

# Study on Auto-Comparison Method Based on Bullet Head Marks

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**Abstract.** This paper proposes a method to compare the marks of two bullet heads. First of all, the correction algorithm of the bullet head marks and the denoising model of the matrix low rank decomposition are proposed for the translation and rotation errors of the measured data. Finally, the method to compare the marks of two bullet heads is proposed based on the definition of the similarity of the matrix and the method of sequence alignment.

## 1 Introduction

In the practice of public security, it is often necessary to judge whether the two bullets are shot by the same gun according to the marks on the bullet heads. The traditional method of judgment is of low efficiency, because the sample is not easy to be preserved, and it is greatly influenced by human factors. The modern high-precision data acquisition equipment and auto-comparison method can improve the accuracy of determining whether the two bullet heads are shot by the same gun, as compared to the traditional method.

## 2 Basic Assumptions

1. It is assumed that the bullet head is a cylinder with a diameter of 7.90mm and a length of 12mm. In the course of the shot, the bullet head will not deform except for the scratch, and it will keep the original posture of the cylinder.

2. When the dimensional data of the bullet head marks is collected, the cylinder center line of the bullet head is parallel to the base plane,  $y$  axis is parallel to the direction of scratches. When the dimensional data of the same curve traces is collected, the fixed shape of the bullet head does not change.

3. When the bullet head is adjusted artificially, there is a translation error of 0.03mm and a rotation error of 0.2°. The change caused by the rotation error is directly equivalent to the change of  $x$ , which is plane coordinate of  $xoy$ . It is considered that the  $z$  does not change.

4. For the name of subfile in the data file,  $c_1 c_2 c_3 c_4$  are the fixed number of the four edges of the same bullet head. The fixed order is the reverse clockwise when looking at the head from the bottom of the bullet head.

## 3 Problem Solving

### 3.1 Deal with measurement error

When collecting 3D data of bullet head marks, the base plane measured is  $xoy$  plane of the space rectangular coordinate fixed on the measuring equipment. The  $z$  axis is perpendicular to the coordinate plane  $xoy$  and points to the outer surface direction of the bullet head. The actual measurement data show the three-dimensional coordinates on the surrounding surface of the scratched bullet head. It is difficult to make the two bullet heads measured in the same position and the same posture because of the manual adjustment of the sensors and bullet head, which will result in the measurement error. It usually causes the translation error of 0.03mm and the rotation error of 0.2°.  $(x, y, z)$  are the points in the space. When there is a translation error, only  $y$  will change,  $x$  and  $z$

will not change. When there is a rotation error, only  $y$  will change,  $x$  and  $z$  will not change. The observed part takes a small proportion of the whole circle area, and the observed part is approximately in a plane, it is assumed that the rotation error only causes the change of the  $x$ , but the  $z$  remains unchanged. Considering the translation error and the rotation error, it is known that the translation and rotation will change the  $x$  or  $y$  in the space, and the  $z$  can be considered not to change. In order to compare the No. 1 bullet head and the No. 2 bullet head, the mark coordinates of one of the bullet heads are required to be moved left and right, back and forth. The specific steps are shown in algorithm 1 and algorithm 2.

**Algorithm 1.** The maximum unit number of the translation of the bullet head mark coordinates.

Step1. Obtain the matrix  $Z$  corresponding to the  $z$  axis coordinates according to the measured data. The relationship of  $(x_i, y_j, z_{ij})$  in the measured data is as follows:  $z_{ij} = f(x_i, y_j)$ , thereinto,  $x_i = 0 + (i-1)h, i = 1, 2, \dots, 564$ ,  $y_j = 0 + (j-1)h, j = 1, 2, \dots, 756$ ,  $h = 0.00275$ ,  $f(\cdot, \cdot)$  is function of two variables.  $z_{ij}$  can constitute matrix  $Z \in \mathfrak{R}^{564 \times 756}$ .

Step2. Determine the rows  $\alpha$  of the translation of a matrix  $Z$  caused by a rotation. The diameter of the bullet head is 7.90mm, so the arc length corresponding to the  $0.2^\circ$  is  $s = 0.2 \times 7.9\pi / 360$  mm. Then  $\alpha = \text{ceiling}(s/h) = 6$ , *ceiling* is the infinite integral.

Step3. Determine the number of columns of the translation of the matrix  $Z$  caused by the left and right translation  $\beta$ .  $\beta = \text{ceiling}(0.03/h) = 11$ .

**Algorithm 2.** Comparison of bullet head marks

Step1. Determine the  $Z1$  and  $Z2$ , which are the coordinate matrix of  $z$  axis of the two bullet heads according to algorithm 1.

Step2. Take partial matrix of  $Z1$ , namely  $\bar{Z}1 = Z1(\alpha + 1 : 564 - 2\alpha, \beta + 1 : 756 - 2\beta)$ .

Step3. For  $i = 1 : 2\alpha$

For  $j = 1 : 2\beta$

take partial matrix of  $Z2$ , namely  $\bar{Z}2 = Z2(i : 564 - 2\alpha + i - 1, j : 756 - 2\beta + j - 1)$ .

Compare  $\bar{Z}1$  with  $\bar{Z}2$ .

End

End

Step4. Output the best result of Step 3.

Summary. This solution puts forward the correction algorithm of bullet head marks for the translation and rotation error of measured data, so that the two bullet heads can be compared in the same position and the same posture.

### 3.2 Denoising model. Analysis and scheme determination

Due to the damage, rust, grease and impurities and marks that are generated randomly, there will be data error and noise. The bullet head marks are divided according to their  $x$  axis or  $y$  axis. Obviously, most of the curves have similar posture, which can be seen in Fig.1~4 (771-1811345) and Fig.5~8 (771-1923252). This means that most rows or columns of matrix  $Z$  have similarity. In another way, the matrix  $Z$  is a low rank or an approximate low rank matrix. In this study, the matrix low rank decomposition method is used to denoise [1,2].

Low rank decomposition model of matrix is  $Z = UV + \varepsilon$ . Thereinto,  $Z \in \mathfrak{R}^{m \times n}$  is measurement matrix,  $\varepsilon \in \mathfrak{R}^{m \times n}$  is noise matrix,  $U \in \mathfrak{R}^{m \times r}$ ,  $V \in \mathfrak{R}^{r \times n}$ . Here,  $r < \min\{m, n\}$ . The decomposition model of matrix can be expressed as follows according to element:

$$Z_{ij} = (UV)_{ij} + \varepsilon_{ij}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

It is presumed that noise is  $\varepsilon_{ij} \sim N(0, \sigma_{ij}^2)$  and relatively independent.

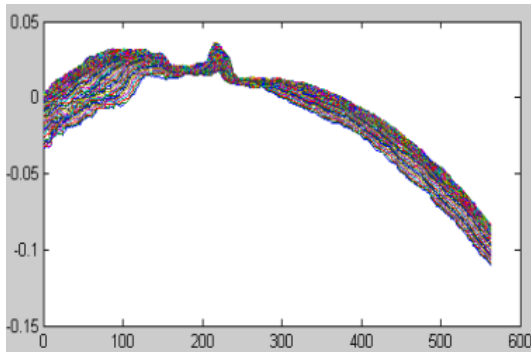


Figure 1. Similarity of Matrix Z column(1~189)

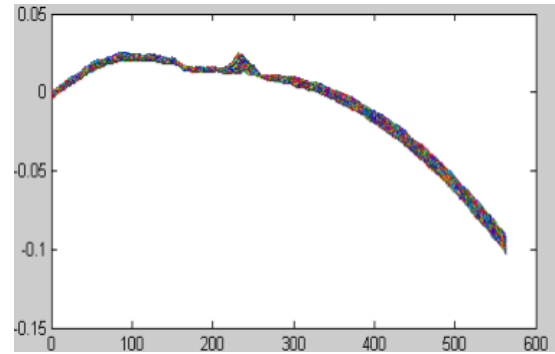


Figure 2. Similarity of Matrix Z column(190~378)

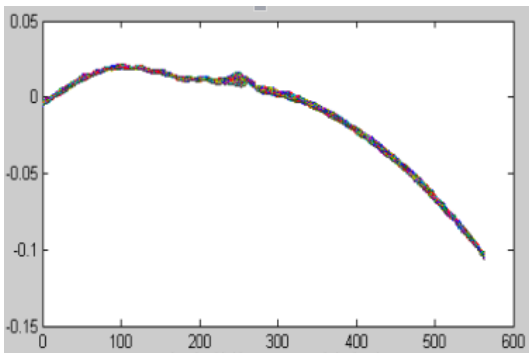


Figure 3. Similarity of Matrix Z column(379~567)

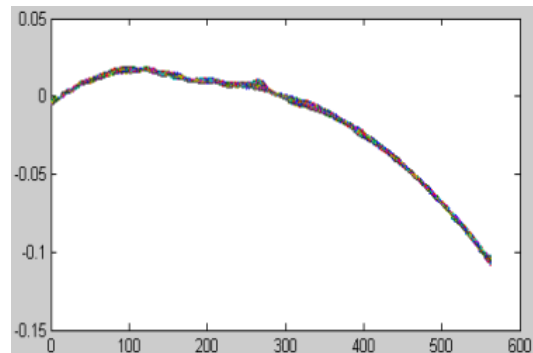


Figure 4. Similarity of Matrix Z column (568~756)

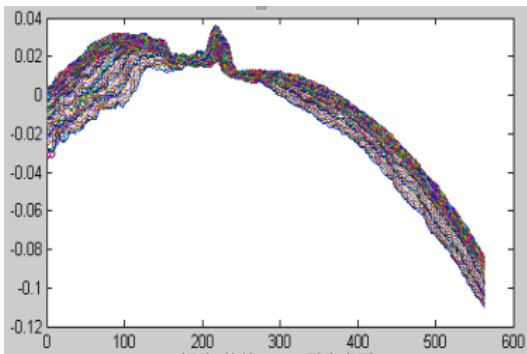


Figure 5. Similarity of Matrix Z column(1~189)

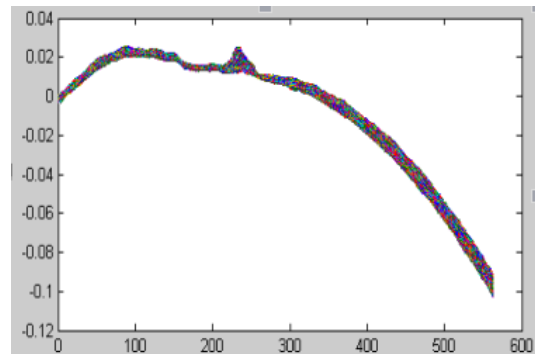


Figure 6. Similarity of Matrix Z column(190~378)

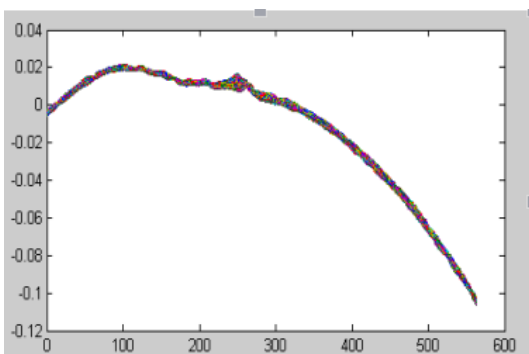


Figure 7. Similarity of Matrix Z column(379~567)

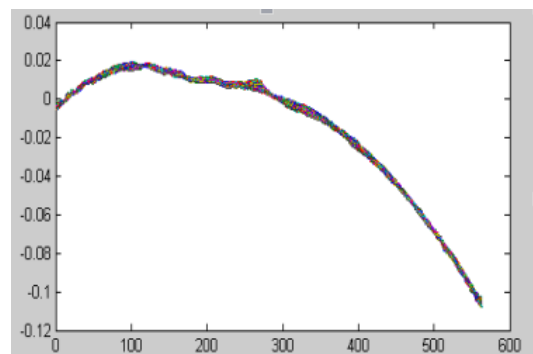


Figure 8. Similarity of Matrix Z column(568~756)

The density function of  $Z_{ij}$  is

$$\varphi(Z_{ij}; (UV)_{ij}, \sigma_{ij}^2) = \frac{1}{\sqrt{2\pi}\sigma_{ij}} e^{-\frac{(Z_{ij} - (UV)_{ij})^2}{2\sigma_{ij}^2}} \quad (1)$$

Then likelihood function is[3]

$$L(U, V) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma_{ij}} e^{-\frac{(Z_{ij} - (UV)_{ij})^2}{2\sigma_{ij}^2}} \quad (2)$$

It is further presumed again that  $\sigma_{ij}^2 = \sigma^2$ , so the log-likelihood function is

$$\ln L(U, V) = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^m \sum_{j=1}^n (Z_{ij} - (UV)_{ij})^2.$$

In order to obtain the optimal low rank decomposition of the matrix, the following optimization problem should be solved

$$\min_{U, V} \ln L(U, V).$$

The optimization problem above is equivalent to

$$\min_{U, V} \|Z - UV\|_F = \left( \sum_{i=1}^m \sum_{j=1}^n (Z_{ij} - (UV)_{ij})^2 \right)^{1/2}. \quad (3)$$

The low rank decomposition of a matrix can usually be obtained by the singular value decomposition of the shear matrix. The singular value decomposition of a matrix can be seen in Theorem 1.

**Theorem 1** If the rank of matrix  $Z \in \mathfrak{R}^{m \times n}$  is  $k$ , there is  $m \times k$  columns of orthonormal matrix  $U$  and  $k \times n$  rows of orthonormal matrix  $V$ , so that  $Z = U\Lambda V$ . There into,  $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_k)$  is diagonal matrix, and  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k > 0$  is singular value of matrix  $Z$  [4,5].

**Theorem 2** The rank of matrix  $Z \in \mathfrak{R}^{m \times n}$  is  $r$ , its singular values are decomposed into  $Z = U\Lambda V$ . If the optimal low rank matrix of  $r$  is approximate  $Z_r$  ( $r \leq k$ ), then  $\|Z - Z_r\|_F = \sum_{i=r+1}^k \lambda_i$  [6,7].

The measurement matrix  $Z$  has noise, so it is an approximate low rank. In this paper, the energy of the matrix is used to obtain the rank  $r$ . For example, the smallest  $r$  can be selected, so that  $\sum_{i=1}^r \lambda_i / \sum_{i=1}^k \lambda_i \geq 0.9$ . All figures for each column after the matrix de-noising can be seen in Fig.9~12 (77T 1-1811345) and Fig.13~16 (77T 1-1923252).

**Verification of scheme.** The implementation of this scheme is based on 771-1811345 data and 771-192352 data. The details are as follows:

1. The coordinates of the  $z$  are extracted from the two groups of data.
2. The matrix  $Z$  is generated by the coordinates of the  $z$ .
3. The singular value decomposition is carried out for each matrix  $Z$ , and the rank  $r$  is determined according to the percentage of the retained energy.
4. The low rank approximation of the matrix  $Z$  is determined by rank  $r$ . The generating diagram of the row of the matrix  $Z$  before and after denoising can be seen in Fig9~12 and Fig13~16. As noted from the diagram, the image after denoising is smoother, so the singular matrix decomposition method has a remarkable denoising effect. This singular matrix decomposition denoising method lays a foundation to accurately solving the third problem and the fourth problem.

**Summary:** In this study, a de-noising model based on matrix low rank decomposition is proposed because of the noise in the measured data. It can be observed from the diagram that the image after denoising is more smooth, so this method can achieve a remarkable denoising effect.

### 3.3 The analysis and establishment of model for the comparison of marks

In order to find out all the lines and marks on the bullet head, it is required to compare whether the partial lines on a certain strip mark are corresponded, and whether they have the same direction and similar outline. In addition, it is required to make a further contrast on the depth of the marks.

The rifling marks on a same bullet head may not be completely damaged.

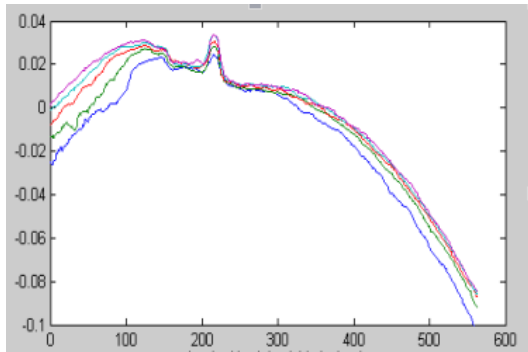


Figure 9. Contrast diagram of denoising effect

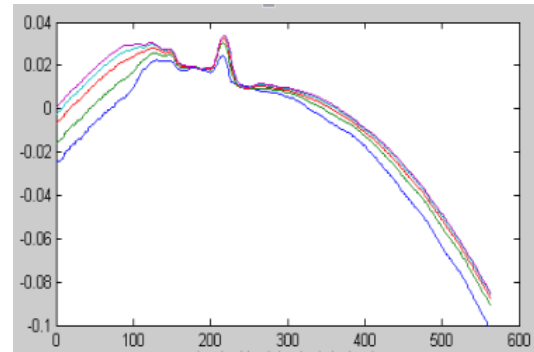


Figure 10. Contrast diagram of denoising effect

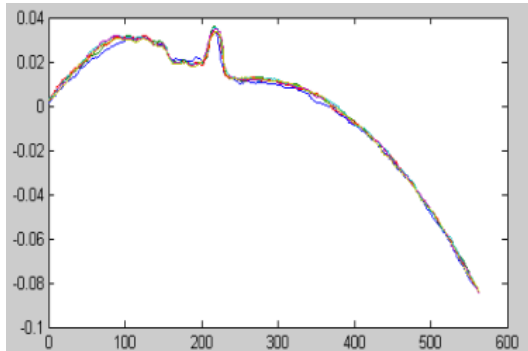


Figure 11. Contrast diagram of denoising effect

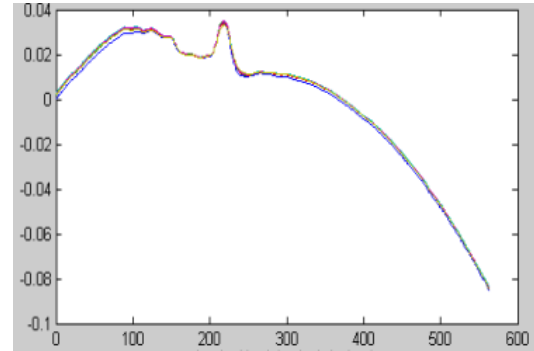


Figure 12. Contrast diagram of denoising effect

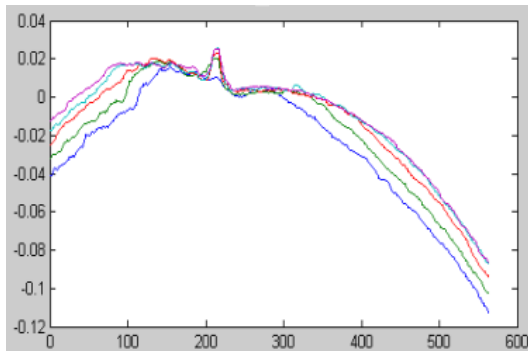


Figure 13. Contrast diagram of denoising effect

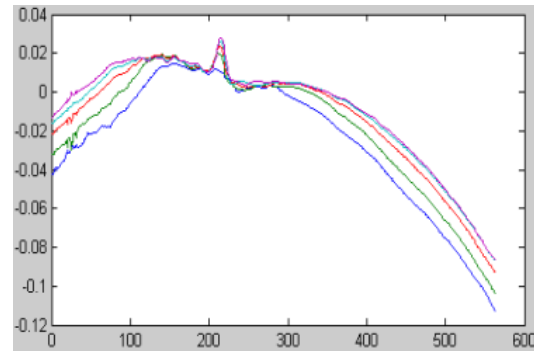


Figure 14. Contrast diagram of denoising effect

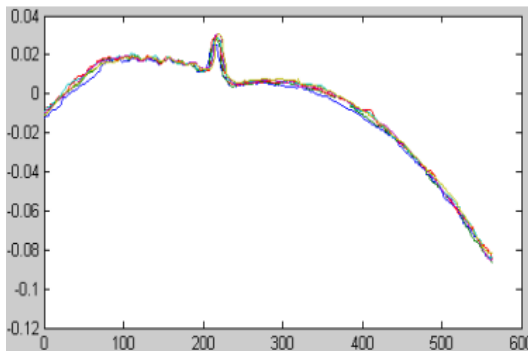


Figure 15. Contrast diagram of denoising effect

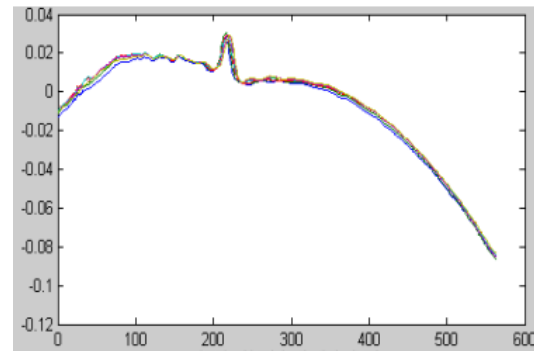


Figure 16. Contrast diagram of denoising effect

Line features which are small, relatively stable and clear can always be found in a line mark or a small area. After a comparison of high magnification, if the small features are all highly consistent, it can be determined whether the two bullets are shot by the same gun. The process of feature extraction is as follow:

(1) Use the data of which the file name begins with 77 to extract matrix  $Z$  :

$$\text{Make } x_i = 0 + (i - 1)h \quad (i = 1, 2, \dots, 564), \quad y_j = 0 + (j - 1)h, \quad (j = 1, 2, \dots, 756).$$

The  $z_{ij}$  can vary with the changes of  $x_i$  and  $y_j$ , so  $564 \times 756$  matrix  $Z$  can be changed.  $z_{ij}$  is the element of  $Z$ , and one matrix  $Z$  can represent one scratch of bullet.

(2) Six guns have a total of twelve bullet heads, and one bullet head has scratches of four edges. There are 48 scratches. So 48 matrix with 48 scratches on six bullets can be set up.

The principle of comparison is to select high-quality parts which can effectively reflect the image features of scratches.

(1) The curve of bullet is fitted on the  $xOz$  cross section in accordance with the data. It can be analyzed from the image that the data of the 51~464 rows in the matrix  $Z$  show the image which is relatively close to scratches. It can reflect the features of the image very well, so this part of data can be preserved. The data on the 51~464 rows of the matrix  $Z$  keep the original arc shape without any change. There is no sign of scratch, so this part of data can be removed.

(2) The curve of bullet is fitted on the  $yOz$  cross section in accordance with the data. It can be analyzed from the image that the data of the 1~50 rows in the matrix  $Z$  show the image which declines rapidly and is not smooth.

The scratch depth is gradually from deep to shallow, so this part of data can be removed.

(3) According to the method of algorithm 2, the data of the 51~464 row in the matrix  $Z$  are compared, and the effect is relatively good.

**Comparison scheme and algorithm.**

Compare matrix  $\{Z1, Z2, Z3, Z4\}$  corresponding to the four  $z$  axis of a bullet head with the four matrix  $\{Z1', Z2', Z3', Z4'\}$  of another bullet head in the following order.

$$\begin{aligned} &\{Z1, Z2, Z3, Z4\} \text{ 与 } \{Z1', Z2', Z3', Z4'\}; \quad \{Z1, Z2, Z3, Z4\} \text{ 与 } \{Z2', Z3', Z4', Z1'\}; \\ &\{Z1, Z2, Z3, Z4\} \text{ 与 } \{Z3', Z4', Z1', Z2'\}; \quad \{Z1, Z2, Z3, Z4\} \text{ 与 } \{Z4', Z1', Z2', Z3'\}; \end{aligned}$$

The similarity of the matrix A and matrix B can be defined as  $sim(A, B) = 1 - \frac{\|A - B\|_F}{\|A\|_F + \|B\|_F}$ .

Obviously, when  $A = B$ , the best result of similarity is 1. In the matrix contrast of each order, four similarities can be obtained. For four similarities, the average value of the maximum 3 similarity is taken as the total similarity. The final similarity of the two bullet head marks is based on the optimal similarity of these four orders. The comparison algorithm is as follow.

Table I. Comparison of two bullet heads marks

T1-1504519	T1-1812492	T1-1814688	T1-1928033	T1-1931817
T1-1504519	T1-1812492	T1-1814688	T1-1928033	T1-1931817
1	0.6980858	0.6726007	0.71004478	0.6927551
0.6980858	1	0.7313898	0.67709515	0.6170889
0.6726007	0.7313898	1	0.75743516	0.6937919
0.7100448	0.6770951	0.7574352	1	0.727632
0.6927551	0.6170889	0.6937919	0.72763205	1
0.6358159	0.6653648	0.6918037	0.67994176	0.749853
0.6739132	0.7232607	0.7504027	0.76889922	0.7559849
0.6690704	0.6827285	0.7638485	0.80384265	0.7444701
0.7034507	0.7252823	0.7627428	0.783919	0.6612651
0.7034507	0.7252823	0.7627428	0.783919	0.6612651

**Algorithm 3.** Comparison algorithm

Step1 Import the data of two bullet heads to generate matrix groups  $\{Z1, Z2, Z3, Z4\}$  and  $\{Z1', Z2', Z3', Z4'\}$ .

Step2 Extract partial characteristics  $Z_i = Zi(51:464,:)$ ,  $Z_i = Zi(51:end)$ ,  $Z_i' = Zi'(51:464,:)$ ,  $Z_i' = Zi'(51:end), i = 1,2,3,4$ .

Step3 Denoise matrix A and B using matrix low rank decomposition,  $i = 1,2,3,4$ .

Step4 Choose the optimal comparison matrix according to the algorithm 2 (the dimension of the

matrix is not consistent, so it cannot be applied directly).

#### 4 Conclusions

According to the experimental results, the method to compare the marks of two bullet heads is obtained based on the similarity of matrix and the method of sequence comparison. Finally, the complete comparison scheme and algorithm are summarized. The experimental results can be seen in Table I. The experimental results show the effectiveness and feasibility of the proposed method.

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