

Dynamic Study of Water Droplets in Two Phase Liquid by Ultrasound

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Abstract. In this paper, the dynamic equation of the simplified model of dispersed droplets in two phase liquid under ultrasonic excitation is derived based on the derivation method of cavitation bubble motion equation. The influence of different parameters on the radius of droplet under ultrasonic action is obtained by MATLAB methods. The results show that ultrasonic wave velocity has very little effect on the variation of c radius in different dispersed phase liquid. The smaller the liquid density is, the larger the radius of water-drop grows. The larger the hydrostatic pressure is, the harder the volume of water-drop will change. The theoretical basis for further study on the mechanism of ultrasonic effect on the reverse phase emulsion is established through the exploration of the change of water-drop radius.

1 Introduction

The mechanism of ultrasonic cavitation has been widely concerned [1], Rayleigh first put forward the kinetic simulation of a single spherical cavitating bubble in an infinite incompressible liquid in 1917, and the results of the calculation are sometimes not consistent with the effect of surface tension, gas content and compressibility on the bubble motion. The R-P equation [2] is modified by introducing the acoustic radiation factor by Prosperetti A., but these are the motions of tiny bubbles in the liquid. In the real life, the two-phase system of liquid-liquid two-phase system or membrane isolation under ultrasonic action is often encountered, such as: ultrasonic catalytic emulsification system [3], ultrasound in the life of the cell membrane in the transport of substances and energy transmission [4], how the ultrasound effect of soft matter droplets (such as: gels, vesicles, etc.) will be change? Ultrasound liposuction in the process [5], the mechanism of ultrasound on fat and fat cells these similar to the role of water droplets matter is expected. So it is very important to study the inverse emulsion in ultrasonic field. This paper refers to the derivation method of cavitation bubble motion equation [6], derives the motion equation of water droplets under the action of ultrasound, studies the change of the water droplet radius in the oil under the ultrasonic action, and establishes the theoretical basis for further studying the mechanism of ultrasonic to inverse emulsion action.

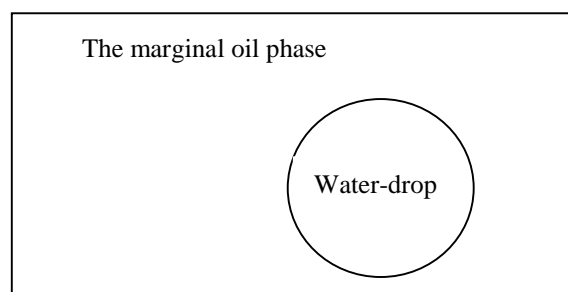


Fig.1 Simplified model of a water-drop in oil phase fluid

2 Theoretical Deduction of the Kinetic Equation of Water Droplets

2.1 Simplification of Model.

In this paper, we study the motion of a water droplet in an infinitely large dispersed phase by using oil as a dispersed phase, as shown in Fig. 1. Liquid compressibility is relatively small, but the addition

of high-strength ultrasound, water droplets due to the external pressure caused by the small volume change and the radius of micron-level initial water droplets volume, the change can not be ignored, that is, the compressibility of water droplets cannot be ignored, The small volume change caused by the same applied pressure acting on the dispersed phase liquid is negligible compared with the whole volume of the dispersed phase, so the dispersed phase liquid can still be assumed as incompressible. It is assumed that the volume expansion and contraction of water droplets under the action of ultrasonic field will certainly be accompanied by energy conversion. The equation of water droplet motion under multiple parameters is deduced by energy conservation, the model is simplified as follows: ① The shape of water droplets is always in the course of motion; ② water droplets wall only do radial movement to avoid the complicated situation caused by the asymmetry of water droplet wall motion; ③ assumes that the dispersed phase liquid is incompressible liquid and the water droplet is a compressible liquid; ④ ignores the influence of gravity, ⑤ water droplets have no energy loss during movement.

2.2 Stress Analysis of Water Droplet Wall

Because of the different molecular properties of the water droplet wall interface, the force field on the interface is not isotropic, so there exists interfacial tension. If we know the surface tension of two-phase matter, we can use the harmonic averaging equation to find the interfacial tension between the two phases, and the harmonic average equation is mainly applied to the interfacial tension between low energy substances (such as polymers, organic liquids, water, etc.) [7]. The additional pressure on the surface of the convex liquid surface is [8]:

$$\Delta P = \frac{2\sigma}{R} \quad (1)$$

Therefore, on the liquid surface, its surface tension coefficient is $\sigma = R\Delta P/2$, So the interfacial tension on the water droplet wall can be obtained by using the harmonic equation:

$$\sigma_{12} = \sigma_1 + \sigma_2 - \frac{4\sigma_1^d \sigma_2^d}{\sigma_1^d + \sigma_2^d} - \frac{4\sigma_1^p \sigma_2^p}{\sigma_1^p + \sigma_2^p} \quad (2)$$

Because the two-phase liquid is oil and water respectively, so after weighing to simplify, the interfacial tension coefficients are uniformly taken $\sigma_{12}=0.0500\text{N/m}$.

Without ultrasonic action, oil and water droplets composed of the system is still fluid system, so its constituent interface is equal pressure surface, water droplets in the oil to maintain balance, water droplets wall internal and external pressure are equal, so there are:

$$P_{in} = P_{out} = P_0 + \frac{2\sigma_{12}}{R_0} \quad (3)$$

The P_0 is the static pressure of the oil around the water droplets, and the R_0 is the initial radius of the water droplets.

2.3 Derivation of Water Droplet Kinetic Equation

Assuming the whole process is isothermal, the droplet volume is only a function of the pressure. The isothermal state equation of the liquid can be obtained by deriving the isothermal compression coefficient and the volumetric elastic modulus:

$$V = A/(P + C)^n \quad (4)$$

For constant temperature, the same liquid A,C,n is a fixed value, the water droplet temperature is 40 °C in this paper, then A=3.6881, C=3653, n=0.15811.

In the absence of ultrasound, the isothermal state equation of water droplets is:

$$V_0 = A/(P_{in} + C)^n \quad (5)$$

When ultrasonic is applied, the corresponding pressure in the water droplet wall is P_{in}' , the isothermal state equation for the water droplets is:

$$V = A/(P_{in}' + C)^n \tag{6}$$

Union (5), (6) is available:

$$\frac{R_0^3}{R^3} = \frac{V_0}{V} = \frac{(P_{in}' + C)^n}{(P_{in} + C)^n} \tag{7}$$

Further finishing can be:

$$\begin{aligned} P_{in}' &= \left(\frac{R_0}{R}\right)^{\frac{3}{n}} (P_{in} + C) - C \\ &= \left(\frac{R_0}{R}\right)^{\frac{3}{n}} \left(P_0 + \frac{2\sigma_{12}}{R_0} + C\right) - C \end{aligned} \tag{8}$$

The external force of the corresponding water droplet wall is

$$P_{out}' = P_0 + \frac{2\sigma_{12}}{R} + P_A \tag{9}$$

For $p_A = p_a \sin(\omega t)$ is the pressure on the water droplet wall by ultrasonic action.

Under the external force, the water droplets change from the radius R_0 to R , and the water droplets work as follows:

$$W = -\int_{R_0}^R P 4\pi R^2 dR \tag{10}$$

The kinetic energy obtained by the liquid droplets contraction is:

$$E_k = \frac{1}{2}mv^2 = \int_R^\infty \frac{1}{2} \rho 4\pi r^2 dr \left(\frac{dr}{dt}\right)^2 = 2\pi\rho R^3 \left(\frac{dR}{dt}\right)^2 \tag{11}$$

According to the conservation of energy, water droplets change from radius R_0 to R under external force, and the work done by the forces on water droplets equals the kinetic energy obtained by water droplets:

$$\int_{R_0}^R P 4\pi R^2 dR = 2\pi\rho R^3 (R')^2 \tag{12}$$

$$P = P_{in}' - P_{out}'$$

On both sides of the r differential, finishing can be:

$$R(\ddot{R}) + \frac{3}{2}(\dot{R})^2 = \frac{1}{\rho}(P_{in}' - P_{out}') \tag{13}$$

If further consideration is given to the energy viscosity loss in the process of water droplet movement, and the radiation damping that exists when the water droplets vibrate to the dispersed phase, the pout ' formula can be amended to:

$$R\ddot{(R)} + \frac{3}{2}\dot{(R)}^2 = \frac{1}{\rho} \left[\left(P_0 + \frac{2\sigma_{12}}{R_0} + C \right) \left(\frac{R_0}{R} \right)^{\frac{3}{n}} - C - \left(P_0 + \frac{2\sigma_{12}}{R} + p_A \right) \right] - \frac{4\mu}{\rho R} \dot{R} + \frac{R}{\rho c} \frac{d}{dt} \left[\left(P_0 + \frac{2\sigma_{12}}{R_0} + C \right) \left(\frac{R_0}{R} \right)^{\frac{3}{n}} - C - p_A \right] \quad (14)$$

The viscosity coefficients of μ as dispersed phase liquid, $\frac{4\mu}{\rho R} \dot{R}$ for the viscous loss term, the damping loss term of acoustic radiation is:

$$\frac{R}{\rho c} \frac{d}{dt} \left[\left(P_0 + \frac{2\sigma_{12}}{R_0} + C \right) \left(\frac{R_0}{R} \right)^{\frac{3}{n}} - C - p_A \right] \quad (15)$$

The motion equation of the water droplet wall under the influence of different parameters belongs to the 2 order nonlinear ordinary differential equation, and the analytic solution is not obtained.,need to be deformed first $\dot{R} = R(t, R)$ a set of first order differential equations, The numerical solution is obtained by numerical iterative method. In this paper, the 4-5-order Runge-kutta algorithm is used to simulate the model equation with MATLAB method.

3 Results and Discussion

3.1 Effect of Sound Velocity on the Change of Water Droplet Radius

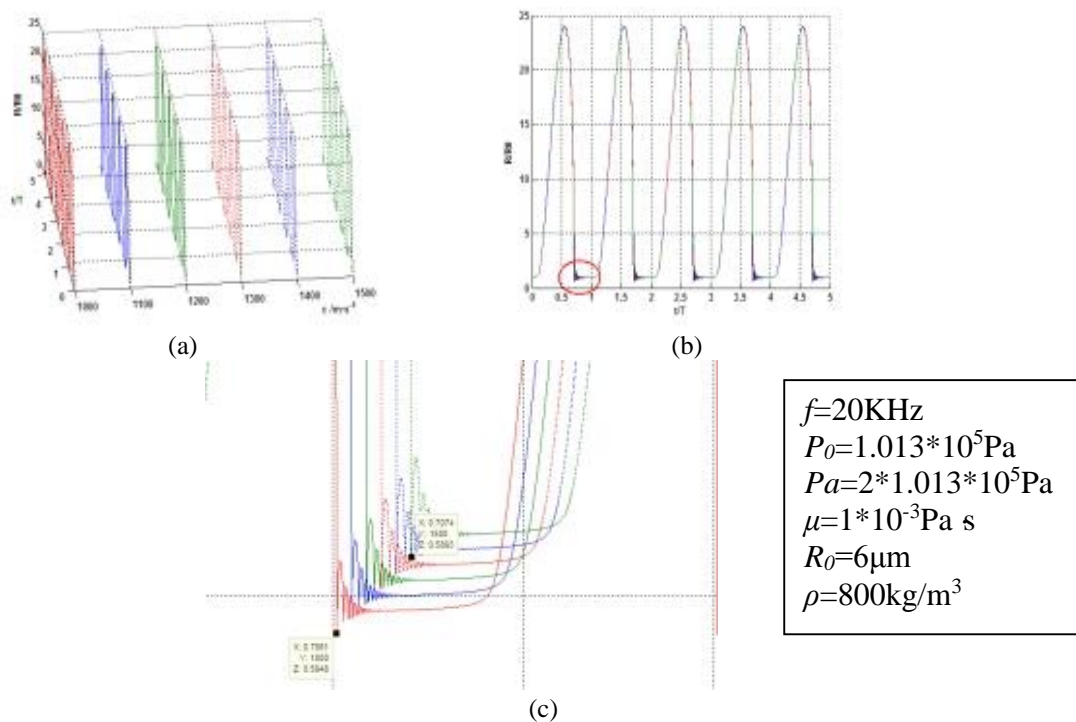


Fig. 2 Variation of water-drop radius at the different sound, (a) three-dimensional coordinates figure; (b) two-dimensional coordinates figure; (c) Magnified view that (b) is circled in red

The velocity of ultrasonic propagation is different when the dispersed phase liquid is different from that of oil. The variation of the droplet radius in 5T (5 cycles) in the dispersed phase liquid of 1000m/s, 1100m/s, 1200m/s, 1300m/s and 1400m/s respectively from Fig.2a , we can see that the change of water droplet radius varies almost exactly the same at different speed of sound. The growth amplitude is about 25 times times the initial radius. Fig. 2b is a two-dimensional graph of 2a, from which water droplets can be obtained to achieve a growth and contraction of about every 0.7T. The minimum value of shrinkage is about 0.6 times times the initial radius, the length is approximately

0.3T after growth and contraction, and the droplets are slightly shaken near the initial radius, as shown in Fig. 2c.

3.2 Effect of Density on the Change of Water Droplet Radius

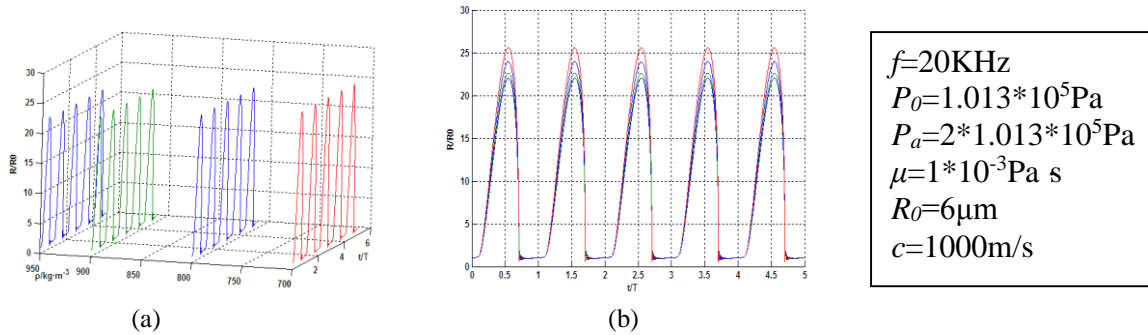


Fig. 3 Variation of water-droplet radius at the different density of oil phase, (a) three-dimensional coordinates figure; (b) two-dimensional coordinates figure

Different types of oil have different densities. Fig. 3a is the change of the droplet radius when the liquid density of the dispersed phase is 700kg/m^3 , 800kg/m^3 , 900kg/m^3 and 950kg/m^3 respectively, and the Fig.3b is the two-dimensional diagram of the Fig.3a. The larger the liquid density of the dispersed phase, the smaller the increase amplitude of the droplets. In the process of increasing density from 700kg/m^3 to 950kg/m^3 , the droplet growth amplitude is about 25.6 times lower than the initial radius of the water droplets to 22.1 times, and the droplet compression amplitude is about 0.6 times to the initial radius. The liquid density of dispersed phase is different, and the length of droplet growth compress is almost the same as that of small shocks, 0.7T and 0.3T respectively.

3.3 The Influence of Static Pressure on the Change of Water Droplet Radius

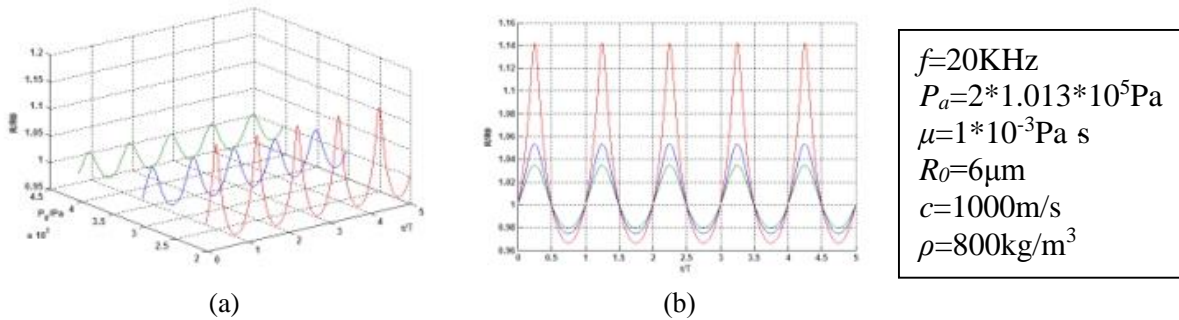


Fig. 4 Variation of water-droplet radius at the different of the static pressure, (a) three-dimensional coordinates figure; (b) two-dimensional coordinates figure

Fig. 4a is the change of the droplet radius when the liquid static pressure of the dispersed phase is taken $2*1.013*10^5\text{Pa}$, $3*1.013*10^5\text{Pa}$, $4*1.013*10^5\text{Pa}$ respectively, Fig.4b is the two-dimensional diagram of the figure 5a. It can be seen that when the static pressure is not less than the acoustic pressure amplitude of ultrasonic excitation ($2*1.013*10^5\text{Pa}$), the water droplets in the vicinity of the initial radius of small amplitude oscillation, and with the increase of static pressure, water droplets growth contraction amplitude is smaller, when the static pressure and ultrasonic excitation sound pressure amplitude is equal, the droplet growth amplitude is about 1.14 times of the initial radius, and the contraction amplitude can only reach 0.97 times of the initial radius. As you can see from figures 2 and 3, when the ultrasound excitation sound pressure amplitude and static pressure respectively take $2*1.013*10^5\text{Pa}$ and $1.013*10^5\text{Pa}$ that is, the ultrasonic excitation sound pressure amplitude is greater than static pressure, water droplets will have a relatively obvious growth and contraction, water droplets growth amplitude can reach the initial radius of about 25 times, the contraction amplitude is

almost 0.6 times of the initial radius. Obviously, compared with the ultrasonic excitation sound pressure amplitude is greater than static pressure, the static pressure is not less than the ultrasonic excitation sound pressure amplitude of water droplets more difficult to increase and compression, in the excitation sound pressure amplitude unchanged, the larger the static pressure water droplets volume is less likely to change.

4 Conclusions

The change of the dispersed water droplets in the two-phase liquid under the action of ultrasonic field depends not only on the water droplets itself and the liquid parameters of the dispersed phase, but also on the external ultrasonic excitation parameters. For the different velocity and density of different dispersed phase liquids, the sound velocity has little effect on the change of the droplet radius, and the viscosity coefficient is, the more obvious the change of the water droplet radius in the 2×10^4 Hz excitation frequency is the $2 \times 1.013 \times 10^5$ Pa. The bigger the static pressure, the less the volume change. Under the ultrasonic excitation, the water droplet growth is more obvious than the droplet compression. By analyzing the simulation results of the radius change of water droplets in the oil phase in ultrasonic field, it can be inferred that the inverse emulsion system or reverse emulsion system, the internal approximate water droplets in the ultrasonic action will occur growth and small shrinkage, and increase the degree of contraction depends not only on the size of the parameters of the ultrasonic field, and depends on some parameters of the system itself. These results will provide guidance for the study of biomedical engineering by ultrasound.

In this paper, some factors in the derivation of droplet kinetic equation are not taken into account, such as the relatively smaller compressibility of the dispersed phase liquid, which awaits further study.

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