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Fuzzy Multi-period Portfolio Optimization Problem of Stock Market

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Abstract. Due to the important effect of the transaction cost, risk, skewness and kurtosis to portfolio returns, the aim of this paper is to simulate the real transactions in stock market by considering the above factors. Firstly, two mean-semi-variance-skewness-kurtosis portfolio optimization models in open-loop and closed-loop are proposed by considering the transaction cost, return, risk, skewness and kurtosis. Secondly, the fuzzy programming approach is used to transform the two models into the corresponding single-objective programming models, and the genetic algorithm with adaptive scale adjustment is designed to solve them. Finally, the real data from the Shanghai Stock Exchange is given to illustrate the advantage of the proposed models and the efficiency of the designed algorithm.

Introduction

The aim of the portfolio management is how to distribute investment funding to maximize the expected return and minimize the risk. Markowitz (1952) originally proposed the M-V model for portfolio selection and optimization [1], which has opened the prelude to the theory of modern portfolio. Since then, many scholars have started to do related research on multi-period portfolio optimization problem in stock market under the Markowitz' mean-variance theory framework, such as Giove(2006)[2], Gupta (2008)[3], Xia (2000)[4], Yu (2014)[5], etc.

Most portfolio optimization models are proposed based on the probability theory, which characterize financial assets as random variables. However, in practice, the real financial market has many non-probabilistic factors affecting the returns of assets. With the continuous application of fuzzy set theory proposed by Zadeh (1965)[6], people began to realize that they can use fuzzy set theory to deal with the ambiguity in the securities market. So far, the study of the portfolio problem in the fuzzy environment has achieved considerable achievements. Östermark(1996) discussed about the dynamic portfolio problem with the risk-free asset and multi-risk asset using fuzzy decision theory and proposed a fuzzy control model[7]. Watada(1997) used fuzzy numbers to describe the investor's expectation for return rate and risk, and proposed a fuzzy portfolio optimization model based on fuzzy decision theory[8]. Qin (2017) employed random fuzzy variable to describe the stochastic return with ambiguous information, and designed random fuzzy simulation and simulation-based genetic algorithm to solve the proposed mean-absolute deviation portfolio optimization model [9]. Yue et al. (2014) proposed a new entropy function based on Minkowski's measure as a new objective function and then established a novel fuzzy multi-objective weighted possibilistic higher order moment portfolio model [10].

The traditional portfolio models were generally followed by the Markowitz model using variance to quantify the risk of portfolio, but there are many scholars have doubted whether variance can quantify the risk appropriately. Swalm(1966) argued that there will be risky only when the investment return is negative, while the risk should be zero when the return is positive, so the risk should be measured in lower semi-variance[11]. Japanese scholars Konno and Yamazai (1991) proposed a mean-absolute deviation model, which use the mean absolute deviation to replace the variance [12]. Pınar (2007) used the downside-risk measure as risk measure to study the multi-period portfolio optimization problem [13].

Another expanding direction of the portfolio model is the relaxation of the assumptions, especially the breakthrough of the assumption that there is no friction in the market. For example, the liquidity of open-end funds is very sensitive to the changes of transaction costs. If the transaction costs in the portfolio model are ignored, the effectiveness of the model will be greatly reduced. Magil and Constantinides (1976) first studied the impact of transaction costs on capital market equilibrium [14]. Eastham and Hastings (1988) used pulse control to study the impact of fixed costs on the portfolio and expanded the connotation of transaction costs [15].

Though great progress has been made in the existing research on the portfolio problem, to our knowledge, very few literatures are available in applying the fuzzy set theory to deal with multi-period portfolio optimization problem. Therefore, the purpose of this paper is to study the multi-period portfolio optimization problem in fuzzy environment. In this paper, the fuzzy multi-period portfolio optimization models in open-loop and closed-loop, which consider the return, risk, transaction cost, skewness and kurtosis of portfolio, are proposed. And the return of portfolio is quantified by the possibilistic mean value of fuzzy variables, the risk is characterized by the possibilistic semi-variance, the skewness and the kurtosis are measured by the third order moment and the fourth order moment about the possibilistic mean value of a return distribution.

The rest of this paper is organized as follows. In Section 2, we will give the details of the modeling process of the fuzzy mean-semi-variance-skewness-kurtosis models in open-loop and closed-loop. In Section 3, the fuzzy programming approach is used to transform the proposed models into the corresponding single-objective programming models, and the genetic algorithm with adaptive scale adjustment is designed to solve them. In section 4, a numerical example is given to illustrate the idea of our models and the effectiveness of the designed algorithm. Finally, some conclusions are given in Section 5.

The Fuzzy Multi-Period Portfolio Optimization Models

In this section, the problem representations and notations used in the following sections are introduced firstly. Then, we discuss the possibility return, the transaction cost, the cumulative risk, the skewness and kurtosis of the portfolio for the multi-period portfolio optimization problem in fuzzy environment. To express investors' preferences more flexible, two fuzzy mean-semi-variance-skewness-kurtosis models are formulated. One of them is an open-loop model and the other is a closed-loop model with dynamic feedback adjustment strategy.

Problem Representations and Notations

Assume that an investor allocates his initial wealth W_1 among n risky assets at the beginning of period 1. The investor is allowed to reallocate his wealth at every beginning of each period and he can obtain the terminal wealth at the end of period T. The returns of assets are denoted as trapezoidal fuzzy numbers. To make it easier to follow our exposition, we first introduce the following notations.$x_{i,t}$ represents the investment proportion of risky asset i at period t; $x(t)$ denotes the vector of the portfolio at period t; $r_{i,t}$ represents the return of risky asset i at period t; $r(t)$ represents the given minimum return level of the portfolio at period t; $c_{i,t}$ denotes the unit transaction cost for risky asset i at period t; c_t represents the transaction cost of the portfolio $x(t)$ at period t.

Maintaining the Integrity of the Specifications

The fuzzy possibilistic return, the transaction costs, the terminal wealth, the cumulative risk, the cumulative possibilisticskewness and kurtosis of the portfolio are introduced. And we quantify return by the possibilistic mean value, risk by possibilistic semi-variance, skewness and kurtosis by the third order possibilistic moment and the fourth order possibilistic moment about the fuzzy return of the asset.

Assume that the entire investment process is self-financing, that is, investors in the entire investment process do not inject new funds or withdraw funds. Based on the foregoing assumptions, $r_{i,t} = (a_{i,t}, b_{i,t}, \alpha_{i,t}, \beta_{i,t})(i = 1,2,\cdots n; t = 1,2,\cdots T)$ are trapezoidal fuzzy numbers. Then, the possibilistic mean value of portfolio $x(t) = (x_{1,t}, x_{2,t}, \cdots, x_{n,t})'$ at period t can be expressed as

$$\text{Eq.1 } E\left(\sum_{i=1}^{n} x_{i,t} r_{i,t}\right) = \sum_{i=1}^{n} \left(\frac{a_{i,t}+b_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right) x_{i,t} \quad (1)$$

We also assume that the transaction cost c_t is a V-shaped function of differences between the portfolio$x(t)$ and the portfolio $x(t-1)$ .So the transaction cost of the portfolio $x(t)$ at period$t$ can be expressed as

$$\text{Eq.2 } c_t = \sum_{i=1}^{n} c_{i,t} \left|x_{i,t} - x_{i,t-1}\right| \tag{2}$$

Then, the possibilistic mean value of the net return of the portfolio $x(t)$ at period$t$ can be expressed as

$$\text{Eq.3 } E(R_t) = E\left(\sum_{i=1}^{n} x_{i,t} r_{i,t} - c_t\right) = -\sum_{i=1}^{n} c_{i,t} \left|x_{i,t} - x_{i,t-1}\right| + \sum_{i=1}^{n} \left(\frac{a_{i,t} + b_{i,t}}{2} + \frac{\beta_{i,t} - \alpha_{i,t}}{6}\right) x_{i,t} \tag{3}$$

So, the expected value of the terminal wealth at the end of period T is

$$\text{Eq.4 } W_T = W_1 \prod_{t=1}^{T-1} E(R_t) = W_1 \prod_{t=1}^{T-1} \left[\begin{array}{c} \sum_{i=1}^{n} \left(\frac{a_{i,t}+b_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right) x_{i,t} \\ -\sum_{i=1}^{n} c_{i,t} \left|x_{i,t} - x_{i,t-1}\right| \end{array}\right] \tag{4}$$

The following conclusion is shown in Saeidifar and Pasha (2009)[16]. The semi-variance of the return is expressed as

$$\text{Eq.5 } Var^{-}(R_t) = \left[\sum_{i=1}^{n} \left(\frac{b_{i,t}-a_{i,t}}{2} + \frac{\alpha_{i,t}+\beta_{i,t}}{6}\right) x_{i,t}\right]^2 + \frac{1}{18}\left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^2 \tag{5}$$

Then, the cumulative risk of the portfolio over T period can be computed as

$$\text{Eq.6 } SV(x) = \sum_{t=1}^{T} \left\{\left[\sum_{i=1}^{n} \left(\frac{b_{i,t}-a_{i,t}}{2} + \frac{\alpha_{i,t}+\beta_{i,t}}{6}\right) x_{i,t}\right]^2 + \frac{1}{18}\left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^2\right\} \tag{6}$$

The cumulative possibilistic skewness of the T period investment can be computed as

$$\text{Eq.7 } PS(x) = \frac{1}{24}\sum_{t=1}^{T} \left\{\begin{array}{c} \frac{19}{45}\left[\left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)^3 - \left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^3\right] \\ +\frac{1}{3}\left[\left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)\left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)^2 - \left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)\left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^2\right] \\ +\left[\sum_{i=1}^{n} x_{i,t}\left(b_{i,t} - a_{i,t}\right)\right]\left[\left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)^2 - \left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^2\right] \end{array}\right\} \tag{7}$$

The cumulative possibilistic kurtosis of the T period investment can be given as

$$\begin{aligned} \text{Eq.8 } K(x) = {} & \frac{1}{72}\left[\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\beta_{i,t}\right]^2 + \frac{3}{8}\left[\sum_{i=1}^{n} x_{i,t}a_{i,t}b_{i,t}\right]^2 + \frac{5}{432}\left[\left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)^4 + \left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^4\right] \\ & + \frac{1}{16}\left[\left(\sum_{i=1}^{n} x_{i,t}a_{i,t}\right)^4 + \left(\sum_{i=1}^{n} x_{i,t}b_{i,t}\right)^4\right] - \frac{1}{18}\left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\beta_{i,t}\right)\left[\sum_{i=1}^{n} x_{i,t}\left(a_{i,t} - b_{i,t}\right)\left(\alpha_{i,t} + \beta_{i,t}\right)\right] + \\ & \frac{5}{432}\left[\left(\sum_{i=1}^{n} x_{i,t}\beta_{i,t}\right)^4 + \left(\sum_{i=1}^{n} x_{i,t}\alpha_{i,t}\right)^4\right] \end{aligned} \tag{8}$$

The Fuzzy Multi-period Portfolio Optimization Model in Open-loop

Assume that the objective of the investor wants to maximize the expected value of the terminal wealth, minimize the cumulative risk, maximize the cumulative skewness and minimize the cumulative kurtosis of the portfolio. Meanwhile, the investor also requires that the return at each period must be not less than the given minimum expected level $r(t)$ and the proportion of the investment allocated must meet a given boundary constraint. Thus, the following fuzzy multi-period portfolio optimization open-loop model (P_1) is formulated.

$$\text{Eq.9}\begin{cases} \max \quad W_T \\ \min \quad SV(x) \\ \max \quad PS(x) \\ \min \quad K(x) \\ s.t. \\ \sum_{i=1}^{n}\left(\frac{a_{i,t}+b_{i,t}}{2}+\frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t}-\sum_{i=1}^{n}c_{i,t}\left|x_{i,t}-x_{i,t-1}\right| \geq r(t)(a) \\ \sum_{i=1}^{n}x_{i,t}=1 \qquad (b) \\ W_{t+1}=W_t\left[\sum_{i=1}^{n}\left(\frac{a_{i,t}+b_{i,t}}{2}+\frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t}-\sum_{i=1}^{n}c_{i,t}\left|x_{i,t}-x_{i,t-1}\right|\right] \\ (c) \\ 0\leq x_{i,t}\leq u_{i,t}, \quad i=1,2,\cdots,n; t=1,2,\cdots T \qquad (d) \end{cases} \tag{9}$$

In the proposed model(P_1), the constraint (9)(a) indicates that the portfolio return must be not less than the given minimum return level $r(t)$ in each period; the constraint (9)(b) represents the investment proportion at period t sum to one; the constraint (9)(c) denotes the wealth accumulation constraint; the constraint (9)(d) represents that the investment proportion of risky asset i at period t, $x_{i,t}$, must not exceed the upper bound $u_{i,t}$. For notational simplicity, we denote the feasible region of the model (P_1) as$x \in D_1$.

The Fuzzy Multi-period Portfolio Optimization Closed-loop Model Based on Dynamic Feedback Adjustment Strategy

Although the formulation of P_1 is clear and easily, it ignores the influence of the historical information about returns of assets on portfolio decision-making. However, in many cases, the process of adjustment on portfolio is a closed-loop structure. In other words, the adjustment on portfolio depends on its historical information. In order to further illustrate the characteristics of closed-loop structure in the process of fuzzy multi-period portfolio optimization, the open-loop model (P_1) is extended to the corresponding closed-loop model (P_2).

In order to show that the multi-period investment decision-making process is a feedback control system, the adjustment sequence of the portfolio will be constructed by using the dynamic feedback adjustment strategy. Assume that the adjustment strategies of portfolios are affine functions about their one-period backwards return deviation [18]. In other words, the adjustment amount of the portfolio at period t depends on the return deviation of the portfolio at period $t-1$. Under this hypothesis, the dynamic feedback adjustment strategy can be expressed as the following causal function:

$$\text{Eq.10 } \Delta\tilde{x}(t)=\Delta\overline{x}(t)+\Theta(t-1)[r(t-1)-\overline{r}(t-1)] \tag{10}$$

Where $\Delta\tilde{x}(t)=(\Delta\tilde{x}_{1,t},\Delta\tilde{x}_{2,t},\cdots,\Delta\tilde{x}_{n,t})'$ denotes the dynamic feedback adjustment proportion of the portfolio at period t; $\Delta\overline{x}(t)=(\Delta\overline{x}_{1,t},\Delta\overline{x}_{2,t},\cdots,\Delta\overline{x}_{n,t})'$ are the nominal adjustment amount of the portfolio at period$t$, and $\Delta\tilde{x}(0)=\Delta\overline{x}(0)=(\Delta\overline{x}_{1,0},\Delta\overline{x}_{2,0},\cdots,\Delta\overline{x}_{n,0})'$ represents the initial adjustment amount. Assume that $\Theta(t)=\left(\theta_{ij}(t)\right)_{n\times n}$ is the market relation matrix at period t. Let $r(t)=(r_{1,t},r_{2,t},\cdots,r_{n,t})'$ represents the vector of the return of the portfolio at period t, and $\overline{r}(t)=(\overline{r}_{1,t},\overline{r}_{2,t},\cdots,\overline{r}_{n,t})'$ represents the vector of the expected level of the return of the portfolio at period$t$.

So, the crisp form investment proportion of the portfolio at period t can be expressed as

$$\begin{aligned}\text{Eq.11 } x(t)&=E\left(\tilde{x}(t)\right)=E[\tilde{x}(t-1)+\Delta\tilde{x}(t)] \\ &=E[\tilde{x}(t-1)]+\Delta\overline{x}(t)+\Theta(t-1)E[r(t-1)-\overline{r}(t-1)] \\ &=x(0)+\sum_{k=1}^{t}\{\Delta\overline{x}(k)+\Theta(k-1)E[r(k-1)-\overline{r}(k-1)]\}\end{aligned} \tag{11}$$

Then, the investment proportion of asset i at period t can be expressed as

$$\text{Eq.12 } x_{i,t} = x_{i,0} + \sum_{k=1}^{t}\{\Delta\overline{x}_{i,k} + \sum_{j=1}^{n}[\theta_{ij}(k-1)E(r_{j,k-1} - \overline{r}_{j,k-1})]\} \quad (12)$$

Moreover, the transaction costs of the portfolio at period t can be expressed as

$$\text{Eq.13 } c_t = \sum_{i=1}^{n} c_{i,t}\left|x_{i,t} - x_{i,t-1}\right| = \sum_{i=1}^{n} c_{i,t}\left|E(\Delta\tilde{x}_{i,t})\right| \quad (13)$$

$$= \sum_{i=1}^{n} c_{i,t}\left|\Delta\overline{x}_{i,t} + \sum_{j=1}^{n}[\theta_{ij}(t-1)E(r_{j,t-1} - \overline{r}_{j,t-1})]\right|$$

The terminal wealth at the end of period T is

$$\text{Eq.14 } W_{T+1} = W_1 \prod_{t=1}^{T} E(R_t) \quad (14)$$

$$= W_1 \prod_{t=1}^{T}\left[\sum_{i=1}^{n}\left(\frac{a_{i,t}+b_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - \sum_{i=1}^{n} c_{i,t}\left|E(\Delta\tilde{x}_{i,t})\right|\right]$$

For the risk, skewness and kurtosis of the portfolio, they are not related to the subjective forecast of the investor, since they are only related to the characteristics of the return of the portfolio. Therefore, the formulas of the cumulative risk, the cumulative skewness and kurtosis under closed-loop decision-making are the same as that in open-loop decision-making.

Assume that the investor's decision-making criteria are exactly the same as the model (P_1). At the same time, it is assumed that the investor takes into account the impact of the deviation between the past portfolio return and the expected return on the current investment adjustment proportion. Then, the following model (P_2) for fuzzy multi-period portfolio optimization in closed-loop is established.

$$\text{Eq.15}\begin{cases} \max \quad W_{T+1} \\ \min \quad SV(x) \\ \max \quad PS(x) \\ \min \quad K(x) \\ s.t. \\ \sum_{i=1}^{n}\left(\frac{a_{i,t}+b_{i,t}}{2} + \frac{\beta_{i,t}-\alpha_{i,t}}{6}\right)x_{i,t} - \sum_{i=1}^{n} c_{i,t}\left|E(\Delta\tilde{x}_{i,t})\right| \geq r(t) \quad (a) \\ \sum_{i=1}^{n}\left\{\Delta\overline{x}_{i,t} + \sum_{j=1}^{n}\left[\theta_{ij}(t-1)\left(\frac{a_{j,t-1}+b_{j,t-1}}{2} + \frac{\beta_{j,t-1}-\alpha_{j,t-1}}{6} - \overline{r}_{j,t-1}\right)\right]\right\} = 0 \quad (b) \\ 1'\Theta(t-1) = 0 \quad (c) \\ 0 \leq x_{i,t} \leq u_{i,t}, \quad i = 1,2,\cdots,n; t = 1,2,\cdots T \quad (d) \end{cases} \quad (15)$$

In the proposed model P_2, the constraint (15)(a) indicates that the return must be not less than the given minimum return level$r(t)$ in each period; the constraint (15)(b) represents the self-financing constraint; the constraint (15)(c) shows the relationship among elements in the market relation matrix at period t; the constraint (15)(d) represents that the investment proportion of risky asset i at period t ,$x_{i,t}$, must not exceed the upper bound $u_{i,t}$. For notational simplicity, we denote the feasible region of the model P_2 as$x \in D_2$.

Solution Algorithm

The Fuzzy Programming Approach

Since the models $P_1$and $P_2$are both multi-objective programming models. The incommensurability between the objectives makes it not easy to find the optimal solution for the multi-objective decision-making problem. In order to solve this problem, the fuzzy programming approach proposed by Zimmermann (1978) will be used to transform them into single objective models [17].

For model P_1, we can obtain its corresponding single objective model P_1'.

$$\text{Eq.16}\begin{cases} \max \lambda \\ s.t.\ \lambda \leq \frac{W_{T+1}-W_{T+1}^{-}}{W_{T+1}^{+}-W_{T+1}^{-}} \\ \lambda \leq \frac{SV^{-}(x)-SV(x)}{SV^{-}(x)-SV^{+}(x)} \\ \lambda \leq \frac{PS(x)-PS^{-}(x)}{PS^{+}(x)-PS^{-}(x)} \\ \lambda \leq \frac{K^{-}(x)-K(x)}{K^{-}(x)-K^{+}(x)} \\ x\epsilon D_1 \end{cases} \tag{16}$$

where $W_{T+1}^{+}=\max_{x\epsilon D_1} W_{T+1}$ and $W_{T+1}^{-}=\min_{x\epsilon D_1} W_{T+1}$, $SV^{+}(x)=\min_{x\epsilon D_1} SV(x)$ and $SV^{-}(x)=\max_{x\epsilon D_1} SV(x)$, $PS^{+}(x)=\max_{x\epsilon D_1} PS(x)$ and $PS^{-}(x)=\min_{x\epsilon D_1} PS(x)$, $K^{+}(x)=\min_{x\epsilon D_1} K(x)$ and $K^{-}(x)=\max_{x\epsilon D_1} K(x)$ represent the ideal solution and anti-ideal solution of W_{T+1}, $SV(x)$,$PS(x)$ and $K(x)$, respectively.

By using the same approach above, the model P_2can also be transformed into corresponding single objective optimization model P_2'.

The Genetic Algorithm

For the single objective optimization models P_1' andP_2' , it is usually difficult to solve them effectively by using traditional optimization algorithms. In consequence, the following genetic algorithm with adaptive scale adjustment is designed. In the new algorithm, the feasibility of each individual in the evolution process is judged by the constraint processing mechanism based on the feasible solution. The procedure of the designed genetic algorithm can be summarized as follows:

Step1. Input the population size *pop_size*, the crossover probability P_c and the mutation probabilityP_m;

Step2.Initialize the randomly generated solutions and code them in real numbers, then express them as the corresponding chromosomes;

Step3. Calculate the fitness function values for all chromosomes and perform scale transformation operations on them;

Step4. According to the fitness function values after the scale adjustment in ***Step3***, perform the selection operation by the roulette strategy;

Step5. Update the chromosomes by crossover and mutation operations;

Step6. Make judgments about the termination condition. If the genetic calculation achieves the maximum allowable algebras or the optimal individuals of successive generations has not improved, he best results will be output and the calculation will be finished, otherwise it will be transferred to ***Step3***.

Numerical Example

In this section,in order to illustrate the idea of the proposed models and the advantage of our designed algorithm, a numerical example based on real world data from Shanghai Stock Exchange will be given. Assume that there are six risky assets for an investor to choose, and he could reallocate his wealth at the beginning of each period. We extracted the historical data from the weekly closing prices of the six risky assets from Jan.2009 to Jan.2016, and set every four years as an observation period to analyze these data. Then, the trapezoidal probability distribution of each risky asset will be estimated by using the simple estimation method proposed by Vercher et al. (2007)[19]. The details can be seen in Table 1.

Table1. The trapezoidal fuzzy return for each asset

Asset i	t=1	t=2
1	(1.0346,1.1118,0.2800,0.7781)	(1.0313,1.1008,0.2804,0.7901)
2	(1.0388,1.0972,0.3036,0.3860)	(1.0390,1.1072,0.3126,0.3891)
3	(1.0241,1.0838,0.2057,0.3573)	(1.0260,1.0853,0.2018,0.3892)
4	(1.0159,1.0852,0.3541,0.3737)	(1.0170,1.0886,0.3556,0.3786)
5	(1.0132,1.0735,0.2951,0.3219)	(1.0118, 1.0782, 0.3017,0.3693)
6	(1.0094,1.0729,0.2791,0.3004)	(1.0082,1.0712, 0.3004,0.3528)

Suppose that an investor holds 10000 yuan at the beginning of period 1, that is, $W_1 = 10000$. And the transaction costs of assets at each period are identical, i.e., $c_{i,t} = 0.003$, for all $i = 1,2,3,4,5,6; t = 1,2$. Let the minimum expected return of the portfolio at the tth period, $(t) = 0.08(t = 1,2)$, and the investment proportion $x_{i,t} \epsilon [0,0.5] (i = 1,2,3,4,5,6; t = 1,2)$. To solve the proposed models with the designed genetic algorithm, the population size pop_size and the maximum iteration number are assumed to be 30 and 1000, the crossover probability P_c is supposed to be 0.8 and the mutation probability P_m is supposed to be 0.01.

Assume that an investor makes his investment decisions by the portfolio optimization model P_1. First, we transform P_1 into corresponding single-objective programming model $P_1{'}$. Then, running the designed genetic algorithm with 1000 generations, the portfolio strategies can be obtained and listed in Table 2.

Table 2. The portfolio strategies of the proposed model P_1

Asset i	t=1	t=2	The Terminal Wealth
1	0.2731	0.2505	14151
2	0.0885	0.1637	
3	0.1981	0.1020	
4	0.1147	0.0832	
5	0.1507	0.2449	
6	0.1750	0.1557	

As can be seen from Table 2, if an investor makes his investment decisions by the model P_1, he will obtain the terminal wealth of 14151.

Suppose that an investor makes his investment decisions by the portfolio optimization model P_2. Similar to the solving method of the model P_1, $P_2$is transformed into corresponding single-objective programming model $P_2{'}$ And we assume that the expected return for the investor at period 1 is $\bar{R}(1) = (1.0732,1.068,\ 1.0792,1.0538,1.0608,1.0521)$. The initial investment proportion $x(0)$ is set as $x(0) = (0.166,0.166,\ 0.167,0.167,0.167,0.167)$. The nominal adjustments proportion of every period investment are $\Delta\bar{x}(1) = (0.1672, -0.1672, 0.25, -0.25, 0.5, -0.5)$ and $\Delta\bar{x}(2) = (0.0002, -0.0001, 0.0084, -0.0086, 0.0093,\ 0.0092)$. So, the following market relation matric can be obtained.

$$\Theta(1) = \begin{pmatrix} -0.0019 & -0.0013 & -0.0003 & 0.0002 & 0 & 0 \\ 0.0005 & 0.0004 & 0.0002 & 0.0001 & 0 & 0 \\ -0.0985 & -0.0621 & -0.0521 & -0.0163 & 0 & 0 \\ 0.0999 & 0.0731 & 0.0219 & 0.0165 & 0 & 0 \\ -0.0031 & -0.0028 & -0.0019 & -0.0001 & 0 & 0 \\ 0.0931 & 0.0329 & 0.0352 & 0.0010 & 0 & 0 \end{pmatrix}$$

Then, solve the model $P_2{'}$ by using the designed genetic algorithm, and obtain the corresponding portfolio strategy as shown in Table 3 below.

Table 3. The portfolio strategies of the proposed model P_2

Asset i	t=1	t=2	The Terminal Wealth
1	0.2522	0.2556	14237
2	0.2563	0.1888	
3	0.0230	0.1533	
4	0.1575	0.1744	
5	0.1687	0.1987	
6	0.1423	0.0292	

From Table 2 and Table 3, we can find that the terminal wealth based on the models P_1 and P_2 are 14151 and 14237, respectively. In others words, the model P_2 performs better than P_1. Therefore, the relatively effective investment strategy is as follows:

The investor should allocate 25.22%, 25.63%, 2.3%, 15.75%, 16.87% and 14.23% of his initial wealth at the beginning of period 1 in risky assets 1, 2, 3, 4, 5 and 6, respectively. And the investor should invest 25.56%, 18.88%, 15.33%, 17.44%, 19.87% and 2.92% of the wealth at the end of period 1 in risky assets 1, 2, 3, 4, 5 and 6, respectively. At the end of period 2, the terminal wealth obtained by the investor is 14237.

Conclusion

In this paper, considering the return, transaction cost, risk, skewness and kurtosis of portfolio, the fuzzy multi-period portfolio optimization problem is studied. First, the fuzzy mean-semi-variance-skewness-kurtosis model in open-loop is proposed. Second, a new fuzzy multi-period portfolio optimization model in closed-loop is constructed by using the dynamic feedback strategy. Since the proposed models are multi-objective nonlinear programming models, the fuzzy programming approach is used to convert them into corresponding single objective programming models. Then, a genetic algorithm is designed to solve them. To prove the practicability of the proposed models and the effectiveness of the algorithm, a numerical example is given. The results show that the model based on dynamic feedback strategy performs better than the general model proposed. In other words, the deviation between the actual return and the expected return should be taken into account when investors make decisions in the stock market.

For the future research in the stock market, the fuzzy multi-period portfolio optimization model in closed-loop will be applied to other asset allocation problems. Asset returns can also be considered as random fuzzy variables. Furthermore, the efficient solution method of multi-objective model will help us deal with more complicated problems.

References

[1]H. Markowitz, Portfolio Selection, Journal of Finance, 7(1): 77–91, 1952.

[2]S. Giove, S. Funari and C. Nardelli, An interval portfolio selection problem based on regret function. European Journal of Operational Research, 170(1): 253-264, 2006.

[3]P. Gupta, M.K. Mehlawat and A. Saxena, Asset portfolio optimization using fuzzy mathematical programming, Information Sciences, 178(6): 1734–1755, 2008.

[4]Y.S. Xia, B.D. Liu, S.Y. Wang and K.K. Lai, A model for portfolio selection with order of expected returns, Computers & Operations Research, 27(5): 409–422, 2000.

[5]J.R. Yu and W.Y. Lee, Portfolio rebalancing model using multiple criteria, European Journal of Operational Research, 209(2): 166–175, 2011.

[6]L.A. Zadeh, Fuzzy sets, Information and control, 8(3): 338-353, 1965.

[7]R. Östermark, A fuzzy control model (FCM) for dynamic portfolio management, Fuzzy Sets and Systems, 78(3): 243-254, 1996.

[8]J. Watada, Fuzzy portfolio selection and its applications to decision making, Tatra Mountains Mathematical Publication, 13: 219-248, 1997.

[9]Z. F. Qin, Random fuzzy mean-absolute deviation models for portfolio optimization problem with hybrid uncertainty. Applied Soft Computing,56: 597-603,2017.

[10]W. Yue and Y.P. Wang, A new fuzzy multi-objective higher order moment portfolio selection model for diversified portfolios. Physica A, 465(1): 124-140, 2017.

[11]R.O. Swalm, Utility Theory—Insights Into Risk Taking. Harvard Business Review, 44(6): 123-136, 1966.

[12]H. Konno and H. Yamazaki, Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. Management Science, 37(5): 519-531, 1991.

[13]M.C. Pinar, Robust scenario optimization based on downside-risk measure for multi-period portfolio selection. OR Spectrum, 29(2): 295–309, 2007.

[14]M.J.P. Magill and G.M. Constantinides, Portfolio selection with transactions costs, Journal of Economic Theory, 13(2): 245-263, 1976.

[15]J.F. Eastham and K. J. Hastings, Optimal impulse control of portfolios, Mathematics of Operations Research, 13(4): 588-605, 1988.

[16]A. Saeidifar and E. Pasha, The possibilistic moments of fuzzy numbers and their applications, Journal of Computational and Applied Mathematics, 223(2): 1028-1042, 2009.

[17]H.J. Zimmermann, Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, 1(1): 45-55, 1978.

[18]W.G. Zhang, Y.J. Liu and W.J. Xu, A possibilistic mean-semivariance-entropy model for multi-period portfolio selection with transaction costs, European Journal of Operational Research, 222(2): 341–349, 2012.

[19]E. Vercher, J.D. Bermúdez and J.V. Segura, Fuzzy portfolio optimization under downside risk measures, American Journal of Pathology, 158(7): 769-782, 2007.