Damage detection of curved member based on the changes in curvatures

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Abstract. A damage location method based on the changes in curvatures was developed for curved structural member. First, the curves of intact and damage members were fitted using least square method. Then, the curvatures of intact and damage members were calculated by the fitted curves. Finally, the damage was located using the changes in the curvatures between intact and damage members. A numerical example of curved beam demonstrated the effectiveness of the developed method.

1. Introduction

The subway brings convenience to people's travel, however, tunnel defects is increasingly prominent [1]. The defects not only affect the use of the tunnel, but also increase the cost of maintaining the tunnel. They can even endanger traffic safety and affect the normal operation of trains [2]. Therefore, early detection of damage to the tunnel structure and timely maintenance are of great social and economic importance. Segments are the main bearing members of the shield tunnel. Unlike conventional structural members, the segments are curved members. Most damage detection methods are developed for straight members [3, 4]. Therefore, it is of great significance to explore effective damage detection methods for curved members.

2. Curve fitting based on least square method

For a given set of data，$(x_i , y_i)(i = 1, 2, \cdots, N)$, the following function can be found

$$f(x) = a_0 f_0 + a_1 f_1 + ... + a_m f_m. \tag{1}$$

Which make the equation, $\sum_{i=1}^{N} \omega(x_i)[y_i - f(x_i)]^2$, minimum. Where $f_0, f_1, ..., f_m$ are a set of linearly independent functions, and $a_0, a_1, ..., a_m$ are corresponding undetermined coefficients. $\omega(x_i)$ is weight function, which reflects the degree of importance of the point (x_i , y_i).

In order to make $\sum_{i=1}^{N} \omega(x_i)[y_i - f(x_i)]^2$ minimum, the following equations must be satisfied

$$\begin{bmatrix} (f_0, f_0) & (f_0, f_1) & \cdots & (f_0, f_m) \\ (f_1, f_0) & (f_1, f_1) & \cdots & (f_1, f_m) \\ \vdots & \vdots & \ddots & \vdots \\ (f_m, f_0) & (f_m, f_1) & \cdots & (f_m, f_m) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} (f_0, Y) \\ (f_1, Y) \\ \vdots \\ (f_m, Y) \end{bmatrix}. \tag{2}$$

Where $Y = (y_1, y_2..., y_n)^T$, and $(f_s, f_t) = \sum_{i=1}^{N} \omega(x_i) f_s(x_i) f_t(x_i)$. The undetermined coefficients of

$a_0, a_1, ..., a_m$ can be solved from the Eq. (2).

Let $f_0 = 1, f_1 = x, ..., f_m = x^m$, Eq. (2) has the following form

$$
\begin{bmatrix}
\sum_{i=1}^{N} \omega_i & \sum_{i=1}^{N} \omega_i x_i & \cdots & \sum_{i=1}^{N} \omega_i x_i^m \\
\sum_{i=1}^{N} \omega_i x_i & \sum_{i=1}^{N} \omega_i x_i^2 & \cdots & \sum_{i=1}^{N} \omega_i x_i^{m+1} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{N} \omega_i x_i^m & \sum_{i=1}^{N} \omega_i x_i^{m+1} & \cdots & \sum_{i=1}^{N} \omega_i x_i^{2m}
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_m
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N} \omega_i y_i \\
\sum_{i=1}^{N} \omega_i x_i \, y_i \\
\vdots \\
\sum_{i=1}^{N} \omega_i x_i^m y_i
\end{bmatrix}.
\tag{3}
$$

When $\omega_i = 1$, Eq. (3) becomes

$$
\begin{bmatrix}
\sum_{i=1}^{N} 1 & \sum_{i=1}^{N} x_i & \cdots & \sum_{i=1}^{N} x_i^m \\
\sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \cdots & \sum_{i=1}^{N} x_i^{m+1} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{N} x_i^m & \sum_{i=1}^{N} x_i^{m+1} & \cdots & \sum_{i=1}^{N} x_i^{2m}
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_m
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i=1}^{N} y_i \\
\sum_{i=1}^{N} x_i \, y_i \\
\vdots \\
\sum_{i=1}^{N} x_i^m y_i
\end{bmatrix}.
\tag{4}
$$

3. Damage index based on the changes in curvatures

Structural damage causes structural stiffness to change, and the changes in stiffness will result in the changes in the curvature of the structure under load. Therefore, the changes in curvature of the structure between intact members and damaged ones can be used as damage index.

The ratio of the curvature of intact member to that of damaged one is

$$
RID = \left| \rho_0(x_i) / \rho_1(x_i) \right|.
\tag{5}
$$

Where $\rho_0(x_i)$ and $\rho_1(x_i)$ are the curvatures of intact and damaged member at x_i. $\rho(x)$ can be calculated using the following equation

$$
\rho(x) = \frac{\left| y'' \right|}{(1 + (y')^2)^{3/2}}.
\tag{6}
$$

Where y is the expression of the fitted curve.

Curvature change rate is defined as

$$
CCR = (\rho_1(x_i) - \rho_0(x_i)) / \rho_0(x_i).
\tag{7}
$$

4. Numerical example

Damage detection was conducted for a curved beam with quarter-arc shape, as show in Fig. 1. The radius, modulus of elasticity, unit weight and Poisson's ratio are 2.0m, 36GPa, $26\,kN/m^3$ and 0.33, respectively. The beam is subjected to vertical uniform load, $1\,kN/m$.
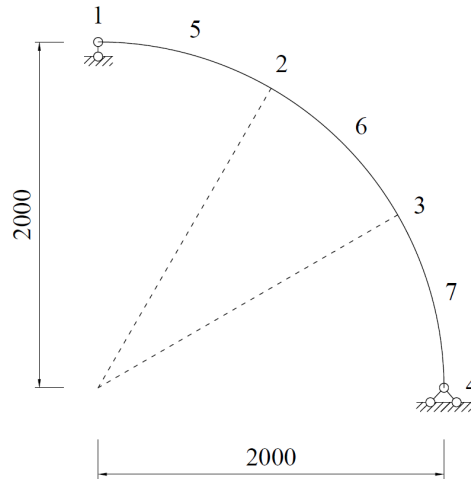
Fig. 1. Diagram of curved beam

The beam was divided into three segments, $P_1 P_2$, $P_2 P_3$ and $P_3 P_4$. Each segment was fitted using quadratic polynomial, which requires three points for every segment. Segment $P_1 P_2$ includes points 1, 5 and 2; Segment $P_2 P_3$ includes points 2, 6 and 3; Segment $P_3 P_4$ includes points 3, 7 and 4.

When the vertical uniform load is applied to the beam, the coordinates of all points can be calculated by finite element analysis. For example, the coordinates of points 1, 5 and 2 are listed in Table 1.

Table 1. The coordinates of points 1, 5 and 2 of intact beam

i	1	5	2
x_i	0	0.39194	0.75482
y_i	2	1.74112	1.42789

Substituting data in Table 1 into Eq. (4), the equation of segment $P_1 P_2$ can be obtained as follows

$$y_1^0(x) = 2 - 0.55528x - 0.26849x^2 \tag{8}$$

Similarly, the equations of segments $P_2 P_3$ and $P_3 P_4$ also can be obtained

$$y_2^0(x) = 2.08740 - 0.80699x - 0.08843x^2, \tag{9}$$

$$y_3^0(x) = 0.86440 + 0.81042x - 0.62131x^2. \tag{10}$$

When segment $P_1 P_2$ is damaged with stiffness loss of 20%, the coordinates of points 1, 5 and 2 are listed in Table 2.

Table 2. The coordinates of points 1, 5 and 2 of damaged beam

i	1	5	2
x_i	0	0.36636	0.71395
y_i	2	1.72504	1.41357

According to the data in Table 2, the equation of segment $P_1 P_2$ of damaged beam was obtained as follows

$$y_1^1(x) = 2 - 0.67583x - 0.20388x^2. \tag{11}$$

The equations of segments $P_2 P_3$ and $P_3 P_4$ have the following form

$$y_2^1(x) = 2.02580 - 0.80456x - 0.07419x^2, \tag{12}$$

$$y_3^1(x) = 0.95499 + 0.63503x - 0.55626x^2. \tag{13}$$

Substituting Eqs. (8), (9), (10), (11), (12) and (13) into Eqs. (5), (6) and (7), *RID* and *CCR* can be obtained. *RID* and *CCR* are show in Fig. 2.
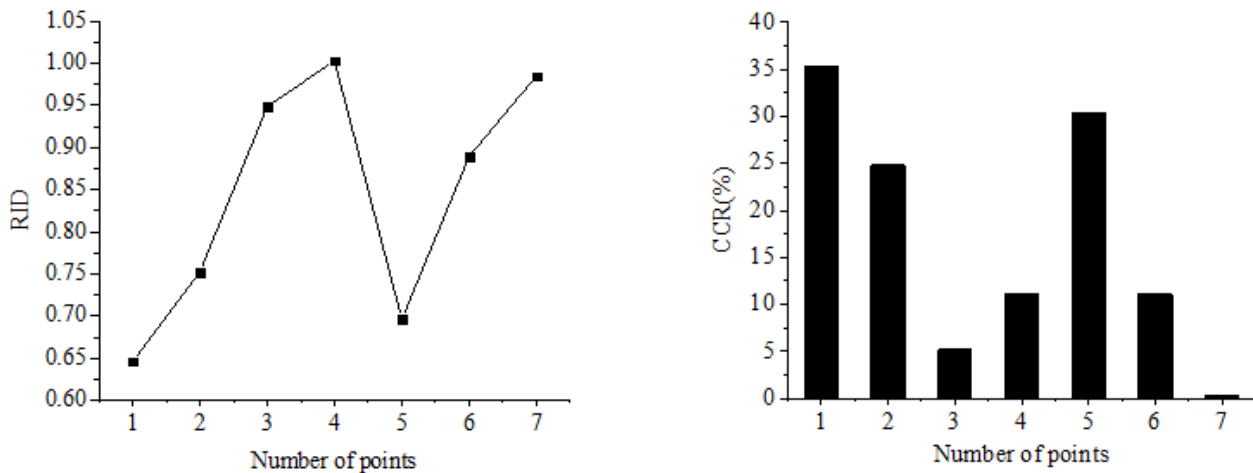

Fig. 2. *RID* and *CCR* of all points

It can be found that the *RID*s of points 1, 5 and 2 are obviously smaller than those of other points, and the *CCR* of points 1, 5 and 2 are obviously greater than those of other points. Therefore, the segment P_1P_2 can be identified as damaged one, which is consistent with actual damage location.

5. Conclusion

The stiffness of structural member decreases due to damage, which makes the curvature of the member change under load. Therefore, the changes in the curvatures can be treated as indices for damage identification, and a damage detection method was developed for curved structural member. The method was demonstrated by identifying the damage location of a curved beam. The results show that the changes in the curvatures are effective indices for damage location.

Acknowledgement

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