

Analysis of pulsation of linear hydraulic motor

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Abstract. To suppress the pulsation problem for the linear hydraulic motor, the pulsation problems of instantaneous angular displacement pulsation and instantaneous angular torque were analyzed. The kinematics equations of the motor crankshaft linkage were established. The kinematics models were simulated and the dynamic was carried out in Matlab/Simulink Simulink environment. The pulsating problem of the linear hydraulic motor in different cylinder numbers were analyzed in detail. At the same times the pulsating law of the hydraulic motor of odd and even cylinder were compared. The results showed that when the number of cylinders is an odd number, the instantaneous angular displacement pulsation and the instantaneous angular torque pulsation of the motor are just half of the hydraulic motor of a contiguous even cylinder. And the pulsation rate of the motor gradually decreases with the increase of the number of the total cylinder. And the decrease is reduced with the increase of the number of the motor's total cylinder.

Introduction

In the hydraulic system, the hydraulic motor as the executive component of the hydraulic system, its displacement pulsation and torque ripple will directly affect the hydraulic system performance^[1]. Larger torque ripple will deteriorate the stability and uniformity of the hydraulic system. Larger pressure pulsations will also cause resonance of the pipeline valve and the entire system, which will have a devastating effect on the entire system, resulting in the entire The system's working life is reduced, more serious, and even lead to the system does not work^[2-5]. Same time as the torque ripple and also due to pressure pulsation of the hydraulic motor speed pulsation, can not achieve accurate speed control^[6-7]. This paper analyzes the displacement and torque pulsation of the linear hydraulic motor, and concludes the torque pulsation and displacement pulsation law.

Kinematics Analysis of Linear Hydraulic Motor

Because each hydraulic cylinder of the linear hydraulic motor has the same structure^[8], only one hydraulic cylinder can be analyzed while analyzing the kinematic of the motor piston. Fig. 1 is the schematic diagram of the linear hydraulic motor:

In the Fig. 1, a means the length of the hydraulic motor rod, b means the eccentric eccentricity, c means the distance from the piston pin axis of the small end of the connecting rod to the top surface of the piston, r means the eccentric radius. O is the eccentric center, O_1 is Crankshaft axis, that is, the center of eccentric rotation, S is the linear distance between the top surface of the motor piston and the crankshaft axis in the axial direction of the cylinder. When the piston moves to the top dead center position, which is located in the position shown in Fig. 1, S can be expressed as:

$$S = a + b + c + r \quad (1)$$

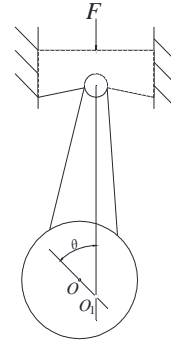
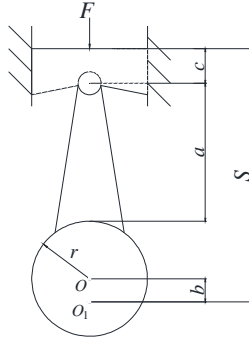


Fig. 1 linear hydraulic motor size diagram Fig. 2 Piston position size (through angle θ)

When the crankshaft rotates through angle θ , the position of piston is shown as Fig.2:

At this point, S can be expressed as:

$$S = b \cos q + \sqrt{(a+r)^2 - (b \sin q)^2} + c \quad (2)$$

If $K = \frac{b}{a+r}$, then expanding Eq. 2 using Taylor series yields:

$$S = b \cos q + c + (a+r) \left\{ 1 - \frac{1}{2} \left(\frac{b}{a+r} \sin q \right)^2 + \frac{1}{2} \times \frac{\left(\frac{1}{2} - 1 \right)}{2!} \left[\left(\frac{b}{a+r} \sin q \right)^2 \right]^2 + \dots \right\} \quad (3)$$

In Eq. 3, since b is the eccentricity of the eccentric, a is the length of the motor link, r is the radius of the eccentric, so the value of K is always less than 1, and the value of K is 0.2 in the routine design^[9]. Therefore, the third and subsequent terms of the Taylor series expansion are all higher order infinitesimals of the preceding paragraph. So it can be neglected and the formula is obtained as follows:

$$S = b \cos q + c + \frac{b}{K} \left[1 - \frac{1}{2} (K \sin q)^2 \right] \quad (4)$$

Piston and crankshaft axis distance shown in Fig. 3:

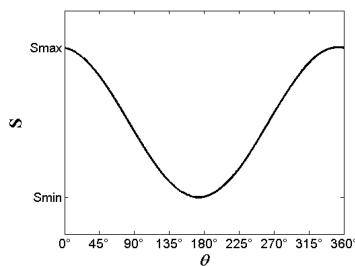


Fig. 3 piston and crankshaft axis distance

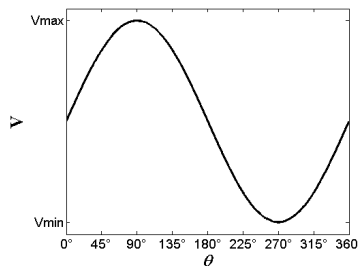


Fig. 4 piston velocity equation of the curve

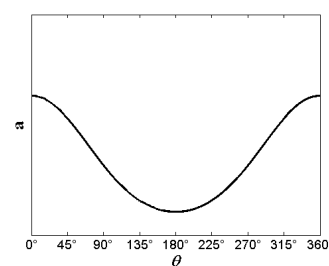


Fig. 5 piston velocity curve of the equation

It can be seen from the figure above that when the hydraulic motor runs for one cycle, that is, when the motor crankshaft rotates 360° and the θ value is 0° , the motor piston is at the top dead center. When the eccentric wheel rotates 180° , that is, when the θ value is 180° , the piston is at the bottom dead center at this time, and the two can obtain the maximum displacement of the piston in a running cycle of $2b$, that is, twice the eccentricity.

According to Eq. 4, the first derivative of S is obtained as the piston velocity equation:

$$v = \frac{dS}{dt} = bw \left(\sin q + \frac{1}{2} K \sin 2q \right) \quad (5)$$

Piston speed equation of the curve shown in Fig. 4:

According to Eq. 5, the first derivative of v for t gives the piston acceleration equation:

$$a = \frac{du}{dt} = bw^2 (\cos q + K \cos 2q) \quad (6)$$

Piston motion acceleration equation curve shown in Fig. 5.

Linear hydraulic motor transient angle displacement pulsation analysis

Hydraulic motor displacement refers to the hydraulic motor without taking into account factors such as leakage, the normal operation, the output shaft per revolution, the total hydraulic fluid suction motor volume. Under normal circumstances, the basic size of the hydraulic motor to determine the displacement of the hydraulic motor will be able to determine the basic, displacement is the hydraulic motor's own properties, not with the outside world interference or changes in the state changes. When the linear hydraulic motor runs one cycle, the total displacement of the motor is:

$$Q = 2MAb = \frac{Mp d^2 b}{2} \quad (7)$$

Where M is the number of motor cylinders, A is the area of the top surface of the piston, and d is the diameter of the piston.

Because the displacement of the motor for the same hydraulic motor is a fixed value, so no research and analysis here, I focus on the analysis of the instantaneous displacement of the hydraulic motor pulsation problems. Eq. 5 by the piston velocity equations available hydraulic motor single cylinder instantaneous angular displacement formula is as follows:

$$q(q) = Au = Ae \left(\sin q + \frac{1}{2} K \sin 2q \right) \quad (8)$$

Therefore, the number of working hydraulic cylinders of a hydraulic motor M , the instantaneous angular displacement of the entire motor formula is as follows:

$$Q(q) = \sum_{i=1}^M Ae \left(\sin q + \frac{1}{2} K \sin 2q \right) \quad (9)$$

The compressibility of the hydraulic oil is not calculated, and because the flow distribution cover covers only a small part of the region (the length is usually $1 \sim 1.5^\circ$)^[10], so in calculating the instantaneous angle of the hydraulic motor Displacement may not consider its impact, that is, when the mathematical singularities. Eq. 9 through the trigonometric function summation and deformation formula can be obtained formula:

$$Q = Ab \left\{ \frac{\sin \left[q + (M-2) \times b \right] \times \sin \left[\left(\frac{M}{2} \right) \times b \right]}{\sin(2b)} + \frac{\sin \left[2q + 2(M-2) \times b \right] \times \sin \left[2 \left(\frac{M}{2} \right) \times b \right]}{\sin(4b)} \right\} \quad (10)$$

Where in $2\beta = 2\pi / M$, a phase angle of the crank on the crank angle of the two adjacent cylinders differ.

Even-numbered cylinder hydraulic motor instantaneous angle displacement pulsation analysis. When the number of working cylinders of the hydraulic motor is even, if the hydraulic motor is in normal operation, the number of motor cylinders in the high pressure chamber is $M/2$. Therefore, the theoretical equation for the instantaneous angular displacement of a linear hydraulic

motor with an even number of cylinders changing with the rotation of the crank angle is:

$$Q = Ab \sum_{n=0}^{\frac{M}{2}-1} \left[\sin(q + 2nb) + \frac{1}{2} K \sin 2(q + 2nb) \right] (0 \leq q \leq 2b) \left(2b = \frac{2p}{M} \right) \quad (11)$$

The curve diagram of the instantaneous angular displacement of an even cylinder linear hydraulic motor with the rotation of the crankshaft angle can be obtained by the Eq. 11, as shown in Fig. 6.

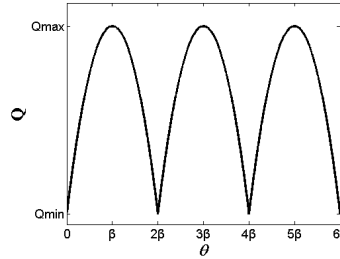


Fig. 6 instantaneous angular displacement of motor of even number cylinder

As can be seen from the above picture, the instantaneous angular displacement is a half sinusoidal periodic function with the crankshaft angle, and the period is 2β . According to the Eq. 11, the maximum value of Q is present at $q = \frac{p}{M}$, $Q(\frac{p}{M})_{\max} = Ab(\frac{1}{\sin \frac{p}{M}})$; At the time

of $q=0$ or $q = \frac{2p}{M}$, the Q has a minimum value, $Q(0)_{\min} = Q(\frac{2p}{M})_{\min} = Ab \cot \frac{p}{M}$

According to the calculation formula of hydraulic motor pulsation rate, the instantaneous angular displacement pulsation rate of the motor is as follows:

$$d_Q = \frac{1 - \cos \frac{p}{M}}{\frac{M}{p} \times \sin \frac{p}{M}} \quad (12)$$

analysis of instantaneous angular displacement pulsation of an odd number cylinder hydraulic motor. The total number of working cylinders when the hydraulic motor is odd, the crank angle adjacent the two hydraulic cylinder crankshaft phase angle difference of $2\beta = 2\pi / M$.

When the hydraulic motor is in normal operation, if the crankshaft rotation angle is $0 \leq q \leq b$, which means the hydraulic motor in the hydraulic cylinder cavity pressure ratio in the low-pressure cavity of the hydraulic cylinder one, the high pressure chamber of the motor hydraulic cylinder number is $(M+1)/2 - 1$. And if the crankshaft rotation angle is $b \leq q \leq 2b$, which means the hydraulic motor in the hydraulic cylinder cavity pressure ratio in the low-pressure cavity of the hydraulic cylinder less moment, the high pressure chamber of the motor hydraulic cylinder number $(M-1)/2 - 1$. Therefore, the theoretical equation for the change of the instantaneous angular displacement of the linear hydraulic motor with the angle rotation of the crankshaft is as follows:

When the angle of rotation of the crankshaft is $0 \leq q \leq b$:

$$Q = Ab \sum_{n=0}^{\frac{M-1}{2}} \left[\sin(q + 2nb) + \frac{1}{2} K \sin 2(q + 2nb) \right] \quad (13)$$

When the angle of rotation of the crankshaft is $b \leq q \leq 2b$:

$$Q = Ab \sum_{n=0}^{\frac{M-1}{2}} \left[\sin(q + 2nb) + \frac{1}{2} K \sin 2(q + 2nb) \right] \quad (14)$$

From the Eq. 13 and Eq. 14 the instantaneous angular displacement of the odd cylinder linear motor can be changed along with the change of the crankshaft angle, as shown in Fig. 7.

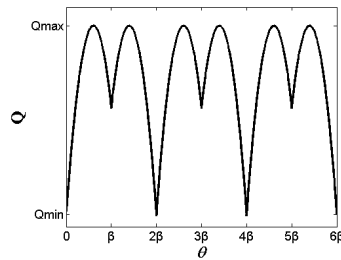


Fig. 7 instantaneous angular displacement curve of motor of odd numbered cylinder

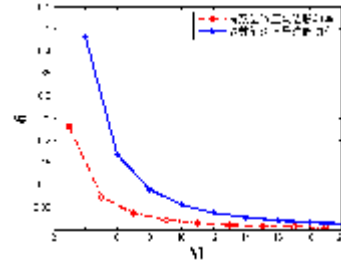


Fig. 8 instantaneous angular displacement pulsation diagram of linear hydraulic motor

As can be seen from Fig. 7, the instantaneous angular displacement of the motor varies periodically with the change of the crankshaft angle, and the period is 2β . According to the Eq. 13- Eq. 14 the

maximum value of Q is found at $q = \frac{b}{2} = \frac{p}{2M}$ or $q = \frac{3b}{2} = \frac{3p}{2M}$,

$$Q\left(\frac{p}{2M}\right)_{\max} = Q\left(\frac{3p}{2M}\right)_{\max} = Ab\left(\frac{1}{\sin \frac{p}{2M}}\right) = Ab\left(\frac{1}{\sin \frac{3p}{2M}}\right); \text{At the time of } q=0 \text{ or } q=\frac{2p}{M}, \text{ the } Q \text{ has}$$

a minimum value, $Q(0)_{\min} = Q\left(\frac{2p}{M}\right)_{\min} = Ab \cot \frac{p}{M}$. And the instantaneous angular displacement pulsation rate

of the motor can be obtained as follows:

$$d_Q = \frac{Ab\left(\frac{1}{\sin \frac{p}{2M}}\right) - Ab \cot \frac{p}{M}}{\frac{1}{2p} \int_0^{2p} Q dq} = \frac{\frac{1}{2 \sin(\frac{p}{2M})} - \frac{\cot(\frac{p}{2M})}{2} + \frac{K \times \tan(\frac{p}{M})}{4}}{\frac{M}{p}} \quad (15)$$

Combining the Eq. 12 and Eq. 15, the number of the hydraulic cylinder of the motor is 4-20, and the instantaneous angular displacement pulsation rate of the linear hydraulic motor of each cylinder is drawn, as shown in Fig. 8.

We can see from figure 8, when the linear type hydraulic motor cylinder has an odd number of cylinders, the motor angular pulsation rate is approximately half and there is a number of even cylinder hydraulic motor instantaneous angular displacement pulse rate, the motor and the instantaneous angular pulsation ratio increases with the total number of cylinders decreases gradually. The increase and the decrease amplitude with the total number of cylinders and reduce motor.

Analysis of Instantaneous Angle Torque Ripple of Linear Hydraulic Motor

The hydraulic motor is an actuator that performs work directly on the external system in the hydraulic system. The most important form of work is the transformation of hydraulic energy into

the mechanical energy of the output shaft. Therefore, for the hydraulic motor, its torque is an important indicator. The following will analyze the torque ripple analysis of the linear hydraulic motor. In general, when discussing the torque of a hydraulic motor, the friction loss of the motor and the system is negligible. Therefore, when the linear hydraulic motor runs for one cycle, the total output torque of the motor is:

$$T = \Delta p * Q = M * \Delta p * \frac{q p d^2 b}{2} = M * F * 2b \quad (16)$$

In the formula, ΔP is the pressure difference between the high pressure chamber and the low pressure chamber of the hydraulic motor distribution plate. It is represented by P afterwards and the unit is Pa. F represents the pressure of the hydraulic oil on the top surface of the motor piston, and the unit is N; the remaining parameters are consistent with the previous text. From Eq. 16, it can be seen that the torque of the hydraulic motor running for one cycle is a fixed value for the same hydraulic motor, so no research and analysis is performed here, and the analysis of the momentary torque ripple of the hydraulic motor is focused on. Since the regularity and characteristics of the torque generated by each hydraulic cylinder of the linear hydraulic motor are the same during operation, only one hydraulic cylinder can be analyzed when the instantaneous torque ripple analysis of the motor piston is performed. Hydraulic motor single cylinder output shaft force diagram as shown below:

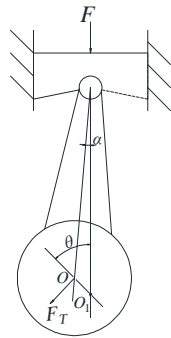


Fig. 9 output shaft force diagram

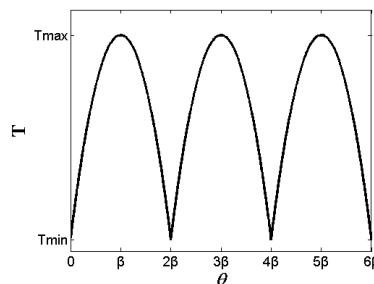


Fig. 10 Instant torque curve of even-numbered

In Fig. 9, α indicates the angle between the axis of the piston pin and the center of the eccentric shaft and the angle between the axis of the piston pin and the axis of the crankshaft when the motor eccentric rotates by θ about the axis of the crankshaft. Decomposing F orthogonally results in a

tangential force $F_T = \frac{F}{\cos \alpha} \sin(\alpha + \theta)$ perpendicular to the connecting line between the center of

the eccentric and the axis of the crankshaft and a normal force $F_N = \frac{F}{\cos \alpha} \cos(\alpha + \theta)$ along the line

connecting the center of the eccentric and the axis of the crankshaft. We can obtain the momentary torque formula for a single cylinder of a hydraulic motor as follows:

$$T = Fb \left(\frac{b}{r + a} \sin \theta \cos \theta + \sin \theta \right) \quad (17)$$

Therefore, for a hydraulic motor with a working hydraulic cylinder number M , the instantaneous angular torque formula for the entire motor is as follows:

$$T_{all} = Fb \sum_{i=1}^M (K \sin \theta \cos \theta + \sin \theta) \quad (18)$$

Analysis of Instantaneous Angle Torque Ripple of Even-Cylinder Hydraulic Motor. When there are an even number of hydraulic cylinders in operation, the instantaneous angular torque of

the linear hydraulic motor is:

$$T_{all} = Fb \sum_{n=0}^{\frac{M-1}{2}} [K \sin(q + 2nb) \cos(q + 2nb) + \sin(q + 2nb)] \quad (0 \leq q \leq 2b) \left(2b = \frac{2p}{M} \right) \quad (19)$$

Eq. 19 can be used to obtain the curve diagram of the instantaneous angular displacement of the even-numbered cylinder linear hydraulic motor as the crankshaft angle rotation changes, as shown in Fig. 10. As can be seen from Fig. 10, the torque is a semi-sinusoidal periodic function with a crank angle and the period is 2β . According to Eq. 19, the instantaneous angular torque rate of the even-numbered cylinder hydraulic motor is:

$$d_r = \frac{1 - \cos(\frac{p}{M})}{\frac{M}{p} \times \sin(\frac{p}{M})} \quad (20)$$

Analysis of Instantaneous Angle Torque Ripple of an Odd-Cylinder Hydraulic Motor.

When the total number of hydraulic motor cylinders is odd, the working principle of the hydraulic cylinder is the same as that of the previous displacement analysis. Therefore, the theoretical equation for the change of the instantaneous angular torque of an odd-numbered linear hydraulic motor with a change in the angular rotation of the crankshaft is:

- ① When the angle of rotation of the crankshaft is $0 \leq q \leq b$:

$$T_{all} = Fb \sum_{n=0}^{\frac{M+1}{2}-1} [K \sin(q + 2nb) \cos(q + 2nb) + \sin(q + 2nb)] \quad (21)$$

- ② When the angle of rotation of the crankshaft is $b \leq q \leq 2b$:

$$T_{all} = Fb \sum_{n=0}^{\frac{M-1}{2}} [K \sin(q + 2nb) \cos(q + 2nb) + \sin(q + 2nb)] \quad (22)$$

Eq. 21 and Eq. 22 can be used to obtain the curve of the instantaneous angular displacement of the odd-numbered cylinder linear hydraulic motor as the crank angle rotates, as shown in Fig. 11:

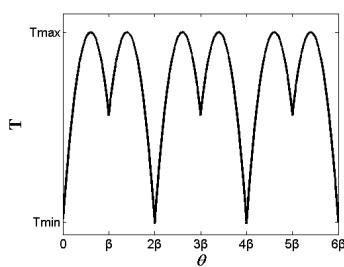


Fig. 11 Instant torque curve of even-numbered

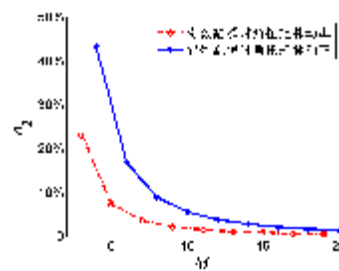


Fig. 12 Instantaneous torque ratio of straight-type hydraulic motor

As can be seen from Fig. 7, the instantaneous angular displacement of the motor changes periodically with the change of the crank angle, and the period is 2β . According to Eq. 21 and 22, the instantaneous angular torque ratio of an odd-numbered cylinder hydraulic motor is:

$$d_r = \frac{\frac{1}{2 \sin(\frac{p}{2M})} - \frac{\cot(\frac{p}{2M})}{2} + \frac{K \times \tan(\frac{p}{M})}{4}}{\frac{M}{p}} \quad (23)$$

With Eq. 20 and Eq. 23, take the number of motor cylinders as 4-20, and plot the momentary angular torque ripple rate of each linear hydraulic motor, as shown in Fig. 12.

When the number of linear hydraulic motor cylinders is an odd number of cylinders, the instantaneous angular torque of the motor is approximately half that of an even numbered cylinder hydraulic motor that differs by one cylinder number, and the instantaneous angular torque of the motor increases with the total number of cylinders. Gradually decrease, and the magnitude of decrease decreases with the increase in the total number of motors.

Conclusion

When the number of linear hydraulic motor cylinders is an odd cylinder, the instantaneous angular displacement of the motor is approximately half that of an even cylinder hydraulic motor with a difference of one cylinder, and the instantaneous angular displacement of the motor is equal to the total number of cylinders. Increasing and gradually decreasing, and the decreasing amplitude decreases with the increase of the total number of motors. When the number of linear hydraulic motor cylinders is an odd number of cylinders, the instantaneous angular torque of the motor is approximately half that of an even numbered cylinder hydraulic motor that differs by one cylinder number, and the instantaneous angular torque of the motor increases with the total number of cylinders. Gradually decrease, and the magnitude of decrease decreases with the increase in the total number of motors.

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