

# To Solve the Load of Beam by Shearing Force Diagram and Bending Moment Diagram—Reverse Thinking in Strength of Materials

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**Keywords:** Forward Problem; Inverse Problem; Statically Indeterminate Constraints

**Abstract.** The paper has a discussion of forward problem and inverse problem for beams in strength of materials. Given loading condition can certainly determine its shearing force diagram and bending moment diagram, but conversely, there may be a variety of statically determinate or statically indeterminate constraint conditions. Furthermore, the solution from statically indeterminate constraint conditions doesn't agree with the given shearing force diagram and bending moment diagram in a general way.

## 1. Introduction

Problems in strength of materials can be roughly divided into two categories, the forward problem and inverse problem. The former demands for results with causes, while the other reverses. Generally the solution to the forward problem exists, which is also unique and stable; instead, the solution to the inverse problems may not exist, or it's not unique, or unique but unstable. Take the beam for example, a problem demanding for the shearing force diagram and bending moment diagram with known loading condition is the forward problem, and the solution is unique; however, a problem demanding for loading condition with the shearing diagram already known as well as bending moment diagram is the inverse problem, the solution of which may not be unique. In this paper we start from a forward problem of an example, discussing the corresponding inverse problem in detail, and illustrate the differences and relations between statically determinate constraints and statically indeterminate constraints

## 2. Forward Problem

The loading condition of the beam spanning 8m is shown in Figure 1. Please draw the shearing force diagram and bending moment diagram.

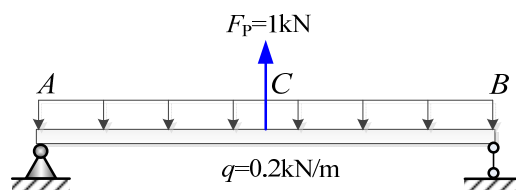


Fig.1. The loading condition

Considering the symmetry, its constraining force in 2 sides is equally 0.3kN in vertical direction without horizontal constraining force. Then we can easily draw the shearing force diagram and bending moment diagram, as shown in Figure 2.

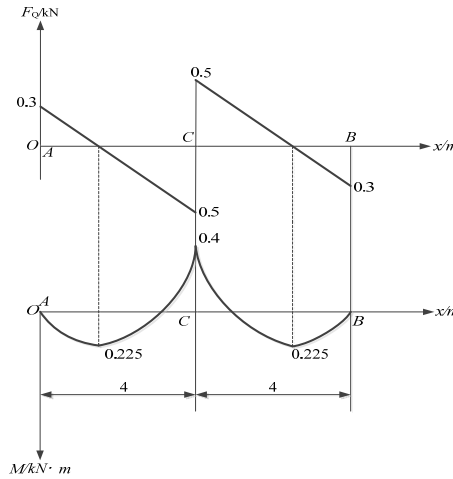


Fig.2. The shearing force diagram and bending moment diagram

Concluded from the contents above, in the case of given external force condition, a beam's internal force condition, namely the shearing force diagram and bending moment diagram, is certainly determined. On the contrary, can we uniquely determine a beam's external force condition if we've known its internal force condition?

### 3. Inverse Problem in Statically Determinate Constraints

A statically determinate beam is under in-plane loading, and its the shearing force diagram and bending moment diagram is shown in Figure 2. Please give the solution to the beam's loading condition.

According to Figure 2, we can work out that there're 0.3kN concentrated force in both sides of the beam and 0.2kN/m uniformly distributed load. The force condition of the beam (without considering whether the force is external or not) is shown in Figure 3.

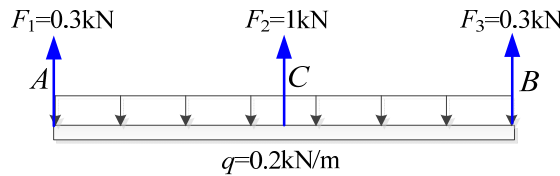


Fig.3. The force condition

Under the statically determinate constraints, a total of 4 kinds of conditions are given in the following when the asymmetry is considered.

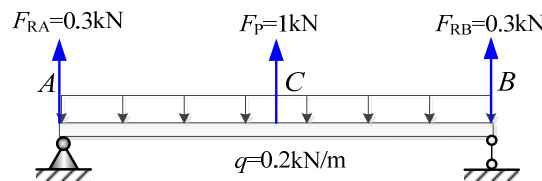


Fig.4. Simply supported beam

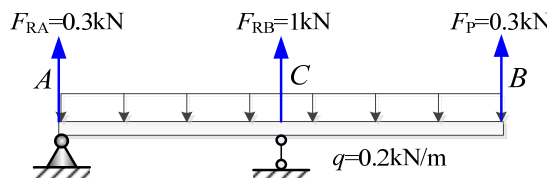


Fig.5. Overhanging beam 1

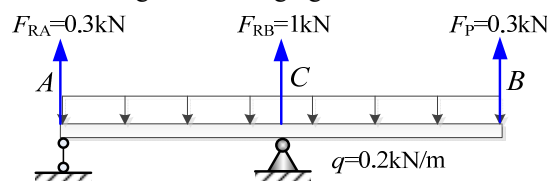


Fig.6. overhanging beam 2

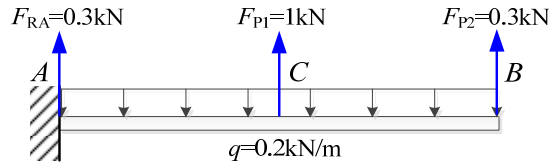


Fig.7. Cantilever beam

It's easy to check that the shearing force diagram and bending moment diagram of all the kinds are consistent with Figure 2. As the horizontal displacement can be neglected, so the fixed hinge bearing and the roller bearings play the same role in this case, namely they are interchangeable. So Figure 5 and 6 can be combined into a same kind overhanging beam, which makes a total of 3 kinds statically determinate constraint conditions.

#### 4. Inverse Problem in Statically Indeterminate Constraints

As the possible conditions of beam's statically determinate constraint have been listed above, following comes the statically indeterminate constraint conditions.

We can infer one of the possible constraint conditions from the beam's force condition, as shown in Figure 8. But it remains checking whether they agree.

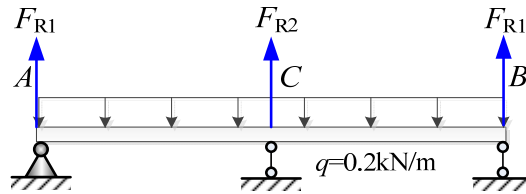


Fig.8. Statically indeterminate constraint

Now we give the solution using diagrammatic multiplication method.

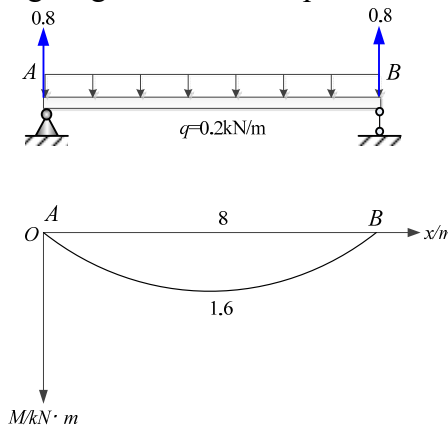


Fig.9. Equivalent system

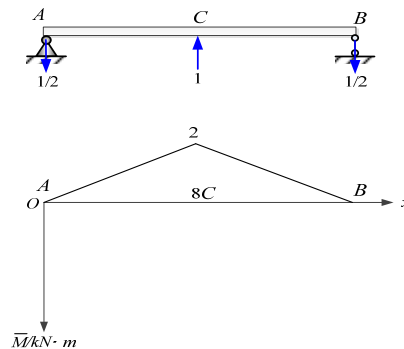


Fig.10. Unit loading system

$$\Delta_{1p} = \frac{1}{EI} \left[ \frac{2}{3} \times 8 \times 1.6 \times (-2) \right] = -\frac{256}{15EI} \quad \delta_{11} = \frac{1}{EI} \left( \frac{1}{2} \times 8 \times 2 \times 2 \right) = \frac{16}{EI}$$

$$X_1 \delta_{11} + \Delta_{1p} = 0 \Rightarrow X_1 = \frac{16}{15}$$

$$F_{R1} = \frac{4}{15}, \quad F_{R2} = \frac{16}{15}$$

The corresponding shearing force diagram and bending moment diagram is shown in Figure 11. From Figure 11 we know that the solution to the inverse problem in Figure 2 is improper, which means the added statically indeterminate constraint here can't replace the external force of the original statically determinate system.

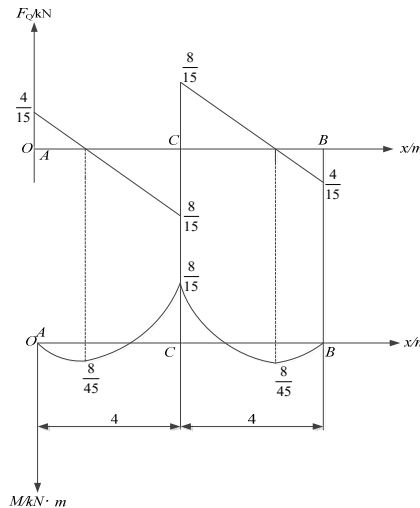


Fig.11. The shearing force diagram and bending moment diagram

Then, under what circumstances will statically indeterminate constraints be equivalent to statically determinate constraints? The answer is obvious. If the shearing force diagram and bending moment diagram are given as in Figure 11, the solution to corresponding inverse problem will have an equivalent statically indeterminate constraints condition, as shown in Figure 12.

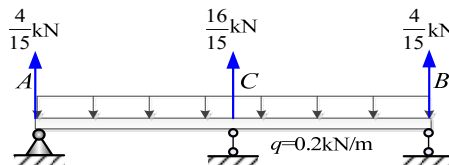


Fig.12. Equivalent statically indeterminate constraints

It's easy to check that the shearing force diagram and bending moment diagram of this condition is consistent with Figure 11. As a possible condition of statically indeterminate constraints is given, we list the rest conditions, as shown in Figure 13. (For convenience, we replace the fixed hinge bearing with the roller bearings.) Inequitable and equivalence conditions analysis are similar to Figure 8, so no more explanation here.

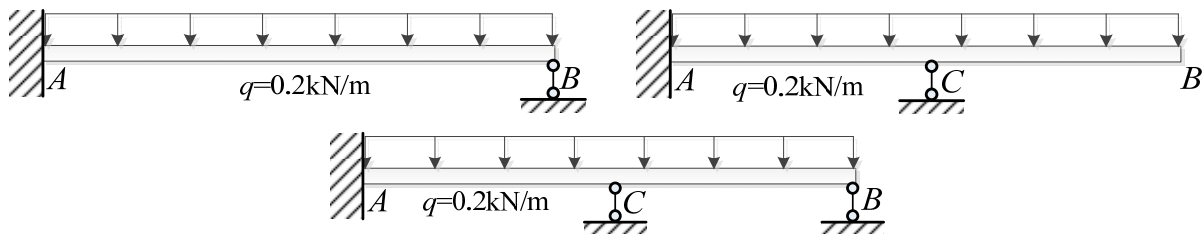


Fig.13. The rest statically indeterminate constraints conditions

### 5. Conclusion

In general speaking, for the forward problems and inverse problem in strength of materials, the solution of forward problem is unique while the other one is not, and it is necessary to discuss on the

conditions, such as distinctions between statically determinate constraints and statically indeterminate constraints of a beam. For the beam with determined internal force condition, this paper provides several different forms of beams, in the case of statically determinate constraints and statically indeterminate constraints, to meet the different needs of engineering practice.

**References**

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