degree of water quality improvement is 1, and the relative advancement degree of water quality deterioration is -1. The range of degree is [-1 ,1].

错误!未找到引用源。. $\hspace{8cm}$ (9)

In the formula, N is the number of divided states.

Model Forecast in Guanting Reservoir

This article uses the multi-index entropy value method to determine the weight, the water quality of Guanting Reservoir from 2006 to 2016 was evaluated by using the Markov model.

Table 1 The water quality monitoring results of Guanting Reservoir from 2006 to 2016

Year	COD_{mn} [mg/L]	NH_3-N [mg/L]	TP [mg/L]	F [mg/L]
2006	5.7	0.69	0.066	1.20
2007	6.3	0.26	0.085	1.33
2008	6.9	0.22	0.092	1.48
2009	6.1	0.18	0.07	1.36
2010	6.2	0.23	0.042	1.39
2011	4.9	0.19	0.073	1.21
2012	4.7	0.19	0.063	1.23
2013	5.5	0.17	0.040	1.27
2014	5.6	0.34	0.040	1.28
2015	5.5	0.27	0.046	1.30
2016	5.0	0.39	0.050	1.33

Determine of State. According to "surface water environment quality standard" (GB 3838-2002), and each state is subdivided into two small states by interpolation. Finally, it is divided into 12 quality states, that is, E={1，2，$\cdots$，12}, and the subordinate matrix of water quality index of Guanting Reservoir is obtained from 2006 to 2016.

$$R = \begin{bmatrix} 6 & 5 & 7 & 7 \\ 7 & 3 & 8 & 8 \\ 7 & 3 & 8 & 8 \\ 7 & 3 & 7 & 8 \\ 7 & 3 & 6 & 8 \\ 5 & 3 & 7 & 7 \\ 5 & 3 & 7 & 7 \\ 6 & 3 & 6 & 8 \\ 6 & 4 & 6 & 8 \\ 6 & 3 & 6 & 8 \\ 5 & 4 & 6 & 8 \end{bmatrix}$$

Determine the Index Weight. The weights of COD_{mn},NH_3-N,TP and F were determined by entropy method by formula (1) ~ (5). Their weights are as follows:错误!未找到引用源。=(0.6969，0.8004,1.000, 0.7179).

Determine the Probability Transfer Matrix. The initial matrix is calculated by the membership matrix and the weight of the water quality index of Guanting reservoir.

$$N = w_i' \times r_{ij} \approx \begin{bmatrix} 4 & 4 & 7 & 5 \\ 5 & 2 & 8 & 6 \\ 5 & 2 & 8 & 6 \\ 5 & 2 & 7 & 6 \\ 5 & 2 & 6 & 6 \\ 3 & 2 & 7 & 5 \\ 3 & 2 & 7 & 5 \\ 4 & 2 & 6 & 6 \\ 4 & 3 & 6 & 6 \\ 4 & 2 & 6 & 6 \\ 3 & 3 & 6 & 6 \end{bmatrix}$$

The probability transfer matrix of Guanting Reservoir is calculated by formula(7):

$$P = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.75 & 0.25 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.33 & 0.34 & 0.33 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.20 & 0.20 & 0.40 & 0.20 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.14 & 0.00 & 0.57 & 0.29 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.09 & 0.82 & 0.09 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.25 & 0.25 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.50 & 0.50 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}$$

Calculate Relative Advancement Degree. The absolute advancement degree of water quality in Guanting Reservoir is 0. 5572 from 2006 to 2016 by formula (8). The relative advancement degree is 0. 0046 by formula (9). The result shows that the change of water quality in Guanting Reservoir from 2006 to 2016 is generally good, but the trend is very slow.
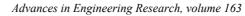
Conclusions

In the application of markov model, the following points should be noted:

Markov model can not accurately predict the specific value of the prediction index in the future time period, but can only give a state interval. The state interval reflects the changing trend of the index, but it can be assumed that if the state interval is subdivided continuously. However, it can be assumed that if the state interval is continuously subdivided, the change state interval of markov model to determine the index will be constantly approaching to the specific value.

When the data volume is small, the transfer matrix can not truly reflect the change trend, the randomness in the state matrix is stronger, the less data, the more susceptible to the influence of the value of a single sample, there may be real change instead of small probability. Therefore, in order to construct the state transition matrix more accurately, data should be collected as much as possible to reflect the changing trend more accurately.

The key to evaluate the water quality of rivers, lakes and reservoirs by Markov model is to construct the corresponding probability transfer matrix, which can be used to predict the future water quality of rivers. The weighted method is not limited to one kind. According to the actual situation and the focus, there can be a variety of different methods. Which kind of weighting method is more scientific and reasonable is worthy of further discussion.

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