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A Fast-Meshless Method Based on GMRES for Two-Dimensional Acoustic Scattering Problem

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Abstract. The method of fundamental solution (MFS) has been known as an effective and simple boundary meshless algorithm. However, the MFS generates dense coefficient matrix and thus requires a lot of computation time for solving large-scale problems by using direct solvers in a personal computer. The generalized minimal residual algorithm (GMRES) is an iterative technique that can reduces computational operations for solving such dense matrix equations. This study combines the traditional MFS with GMRES iterative solver to calculate the two-dimensional acoustic scattering problems. With this approach, the operations are reduced to O (N^2) while O (N^3) operations are required for the traditional MFS using the direct solvers. Numerical examples with up to 20800 DOF are solved successfully on a laptop using the developed GMRES-MFS code. These results clearly demonstrate the efficiency and accuracy of the GMRES-MFS for solving two-dimensional acoustic scattering problems.

Key words: MFS; O (N^2); O (N^3); DOF; GMRES-MFS; meshless algorithm; matrix equations.

INTRODUCTION

Numerical simulation of the Helmholtz-type equations of time-harmonic acoustic waves has been an important topic of study in science and engineering. Since moving boundary in high dimension makes mesh generation a troublesome task, great attention and effort have been paid in recent years to meshless methods where neither meshing of the boundary nor domain is required [1], such as element-free galerkin method [2], method of fundamental solutions (MFS) [3], boundary knot method [4], Cartesian grid method [5], local boundary integral equation [6]. Among these methods, the MFS is the most effective and popular boundary-type meshless method due to its high accuracy, rapid convergence and easy implementation. In particular, the MFS is suitable for scattering and radiation problems by choosing appropriate fundamental solutions satisfying the radiation condition at infinity. These advantages with the MFS have attracted continued interests from researchers. However, the conventional MFS in general produce dense and non-symmetric matrices that require O (N2) memory storage and another O (N3) operations to solve the system with direct solvers, such as Gauss Elimination solver, LU Decomposition solver, where N is the number of equations of the linear system.

Therefore, the scale of calculation is often limited to very small number of nodes by using traditional MFS. The efficiency in solving the MFS equations has been a serious problem for large-scale models. Although, many accelerated methods have been developed for large-scale acoustics problems, such as the fast multipole boundary element method [7, 8], but its complexity discourages beginners. A generalized minimal residual algorithm (GMRES) [9] can reduces the O (N^3) operations to O (N^2) operations, and with the continuous improvement of personal computer performance, memory storage is not a problem for models which are not very large. The purpose of this paper is to investigate how to apply the MFS to solve a model that is not very large with a personal computer.

This paper is organized as follows: Section 2 introduces the conventional MFS formulation for 2D acoustic scattering problems. Section 3 introduces the procedures of generalized minimal residual algorithm. Section 4 shows some example problems solved by using the provided code to demonstrate the efficiencies of the GMRSE-MFS with personal computer. Section 5 concludes this paper with some discussions.

SCATTERING THEORY OF MFS

Consider a time-harmonic acoustic wave in an infinite, homogeneous acoustic medium of mean density ρ and sound speed c_0. The acoustic medium domain is Ω. The wave incident upon one (or more) rigid, fixed obstacle occupying the region Ω_i with boundary $\partial\Omega$ in 2D as shown in Fig.1. The acoustic pressure $\boldsymbol{P}$at point $x\in\Omega$ obeys the governing differential equation for steady-state linear acoustics, as well as the following well known Helmholtz equation [10, 11].

$$\nabla^2 P(x) + k^2 P(x) = 0 \quad x \in \Omega \tag{1}$$

Here, $\Omega=\Omega_i\cup\Omega_e$, and $k=\omega/c_0$ is the classical wave number defined as the radio of the angular frequency ω and the sound speed c_0.

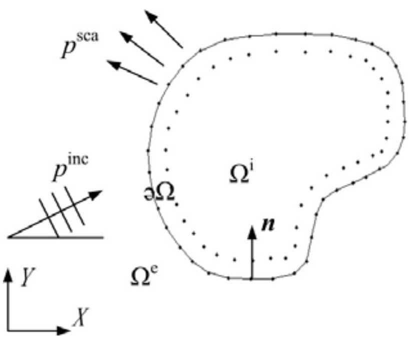

FIGURE 1. Scattering geometry

The scattered wave is defined by

$$P^{\text{sca}} = P - P^{\text{inc}}, \tag{2}$$

Where $\boldsymbol{P}^{\text{inc}}$ and $\boldsymbol{P}$ represents the incident acoustic wave and total acoustic wave. In the case the incident wave is a plane wave defined as $P^{\text{inc}}=A^{\text{inc}}\exp(ik\zeta\cdot x)$ (with time convention $\boldsymbol{e}^{-i\omega t}$), where the $\zeta\cdot x$ controls the direct of the incident wave, and A^{inc} is acoustic pressure amplitude.

On $\partial\Omega$, the boundary of the rigid scatterer, $v\cdot n$=0, where n is the unit normal to $\partial\Omega$, that is, $\partial\boldsymbol{P}/\partial n$=0, rewriting the problem in terms of P^{sca} gives:

$$\nabla^2 P^{\text{sca}}(x) + k^2 P^{\text{sca}}(x) = 0 \quad x \in \Omega^{\text{e}} \tag{3}$$

$$\frac{\partial P^{\text{sca}}}{\partial n}(x) + \frac{\partial P^{\text{inc}}}{\partial n}(x) = 0 \quad x \in \partial\Omega \tag{4}$$

In addition, P^{sca} must satisfy the Sommerfeld radiation condition at infinity

$$\lim_{r\to\infty} r^{1/2}\left(\frac{\partial P^{\text{sca}}}{\partial r} - ik\, P^{\text{sca}}\right) = 0 \tag{5}$$

Where $r = \|x\|, x \in \Omega^{e}$

In the method of fundamental solutions, we place N collocation points (x_i, i=1, 2 … N) on the boundary and another N auxiliary or source points (y_j, j=1, 2 … N) outside the domain Ω_e (Fig.1). The locations of the sources and the source intensities are solved simultaneously using a nonlinear least-squares approach, which can remove the uncertainty in determining the distance between the sources and the boundary. The scattered field P^{sca} at each point xi is approximated by the following expression which satisfies Eq. (1) or Eq. (3):

$$P^{sca}(x_i) = i\rho\omega\sum_{j=1}^{N} G(x_i, y_j)u(y_j) \quad x_i \in \Omega^{e} \tag{6}$$

Where

$$G(x_i, y_j) = \frac{i}{4} H_0^1(kr) \tag{7}$$

Is the free space Green's function for 2D acoustic problems, $i = \sqrt{-1}$, $r = \|x_i - y_j\|$ is the distance between the collocation point x_i and the field point y_j, H_0^1 is the Hankel function of the first kind with 0th order. $U(y_j)$ is the unknown intensity of the sources at the auxiliary point's y_j. By substituting Eq. (6) into Eq. (4), we have

$$\frac{-1}{i\rho\omega}\frac{\partial P^{inc}}{\partial n}(x_i) = \sum_{j=1}^{N} F(x_i, y_j)u(y_j) \tag{8}$$

Where $F(x_i, y_j) = \dfrac{\partial G}{\partial n(x_i)} = \dfrac{-ik}{4} H_1^1(kr)\dfrac{\partial r}{\partial n(x_i)}$ (9)

In the conventional MFS approach, the following standard linear system of equations is formed after applying Eq. (8) at all the collocation point's xi (i=1, 2 … N):

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{Bmatrix}, \text{ or } \boldsymbol{Au} = \boldsymbol{b} \tag{10}$$

Where A is the coefficient matrix, *u* the unknown vector and b the right-hand side vector.

Once all the values of *u* are determined by solving this equation, the acoustic pressure at any point inside the domain Ω_e or on the boundary $\partial\Omega$ can be evaluated using Eq. (2) and Eq. (6).

In the conventional method of fundamental solutions, the solution of the system in Eq. (10) using direct solvers such as Gauss elimination requires O (N^3) operations. Such characteristics of the conventional MFS have limited its applications in solving large-scale problems.

THE PROCEDURES OF GMRES

For the conventional MFS, the discretization of integral equation leads to a dense, non-symmetric system matrix which is expensive to store and solve. For an acoustic problem with N unknowns, a direct solution of such linear system requires O (N^3) operations and O (N^2) memory storage. However, for iterative solver, only the multiplication of matrix and vector is required in iteration process, there is no need to decompose the coefficient matrix of linear system. A iterative solver called generalized minimal residual algorithm (GMRES) is the most popular to solve the linear system equation

For the linear system equations, such as Eq. (10), the basic steps of the GMRES iterative solver are as follows.

Step 1. Initialization: Choose u_0, and compute $\eta_0 = b - Au_0$, and $v_1 = \eta_0 / \beta$ where $\beta = \|\eta_0\|$.

Step 2. Iterate: For j=1, 2, m, until satisfied do:

$$h_{ij} = \left(Av_j, v_i\right), \quad v_{j+1} = Av_j - \sum_{i=1}^{j} h_{ij} v_i\ , \quad i = 1, 2, \cdots, j\ , \tag{11}$$

$$h_{j+1,j} = \left\|\hat{v}_{j+1}\right\|, \quad v_{j+1} = \hat{v}_{j+1} / h_{j+1,j}\ , \tag{12}$$

To update V_{j+1} and $\bar{H}_j$, $V_{j+1} = \left(V_j, v_{j+1}\right)$, $\bar{H}_j = \begin{bmatrix} \bar{H}_{j-1} & h_{ij} \\ 0 & h_{j+1,j} \end{bmatrix}_{(j+1)\times j}$, here, $\bar{H}_j$ is Hessenberg matrix.

Step 3. To solve the least squats problem:

$$\|\eta_m\| = \min_{Y=R^m} \left\|\beta e_1 - \bar{H}_m Y_m\right\|. \tag{13}$$

Hence the solution is $u_m = u_0 + V_m Y_m$, where $e_1 = \left[1, 0, 0, \cdots\right]^T$, $V_m = \left[v_1, v_2, \cdots, v_m\right]$.

Step 4. To compute $\eta_m = b - Au_m$,if $\|\eta_m\| / \beta < \varepsilon$ is satisfied, where ε is converges, then stop, and u_m is the final solution. If not, update u0 to u_m, compute $v_m = \eta_m / \|\eta_m\|$ and go to Step 2.

In order to accelerate the solution, preconditioners for the GMRES-MFS are crucial for its convergence and computing efficiency. We use a block diagonal preconditioner, which is form by adjacent collocation points and source points. The application of the sparse approximate inverse preconditioned GMRES to the linear system for solving acoustic problems is presented by Chen and Harris [12].

NUMERICAL EXAMPLES

Numerical examples are presented to demonstrate the feasibility, accuracy and efficiency of the GMRES-MFS for 2D acoustic problems. We compare the efficiency and accuracy of the developed GMRES-MFS with those of the conventional MFS based on direct solvers. The algorithm is implemented in Fortran 90 and tested on a laptop with an Intel Dual Core 2.2 GHz CPU, 3 GB RAM. The iterative tolerance is set to 10^{-3}.

Scattering from a Rigid Cylinder

As an example, to test the accuracy of the program, we compute acoustic wave scattered by an infinite rigid cylinder of radius a=0.5 with a plane incident wave of unit amplitude travelling along the positive x-axis (θ= 0) in a direction perpendicular to the axis of the cylinder. Wave number is set to 5. Sample filed points are evenly distributed on a circle of $r = 2a$, as shown in Fig. 2. Theoretical sound pressure at point (r, θ) is given as

$$p(r,\theta) = p^{inc} + p^{sca} = p^{inc} - \sum_{m=0}^{\infty} \varepsilon_m i^m \frac{J_{m'}(ka)}{H^1_{m'}(ka)} H^1_m(kr) cos(m\theta) \quad (14)$$

where ε_m is Neumann constant, $J_{m'}$ and $H^1_{m'}$ mean derivative of J_m and H^1_m with respect to *ka*. Field pressure plotted in that the accuracy of the program.

Fig. 2 shows the field pressure contour plot for scattering of rigid cylinder with wave number k=5. The field pressures, on a circle of $r = 2a$, given by GMRES-MFS and analytical way are compared as shown in Fig. 3.

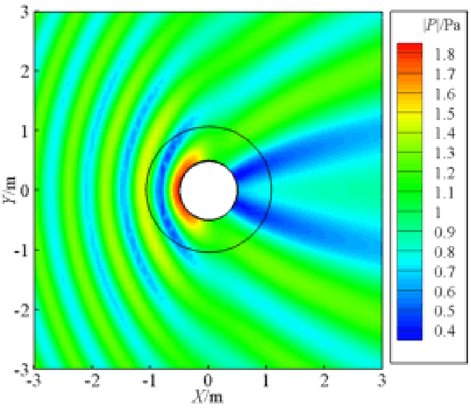

FIGURE 2. Sound pressure contour plot for scattering of a rigid cylinder

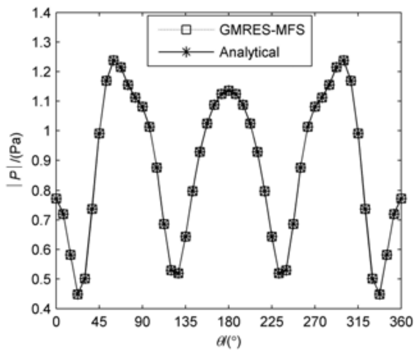

FIGURE 3. Sound pressure given by analytical and GMRES-MFS

Scattering from Four Rigid Cylinders

Fig. 4 shows the field pressure contour plot due to the incident plane wave with radius a=0.5 m and wave number k=2π/5 on a group of four rigid cylinders arranged at the vertices of a square. The plane wave of unit amplitude is travelling along the positive x-axis (θ = 0). Compared to the results shown in the Fig.9 of Ref [13], Fig.4 shows the same results.

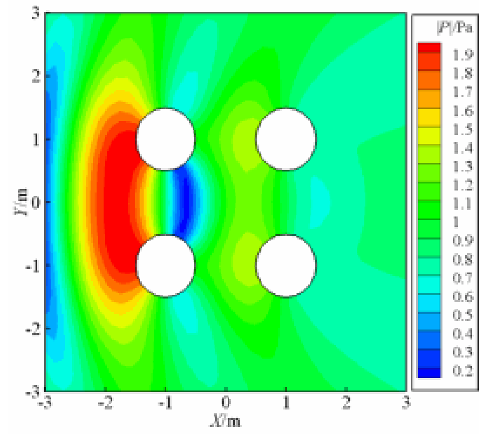

FIGURE 4. Sound pressure contour plot for scattering of four rigid cylinders

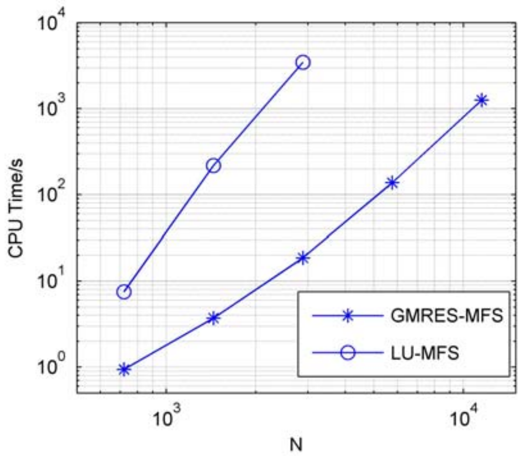

FIGURE 5. CPU time consuming with different methods

With the same numerical model of four rigid cylinders scattering and wave number k=4. Fig.5 shows the CPU time for the GMRES-MFS in solving these relatively large models with increasing degree of freedom, as compared with the conventional MFS by using LU Decomposition solver. It is being seen that the total CPU time increases two curves for the GMRES-MFS and LU-MFS show the order $O(N^2)$ and $O(N^3)$ computational efficiencies, respectively. This numerical example clearly demonstrates the developed algorithm is efficient for solving relatively large-scale acoustic problems in personal computer.

MECS-Shaped Model Analysis

Fig.9. shows the rigid MECS-shaped model scattering problem with wave number k=5, and the plane incident wave of unit amplitude travelling along the negative y-axis (θ= $3\pi//4$). The MECS-shaped scattering model contains 20800 collocation and source points. It takes 32 iterations and 4036 seconds; the scattering problem of rigid MECS-shaped model is solved successfully on a laptop PC.

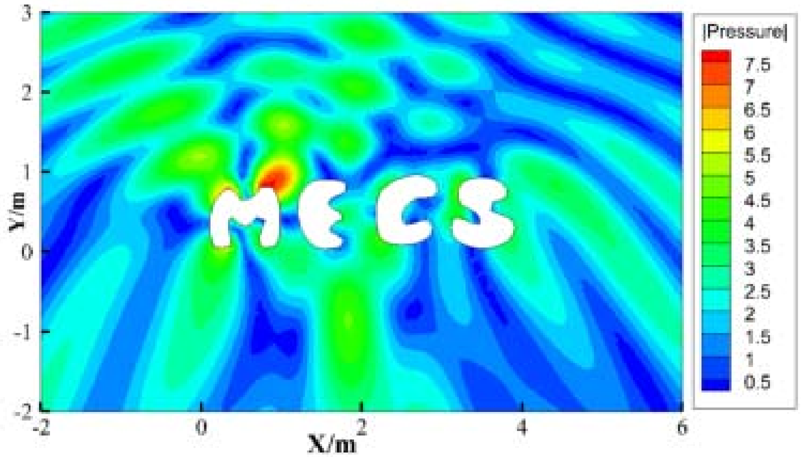

FIGURE 6. Sound pressure contour plot for scattering of MECS-Shaped Model

CONCLUSION

In this paper, we proposed a GMRES-MFS with preconditioning for solving 2D acoustic scattering problem. The computational efficiency of the developed algorithm is improved by adopting the approximate inverse preconditioned GMRES iterative solver to solve system equation. Numerical results show that computational time of the proposed GMRES-MFS are at the rate of O (N^2) while the traditional MFS requires O (N^3) operations using direct solvers. The numerical examples including a model with 20800 DOF are solved successfully on a laptop PC. These examples clearly demonstrate the accuracy and efficiency of the developed fast algorithm for solving 2D acoustic scattering problems.

It notices that the method proposed in this paper also has shortcomings. Because of the O (N^2) computational time and memory storage, the application of the proposed GMRES-MFS for the computation of very large-scale models is limited. However, because of the similarity of the BEM and MFS, it is natural to apply the fast multipole method (FMM) to accelerate the solutions of the MFS, and this will be an important improvement in the efficiency of the MFS for large scale acoustic problems.

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