The Cold Air Incursion Prediction Based on the Behavioral Model of Opening/Closing a Door of Classroom Established by Matlab

Zi Ouyang[a], Haorui Zhang, Yike Chen, Haoyu Chao and Weiwei Yu

Xi'an Jiao Tong University, 28 West Xianning Rd. Beilin District, Xi'an 710049, China

[a]ouyangzi002@163.com

Keywords: Computational methods. Convection. Thermal management. The behavioral model of opening/closing a door. MATLAB. Cold air incursion prediction.

Abstract. Through our observation of the classmates' behavior when entering or leaving the classroom, we find that it is more probable that they will keep the door close than leave it open when the door is already close. And the larger the number of classmates entering or leaving the classroom per unit time is, the less probability that they will close the door. Additionally, if the door has been kept open for a period of time, one or two classmates will close the door with initiative. Then the classmates having seen their behaviors will also have the consciousness of closing the door. So we use the software MATLAB to establish the mathematical model of classmates' closing or opening the door when entering or leaving the classroom on the base of our observation. Furthermore, since the cold air incursion is related to the state whether the door is close, we also use our model to predict about the heat loss from the cold air incursion.

Introduction

Introducing Phenomenon. Our politics teacher requires that we investigate uncivilized phenomenon, so we investigate the classmates' behavior of opening/closing a door of the classroom when they enter or leave this classroom. During the course of our observation and investigation, the data collection can be completed by using the video capture method. Fig.1 shows some main photographs obtained by using clipping video.

Analyzing Phenomenon. Through our observation of the classmates' behavior when entering or leaving the classroom, we find that it is more probable that they will keep the door close than leave it open when the door is already close. And the larger the number of classmates entering or leaving the classroom per unit time is, the less probability that will close the door. Additionally, if the door has been kept open for a period of time, one or two classmates will close the door with initiative. Then the classmates having seen their behaviors will also have the consciousness of closing the door.

Fig. 1 Main photographs obtained by using clipping video.

Establishing Model and Making Mathematical Experiment

Establishing Model. We establish the following mathematical model in order to provide a more concrete analysis on this phenomenon, and also to promote the method of the research on uncivilized behavior.

In the initial stages, for most of classmates when entering or leaving the classroom, we suppose that

(1) p_{10} is the probability of closing a door not been opened yet;

(2) p_{20} is the probability of closing a door opened already;

(3) $p(n)$ is the probability of the door closed when the nth person enters this classroom;

(4) p_1, is the probability of closing a door when the (n-1) th person enters this classroom, and at the time the door of this classroom is closed still. But p_2 is the probability of closing a door opened already when the (n-1) th person enters the classroom.

Then, the following relationship is obtained

$$p(n) = p_1 \times p(n-1) + p_2 \times (1 - p(n-1)) \tag{1}$$

And, because both p_1 and p_2 decrease with increase on the flow rate of people, we assume that the flow rate of people follows normal distributions for time, as follows:

$$f(t) = a_1 \times \exp\left(-\left(\frac{(t-b_1)}{c_1}\right)^2\right) \tag{2}$$

where a_1, b_1 and c_1 are constants. So, the probability of closing a door at time "t" may be fitted with index equations, we have

$$p_1 = p_{10} \times \exp(-kf(t)) \tag{3}$$

and

$$p_2 = p_{20} \times \exp(-kf(t)) \tag{4}$$

where the symbol, k denotes constant.

Next, the probability of the door is closed by the nth person, which can be solved by using the following equations (Note: $f(t_0)$ should be close to zero as far as possible, t_0 is the initial time).

$$\int_{t_0}^{t} f(t)dt = n \tag{5}$$

and

$$p_1 = p_{10} \times \exp(-kf(t)) \tag{6}$$

$$p_2 = p_{20} \times \exp(-kf(t)) \tag{7}$$

Mathematical Experiment Assumptions.

(1) There are altogether N classmates.

(2) The symbol p_0 denotes the proportion of classmates who are able to close the door consciously. p_0 represents these classmates sitting at the door and feeling chilly, and also those sitting at the door before then, have been influenced by the conscious behavior of closing the door. Obviously, they are different from most of classmates, they will close the door based on the above two reasons mentioned, the probability of closing a door is higher than most of other classmates.

(3) The behavioral model gives the assumption that the probability of closing the door equals to "1" when the classmate mentioned above appears every time. Then, the boundary probability, q is also assumed, additionally, the door is considered to be open when p<q, on the contrary, the door is close.

Based on the model established and assumptions above, we make mathematical experiment by introducing MATLAB.

Mathematical Experiment Results and Conclusions. According to our mathematical model, it is easy to know that the variable influencing on generation and diffusion of uncivil behaviors can be given by

$$p_{30} = p_{10} + p_{20} \tag{8}$$

The symbol p_{30} is proportion value of person who can close the door without anyone telling them to do it. The p_0 mentioned above represents these kinds of persons can close the door consciously. The two variables, p_{30} and p_0 seem to be the same, but they have essential difference.

The symbol p_{30} represents the behavior of closing a door of people, which is an absolute spontaneous behavior with independence and specificity. But, p_0 expresses that the behavior of closing a door tends to a behavior "simulated", namely the activity according with live in group, is influenced by other's behavior.

In general, we may not discriminate between p_{30} and p_0, or, the differences between p_{30} and p_0 are considered to have not an influence on outputting results. But, we give a certain condition, i.e., we give an influence of others on the behavior model, at this time, $p_0 > p_{30}$, the behavior of closing a door will occur easily. So, we have sought a solution to the problem.

In this paper, the mathematical experiment results highlight the importance of the environmental influences, and the environmental influences are the behavior of others, also can be the supervision from outside, for example, someone stands nearer to the door and reminds him or her to close the door once in a while.

Increase in the value of the p_{30} remains to heighten the quality of everyone and is also a fundamental solution, but is slower than increase in the value of the p_0. By contrast, it is easier to increase the valve of the p_0.

Cold Air Incursion Prediction

When the door is open, which will lead to energy consumption, because of the natural convection resulted from the difference of air temperature between indoor and outdoor, for example, the cold air infiltration through the door of heating room in the winter. In this paper, the mathematical model is established to calculate the cold air incursion and thermal dissipation preliminarily.

Geometric Model. In this paper, the size of the door is 2 meters high, H=2m, 1.4 meters wide, W=2×0.7=1.4m. Fig. 2 shows the geometric model of the door opened. And the angle displacement, θ_1 and θ_2 also denote the open degree of door illustrated in Fig. 2.
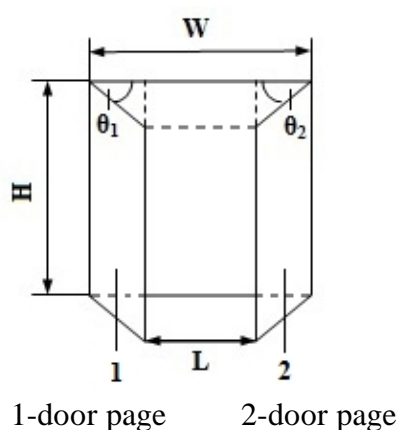

1-door page   2-door page
Fig. 2 Illustration of geometric model of the open double page door.

Cold Air Incursion Mathematical Model. The cold air incursion under consideration is the sum, G, as follows

$$G = G_1 + G_2 \tag{9}$$

where the symbols G_1 and G_2 are the cold air incursion from the front and top of the open door as shown in Fig. 3, respectively.
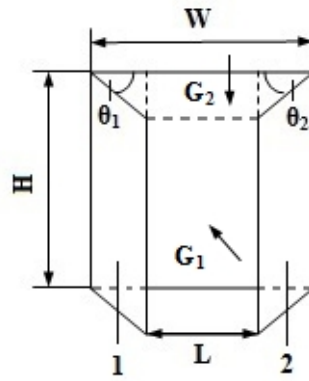
Fig. 3 Illustration of cold air incursion from the front and top of the open door, G_1 and G_2.
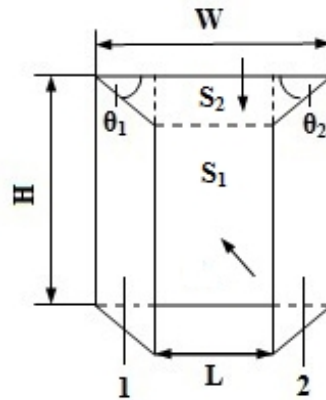

Fig.4 Illustration of cross-sectional area of the cold air incursion from the front and top of the open door, S_1 and S_2.

The total cross-sectional of area of the cold air incursion under consideration can be expressed as

$$S = S_1 + S_2 \qquad (10)$$

where the symbols S_1 and S_2 are the cross-sectional area of the cold air incursion from the front and top of the open door as shown in Fig.4, respectively.
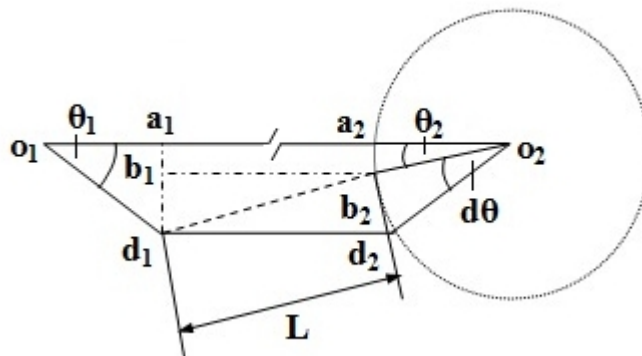

Fig. 5 Illustration of geometrical model of the top of the door being open ($\theta_1 \neq \theta_2$).

Fig.5 shows the geometrical model of the open door, $o_1 o_2 = W$. Supposing, $a_1 a_2 = \Delta x$, $b_1 d_1 = \Delta y$, and $\theta_1 = \theta_2 + d\theta$, then

$$\Delta x = W - \frac{W}{2} \times \cos\theta_1 - \frac{W}{2} \times \cos\theta_2 = W - \frac{W}{2} \times \cos(\theta_2 + d\theta) - \frac{W}{2} \times \cos\theta_2 \qquad (11)$$

$$\Delta y = \frac{W}{2} \times \sin\theta_1 - \frac{W}{2} \times \sin\theta_2 = \frac{W}{2} \times \sin(\theta_2 + d\theta) - \frac{W}{2} \times \sin\theta_2 \qquad (12)$$

and

$$L = \sqrt{\Delta x^2 + \Delta y^2} \qquad (13)$$

From Fig. 4, the cross-sectional area of the cold air incursion from the front of the open door, S_1, is then

$$S_1 = L \times H \qquad (14)$$

In particular, letting $d\theta=0$, and $\theta_1=\theta_2=\theta$, the geometric size, L takes the following form.

$$L = \sqrt{\Delta x^2} = \Delta x = W - W\cos\theta = (1 - \cos\theta)W \qquad (15)$$

Hence,

$$S_1 = (W - W\cos\theta) \times H = (1 - \cos\theta)W \times H \qquad (16)$$

The cold air incursion from the front of the open door, G_1 is

$$G_1 = v_{w1}S_1 \qquad (17)$$

where v_{w1} represents the velocity of cold air incursion from the front of the open door. Refer to Fig.4 and Fig.5, the top cold air flow area S_2 can be written as

$$S_2 = S_{o_1a_1b_1} + S_{a_1a_2b_1b_2} + S_{o_2a_2b_2} + S_{b_1b_2d_1}$$

$$= \frac{1}{2} \times \frac{W}{2}\cos\theta_1 \times \frac{W}{2}\sin\theta_1 + \left(W - \frac{W}{2}\cos\theta_1 - \frac{W}{2}\cos\theta_2\right) \times \frac{W}{2}\sin\theta_2 \qquad (18)$$

$$+ \frac{1}{2} \times \frac{W}{2}\cos\theta_2 \times \frac{W}{2}\sin\theta_2 + \frac{1}{2} \times \left(W - \frac{W}{2}\cos\theta_1 - \frac{W}{2}\cos\theta_2\right) \times \left(\frac{W}{2}\sin\theta_1 - \frac{W}{2}\sin\theta_2\right)$$

In particular, with $\theta_1=\theta_2=\theta$, S_2 can be reduce to

$$S_2 = \frac{1}{2} \times \left(W - \left(2 \times \frac{W}{2}\cos\theta\right) + W\right) \times \frac{W}{2}\sin\theta$$

$$= \frac{1}{2} \times (2W - W\cos\theta) \times \frac{W}{2}\sin\theta = \left(W - \frac{W}{2}\cos\theta\right) \times \frac{W}{2}\sin\theta = (2 - \cos\theta)\sin\theta \times \frac{W^2}{4} \qquad (19)$$

The cold air incursion from the top of the open door, G_2, in m^3/s, is

$$G_2 = v_{w2}S_2 \qquad (20)$$

where the symbol v_{w2} represents the velocity of cold air flow from the top of door.

Calculation Results of Cold Air Incursion. Let $\theta_1=\theta_2=\theta=\pi/6$, $\pi/4$, $\pi/3$, respectively, and $v_{w1}=v_{w2}=1$m/s, the values of S_1, S_2, and G_1, G_2 are calculated, the calculating date are listed in Table 1. Additionally, we evaluate the total cross-sectional area of the cold air incursion, S, and the total cold air incursion, G, which are also given in the Table 1.

Table 1 S and G of the open door for $v_{w1}=v_{w2}$ (Let $v_{w1}=v_{w2}=1$m/s).

θ_1 (rad)	θ_2 (rad)	S_1 (m^2)	v_{w1} (m/s)	G_1 (m^3/s)	S_2 (m^2)	v_{w2} (m/s)	G_2 (m^3/s)	S (m^2)	G (m^3/s)
$\pi/6$	$\pi/6$	0.3751	1	0.3751	0.2778	1	0.2778	0.6529	0.6529
$\pi/4$	$\pi/4$	0.8201	1	0.8201	0.4480	1	0.4480	1.2681	1.2681
$\pi/3$	$\pi/3$	1.4	1	1.4	0.6365	1	0.6365	2.0365	2.0365

Table 1 shows S_1 increases with increasing θ_1 from $\pi/6$ to $\pi/3$, so G_1 increases. As such, G_2 also increases with increasing S_2. From Table 1, we know that S_1 is about 0.4 m^2, 0.8 m^2 and 1.4 m^2 for the angular rotation θ_1, $\pi/6$, $\pi/4$ and $\pi/3$, respectively. S_2 is about 0.3 m^2, 0.4 m^2 and 0.6 m^2 for the angular rotation θ_2, $\pi/6$, $\pi/4$ and $\pi/3$, respectively. So the results illustrate that decreasing θ_1 and θ_2 can reduce the cold air incursion.

Furthermore, based on the date in the Table 1, we can also calculate that the average value of increasing G_1, $\Delta G_{1,average} \approx 0.5$m^3/s when θ_1 and θ_2 are raised, and the increasing value, $\Delta\theta_1=\Delta\theta_2=\pi/12$,

the average value of increasing of G_2, $\Delta G_{2,average} \approx 0.25 m^3/s$. That means $G=G_1+G_2$, is more affected by the increasing of G_1. It is clear that the growth rate of S_1 is higher than the one of S_2. Accordingly, the growth rate of G_1 is also higher than the one of G_2. From the calculating results shown in Table 1, we can see that the growth rate of S is the same as the one of G when we give $v_{w1}=v_{w2}=1m/s$. Thus the above results also prove that decreasing θ_1 and θ_2 can reduce the cold air incursion.

According to the basic principle of natural convection, the velocity of cold air flow from the top of the open door is different to the one from the front of the open door, for ordinary heating room, for example, our classroom. In normal conditions, $v_{w1}>v_{w2}$, hence, we let $v_{w2}=0.5m/s$, and $v_{w2}/v_{w1}=1/2$. Then S and G are calculated further, and the results are shown in Table 2. The data in the Table 2 shows the reduction of growth rate of G_2 is about $0.1m^3/s$, $0.2m^3/s$ and $0.3m^3/s$ for the angular rotation θ_2, $\pi/6$, $\pi/4$ and $\pi/3$, respectively. And for the total cold air incursion, G, its growth rate also reduces similarly.

Table 2   S and G of the open door for $v_{w1} \neq v_{w2}$ (Let $v_{w1}>v_{w2}$, $v_{w2}/v_{w1}=1/2$).

θ_1 (rad)	θ_2 (rad)	S_1 (m^2)	v_{w1} (m/s)	G_1 (m^3/s)	S_2 (m^2)	v_{w2} (m/s)	G_2 (m^3/s)	S (m^2)	G (m^3/s)
$\pi/6$	$\pi/6$	0.3751	1	0.3751	0.2778	0.5	0.1389	0.6529	0.5140
$\pi/4$	$\pi/4$	0.8201	1	0.8201	0.4480	0.5	0.2240	1.2681	1.0441
$\pi/3$	$\pi/3$	1.4	1	1.4	0.6365	0.5	0.3183	2.0365	1.7183

Calculating Heat Loss. Let us evaluate the heat loss due to cold air incursion using date in Table 1 and Table 2. The expression for the heat Q, kW, due to the cold air incursion loss, can be written as

$$Q = \rho G c \Delta t_1 \tag{21}$$

where let the density of air, $\rho=1.29 kg/m^3$, the specific heat capacity, $c=10^3 J/(kg \cdot \square)$, and the temperature difference of the cold and hot air $\Delta t_1=1\square$. The heat loss from the cold air incursion of the open door is calculated and shown in Table 3 with $v_{w2}=1m/s$, Table 4 with $v_{w2}=0.5m/s$.

Table 3   Heat loss from the open door (Let $v_{w2}=1m/s$).

θ_1 (rad)	θ_2 (rad)	S_1 (m^2)	v_{w1} (m/s)	S_2 (m^2)	v_{w2} (m/s)	S (m^2)	G (m^3/s)	Δt_1 (°C)	Q (kW)
$\pi/6$	$\pi/6$	0.3751	1	0.2778	1	0.6529	0.6529	1	0.842
$\pi/4$	$\pi/4$	0.8201	1	0.4480	1	1.2681	1.2681	1	1.636
$\pi/3$	$\pi/3$	1.4	1	0.6365	1	2.0365	2.0365	1	2.627

Table 4   Heat loss from the open door (Let $v_{w2}=0.5m/s$).

θ_1 (rad)	θ_2 (rad)	S_1 (m^2)	v_{w1} (m/s)	S_2 (m^2)	v_{w2} (m/s)	S (m^2)	G (m^3/s)	Δt_1 (°C)	Q (kW)
$\pi/6$	$\pi/6$	0.3751	1	0.2778	0.5	0.6529	0.5140	1	0.663
$\pi/4$	$\pi/4$	0.8201	1	0.4480	0.5	1.2681	1.0441	1	1.347
$\pi/3$	$\pi/3$	1.4	1	0.6365	0.5	2.0365	1.7183	1	2.217

Table 5   Influence of the change in v_{w2} on the cold air incursion and heat loss.

θ_1 (rad)	θ_2 (rad)	Δv_{w2} (m/s)	G' (m^3/s)	G'' (m^3/s)	ΔG (m^3/s)	Q' (kW)	Q'' (kW)	ΔQ (kW)
$\pi/6$	$\pi/6$	0.5	0.6529	0.5140	0.1389	0.842	0.663	0.179
$\pi/4$	$\pi/4$	0.5	1.2681	1.0441	0.2240	1.636	1.347	0.289
$\pi/3$	$\pi/3$	0.5	2.0365	1.7183	0.3182	2.627	2.217	0.410

Table 5 gives the influence of v_{w2} on the cold air incursion and heat loss from the open door. In Table 5, the symbol G' and G'' denote the total cold air incursion for the velocity, v_{w2}, 1m/s, 0.5m/s, respectively. And, Q' and Q'' denote heat loss for the velocity, v_{w2}, 1m/s, 0.5m/s, respectively.

From the above calculating results, we can know that the average value of decreasing of cold air incursion, $\Delta G_{average}$ is about $0.227 m^3/s$, and the heat loss also reduces correspondingly, the average value, $\Delta Q_{average}$ is about 0.293 kW when the velocity of cold air moving through the top section of the open door decreases, $\Delta v_{w2}=0.5 m/s$. The results also illustrate that the influence of velocity v_{w2} is smaller than the one of v_{w1}. So we can conclude that the main cold air incursion and heat loss are caused by the cold air moving through the front section of the open door.

Conclusions

Based on the behavioral mathematical experiment of opening and closing the door, we can know that during the same period of time, $\Delta \tau$, $G \times p_0 < G \times p_{30}$ because of $p_0 > p_{30}$ when the model is given "certain influences", obviously, its corresponding result is $Q \times p_0 < Q \times p_{30}$, which also illustrate that the increasing of p_0 is good for saving energy in short time.

References

[1] X.P. Li: *Probability* (Higher Education Press, China 1997).