# Low Sidelobe Thinned Arrays by Means of Time Modulated Technique

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**Keywords:** Time modulated; Thinned array; Iterative Fourier technique; Sidelobe level

**Abstract.** Being inspired by the time modulated technology, an approach for synthesizing linear thinned array with low sidelobe level (SLL) is presented. In the proposed method, the thinned array is obtained through replacing the static element weighting by the time duration of a prescribed modulation cycle, and the iterative Fourier technique (IFT) is performed to explore low sidelobe array. Furthermore, for the un-neglected higher sideband level (SBL) of the harmonics, the algorithm of differential evolution (DE) is adopted to lower the SBL through optimizing the on-off time of time modulated elements. Numerical instances validate that the obtained thinned array has considerable reduced sidelobe level as well as reduced SBL, as is compared to the conventional thinned array.

## Introduction

Thinned arrays had attracted the attentions of many researchers for its good features in lowering antenna cost and relaxing feeding network. Until recently, a number of tools, such as the genetic algorithm (GA) [1], the statistical method [2], the ant colony optimization (ACO) [3], the iterative Fourier technique [4], and so on, had been adopted for the synthesis of thinned arrays. The common characteristic of those methods is to explore thinned arrays with lower sidelobe levels by purely density tapering, but it is not always effective especially in the case of ultra-low sidelobe requirement. For the above reason, another way to arrive at ultra-low sidelobe levels is usually performed through resorting to the methods of amplitude tapering such as Taylor and Chebyshev. However, the feeding network would in exchange become complex and even unable to be realized by practical engineering.

The time modulated array (TMA), which takes time as an additional degree into traditional arrays, was first proposed by Shanks and Bickmore in 1958 [5]. According to theory of TMA, a number of high speed RF switches are periodically switched on and off by an electronic control circuit so that the excitation weighting of static element can be replaced by the time duration of a prescribed modulation cycle. In other words, the amplitude weighting function of conventional antenna arrays can be synthesized in a time-average sense. Owing to the above reason, TMA has been deeply studied for the synthesis of low sidelobe arrays in the past decades. However, the major disadvantage of TMA is that many sideband signals spaced at multiples of the modulation frequency existed, which make the electromagnetic energy radiated into harmonic beams [6]. Therefore, many researchers focus their attention mainly on the way how to suppress the sideband level (SBL) of TMA [6-10].

In this paper, a method for exploring low sidelobe thinned arrays is presented by means of time modulated technique. In the method, the thinned array with each element fed by non-uniform amplitude is achieved firstly by the IFT as is described in [4]. Then, each turned ON element is switched on for a normalized time duration that is equal to its static amplitude weighting. As a result, the sidelobe level of the thinned array is considerably reduced. Numerical result validates the effective of the proposed method.

## The Model of Time Modulated Array

Consider a linear array including N uniform excited elements spaced at half wavelength, the array factor can be represented by

$$F(\theta) = \sum_{n=0}^{N-1} A_n e^{jn\pi\cos\theta} \tag{1}$$

where $\theta$ is the angle measured with respect to the direction of linear array. $A_n$ is the nth element excitation. For convenience, all the element excitations are labeled as $\{A_n\}$. According to the theory of time modulating, when all the above N elements are time modulated, Eq.1 can be written as

$$F(\theta) = \sum_{n=0}^{N-1} A_n \gamma_n(t) e^{jn\pi\cos\theta} \tag{2}$$

and $\gamma_n(t)$ satisfies the relationship as below

$$\gamma_n(t) = \begin{cases} 1 & 0 \leq \tau_{nON} < t < \tau_{nOFF} \leq T_0 \\ 0 & 0 \leq t < \tau_{nON} \text{ 且 } \tau_{nOFF} < t \leq T_0 \end{cases} \tag{3}$$

Eq.3 describes the switched state of nth element, where $\tau_{nON}$ and $\tau_{nOFF}$ respectively denote the turned on as well as turned off time, $T_0$ is the modulated cycle. Due to the periodic time modulation, the expression of $\gamma_n(t)$ in the form of frequency domain according to the theory of Fourier series, can be describes as

$$\gamma_n(t) = \sum_{m=-\infty}^{+\infty} \frac{\sin[m\pi(\tau_{nOFF}-\tau_{nON})/T_0]}{m\pi} e^{-j\pi m(\tau_{nOFF}+\tau_{nON})/T_0} e^{jm\omega_0 t} \tag{4}$$

where $\omega_0$ is the modulation frequency. The far field pattern can be obtained by combining Eq.1 and Eq.4, therefore

$$F(\theta) = \sum_{m=-\infty}^{+\infty} e^{jm\omega_0 t} \sum_{n=0}^{N-1} A_n \frac{\sin[m\pi(\tau_{nOFF}-\tau_{nON})/T_0]}{m\pi} e^{-j\pi m(\tau_{nOFF}+\tau_{nON})/T_0} e^{jn\pi\cos\theta} \tag{5}$$

As can be seen from the above equation, the pattern of TMAA can be divided into two parts. One part is main radiation pattern than operated at center frequency represented by m=0. Accordingly, m≠0 corresponds to the mth harmonic pattern. Eq.5 shows that the main pattern depends only on the on-duration of the elements within a modulation cycle, and the harmonic pattern is not only related to the time duration of each element, but also determined by the on-off state of the element. Generally, the first harmonic pattern has higher level of sideband level, therefore, a common approach to suppress the harmonic radiation is to use the strategy of pulse shifting to minimize the SBL of first harmonic.

## The Method of the Iterative Fourier Transform

For the linear array with N isotropic elements, the proposed method can be described below.
1) Randomly initialized the element excitations $\{A_n\}$ within (0,1).
2) Perform K point IFFT over the element excitations $\{A_n\}$ to get the far field pattern $F(\theta)$ with $K > 2N$ to meet the requirement of the sampling theory.
3) Adjust the value of $F(\theta)$ in sidelobe region to match the pre-specified value.
4) Compute $\{A_n\}$ from the matched pattern using K point FFT.
5) Truncate the K points of $\{A_n\}$ to M points that coincide with the antenna array while set the value of rest points to be zero.
6) Select the $M \cdot f_0$ sampling points of $\{A_n\}$ which have larger amplitudes and set their values equal to one, while the values of rest points equal to zero where $f_0$ represents the filling factor.
7) Repeat steps 1)-6) until the same element distribution is obtained in two consecutive iterations, or the maximum number of iteration is arrived.

## Numerical Examples

Consider a symmetrical linear array that includes 100 lattices spaced at half wavelength. The filling factor is equal to 76%. Fig. 1 shows the far field pattern at the center frequency. The SLL is equal to -26.24dB, which is about 5.71dB lower than the synthesis result presented by the GA [11].
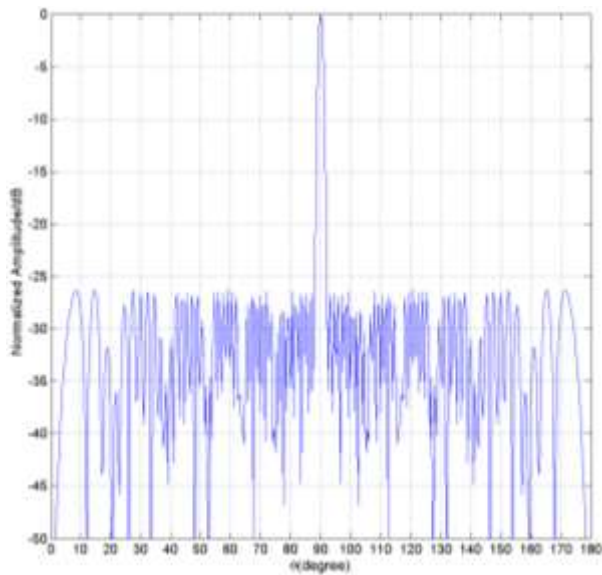


Figure 1. Finite The far field pattern at the center frequency for thinned TMA with filling factor equal to 76%
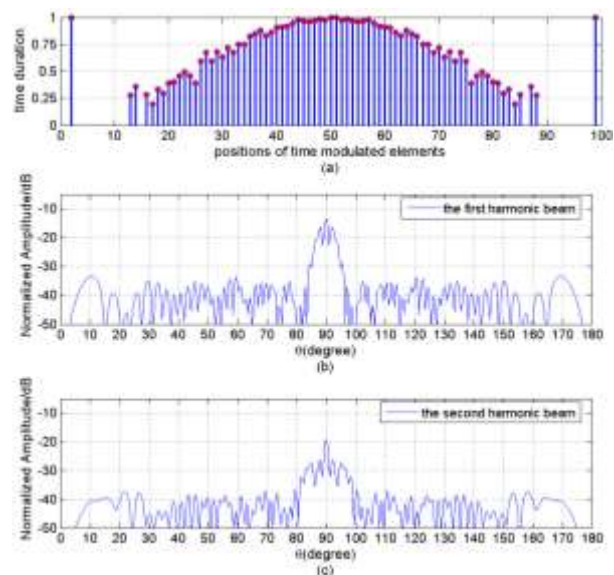


Figure 2. Finite (a) The element time duration for thinned TMA with filling factor equal to 76%. (b) the first harmonic beam; (c) the second harmonic beam

However, as a tradeoff, the directivity of this thinned array is reduced from 18.92dB [11] to 17.24dB because of the time modulation. More specifically, the reason lies in the fact that part of the radiation power illuminate into the harmonic patterns. The calculation for the power loss shows that about 21.86% of total power has been lost in this instance. Furthermore, Fig. 2(a) shows the time durations of turned ON element versus the element positions. It can be seen that the distribution of time durations well matches the tapering scheme when excluding the elements near
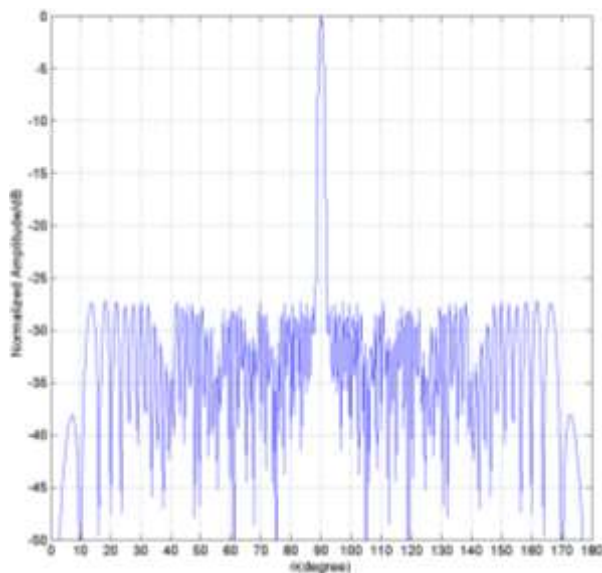


Figure 3. Finite The far field pattern at the center frequency for thinned TMA with filling factor equal to 80%
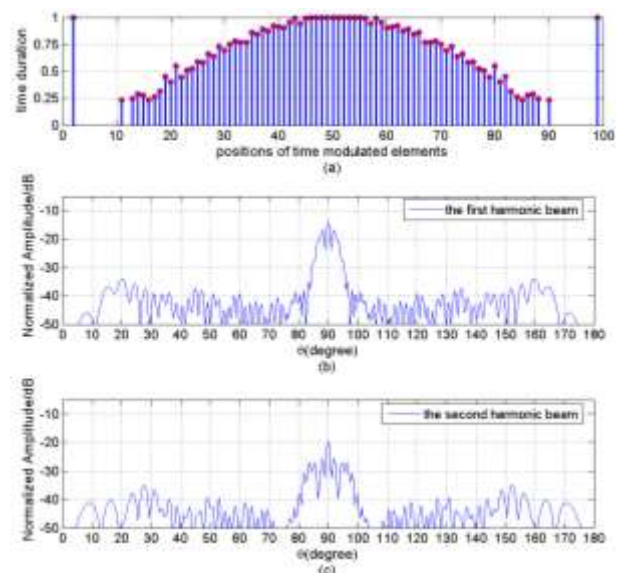


Figure 4. Finite (a) The element time duration for thinned TMA with filling factor equal to 80%. (b) the first harmonic beam; (c) the second harmonic beam

The two ends. Fig. 2(b) and Fig. 2(c) depict the first and the second harmonic beams caused by time modulation. The corresponding SBL is equal to -13.41dB and -19.11dB, respectively. Similarly, we further consider a symmetrical linear array with filling factor equal to 80%. The obtained

thinned time modulated array have the sidelobe level equal to -27.11dB, about 6.59dB reduced as compared to the value obtained by the ACO [3]. Fig. 3 shows the far field pattern that operates at the center frequency. Comparing with the conventional array, this array has a power loss about 20.7% because of the influence of time modulation.

## The Sideband Suppression by the Method of Differential Evolution

Although the far field pattern at the center frequency has ultra-low sidelobe level, as compared to conventional thinned array, the high SBL of harmonic beams, which should be avoided in most cases, is unacceptable. Therefore, in this section, the algorithm of differential evolution (DE) is used to optimize the on-off state of the dynamic elements, so that the sideband level of the harmonic
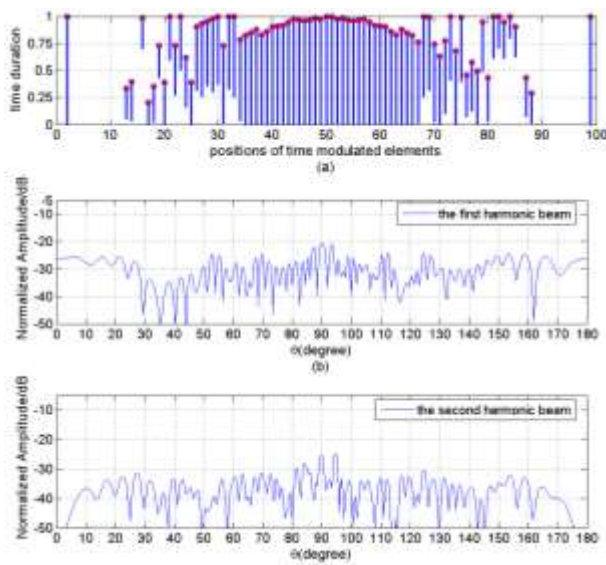


Figure 5.   Finite (a) The element time duration optimized by DE for thinned TMA with filling factor equal to 76%. (b) the first harmonic beam; (c) the
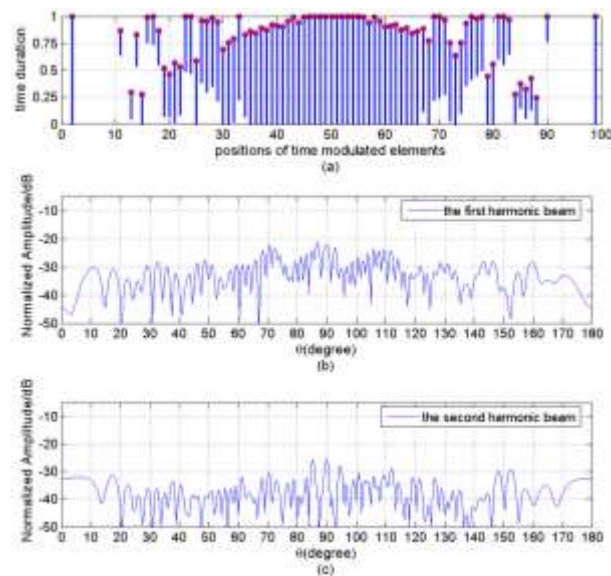


Figure 6.   Finite (a) The element time duration optimized by DE for thinned TMA with filling factor equal to 80%. (b) the first harmonic beam; (c) the

beams is effectively suppressed. The mutation strategy of rand/1/0 is adopted. The initial parameters of DE, including the population size, the scaling factor, the maximum of generation, and the mutation probability, is respectively equal to 20, 0.7, 20 and 0.9. The final optimization results for the above mentioned array are described by Fig. 5 and Fig. 6. To be detail, Fig. 5(a) and Fig. 6(a) give the on-off state of the dynamic elements after pulse shifting, and the time duration for each element of thinned array is not changed comparing with that presented in Fig. 2 and Fig. 4. The first and second harmonic patterns in Fig. 5 and Fig. 6 show that at least 7dB SBL reduced for the first harmonic beam, and about 5.5dB SBL reduced for the second harmonic beam. Therefore, we can see that the sideband level of the first and the second harmonic beam is considerably suppressed.

## Conclusion

The synthesis results for the two presented instances show the advantageous of the proposed thinned TMAA by means of lower sidelobe level. Furthermore, based on the theory of pulse shifting, the global algorithm of DE is adopted to suppress the first and second harmonic beams. The optimization results show the effectiveness of DE in sideband suppression. The proposed method can be also extended to planar thinned array by minor modification. Therefore, the method provides an effective way for the pattern synthesis of the low sidelobe thinned arrays.

## Acknowledgements

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