

# Fuzzy Identity-Based Threshold Key-Insulated Encryption with Ciphertext Policy

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**Abstract.** To solve the signing key exposure problem in fuzzy identity-based encryption systems with ciphertext policy, we propose a fuzzy identity-based threshold key-insulated encryption scheme with ciphertext policy (FIBTKIE-CP) which is provably secure. Our scheme is key-insulated and strongly key-insulated. Even if temporary private keys for up to  $N-1$  time periods are compromised, an adversary is still unable to obtain this user's temporary private key from the remaining time period. Even if up to  $k-1$  helpers are exposed and all temporary private keys are compromised, the adversary still can not harm the security of the non-exposed periods..

## Introduction

Security is harmed by inadvertent loss of private keys. In 2002, Dodis et al. [3] introduced a key insulation mechanism, which can protect secret keys in public key cryptosystems. Weng et al. [5] proposed the threshold key-insulation in which for at least  $k$  out of  $n$  helpers are used to refresh the user's temporary private keys. Ciphertext policy FIBE (FIBIE-CP) [2] is a variant of FIBE [4]. In a FIBIE-CP system, attributes are associated with user secret keys and access structures with ciphertexts. To deal with the key exposure problem in FIBE systems, Chen et al. gave a fuzzy identity-based parallel key-insulated encryption (FIBPKIE-CP) [1] scheme. But Chen et al. used two different helpers to refresh the private keys. There are some scenarios in which at least  $k$  out of  $n$  helpers are needed to update the user's temporary private keys. To strengthen the security and flexibility of Chen et al.'s scheme, we give a fuzzy identity-based threshold key-Insulated encryption scheme with ciphertext policy (FIBTKIE-CP) in which decryption is enabled if and only if the user's identity (attribute set) satisfies the access structure.

## Model of FIBTKIE-CP

### Definition

Throughout this paper, we use bilinear pairings, DBDH assumption and PRF[1]. We let  $Z_p^*$  denote the set  $\{0, 1, 2, \dots, p-1\}$  and denote  $Z_p / 0$ . For a finite set  $S$ ,  $x \in_R S$  means choosing an element  $x$  from  $S$  with a uniform distribution. A FIBTKIE-CP scheme consists of six algorithms: (1) Setup( $k$ ): Given a security parameter  $k$ , the authority runs this algorithm to output a master secret key  $msk$  and a public key  $pk$ ; (2) KeyGen( $w, msk$ ): Given the user's identity  $w$ , as a set representing a user's attributes, and the master-key  $msk$ , the authority runs this algorithm to output an initial private key  $TK_{w,0}$  and  $n$  helper keys  $\{HK_{w,i'}\}_{1 \leq i' \leq n}$  corresponding to  $w$ . Each helper key  $HK_{w,i'}$  is kept by the  $i'$ -th helper and the user with identity  $w$  keeps the initial private key. (3) HelperUpt( $t, w, HK_{w,i'}, pk$ ): The helper key-update algorithm takes as input a period index  $t$ , an identity  $w$  and his  $i'$ -th ( $1 \leq i' \leq n$ ) helper key  $HK_{w,i'}$ . It outputs the  $i'$ -th key-update information share  $UI_{w,t,i'}$  with respect to identity  $i'$  and period  $t$ . (4) UserUpt( $t, w, TK_{w,t'}, UI_{w,t',i'}, PK$ ): The user key-update algorithm takes as input an identity  $w$ , his temporary private key  $TK_{w,t'}$  for period  $t'$ , and a set  $\{UI_{w,t',i'}\}_{i' \in S'}$  of key-update information shares, where  $S \subseteq \{1, \dots, n\}$  and  $|S'| \geq k$ . It returns this user's temporary private key  $TK_{w,t}$  for period  $t$ , and deletes  $TK_{w,t'}$  and  $\{UI_{w,t',i'}\}_{i' \in S'}$ ; (5) Encryption( $t, M, W, pk$ ): The Encryption algorithm takes as input the public key  $pk$ , the time period index  $t$ , a message  $M$  and an access structure  $W$ . It returns a ciphertext

$(t, E)$  such that a temporary private key generated from attribute set  $w$  for period  $t$  can be used decrypt  $(t, E)$  if and only if  $w \models W$ ; (6)  $\text{Decryption}(t, E, w, TK_{w,t}, pk)$ : The Decryption algorithm takes as input a ciphertext  $(t, E)$  and a temporary private key  $TK_{w,t}$ . It returns the message  $M$  if  $w$  satisfies  $W$ , where  $S$  and  $t$  are the identity (attribute set) and the time period index respectively used to generate  $TK_{w,t}$ .

### Security notions for FIBTKIE-CP

A FIBTKIE-CP scheme is said to be secure against chosen plaintext attacks (CPA) in the sense of key-insulation if no probabilistic polynomial-time adversaries have non-negligible advantage in the following game. For convenience, we give the definition of a restricted identity as below: the attribute set of the restricted identity satisfies challenge access structure  $W^*$ .

**Init.** The adversary declares the access structure  $W^*$  and the time period index  $t^*$  that he wishes to be challenged upon.

**Setup.** The challenger runs the setup phase of the algorithm and tells the adversary the public parameters.

**Phase 1.** The adversary adaptively issues a set of queries as below: (1) Key Generation Query  $\langle g \rangle$ : The challenger first runs algorithm  $\text{KeyGen}$  to obtain the initial private key  $TK_{g,0}$  and  $n$  helper keys  $\{HK_{g,i'}\}_{1 \leq i' \leq n}$ . It then sends these results to the adversary; (2) Helper Key Query  $\langle g, i' \rangle$ : The challenger responds by running algorithm  $\text{KeyGen}$  to generate  $HK_{g,i'}$  and sends it to the adversary; (3) Temporary Private Key Query  $\langle g, t \rangle$ : The challenger responds by running algorithms  $\text{HelperUpt}$  and  $\text{UserUpt}$  to generate  $TK_{g,t}$ . It then returns it to the adversary.

**Challenge.** The adversary submits two equal length messages  $M_0, M_1$ . The challenger flips a random coin,  $b$ , and encrypts  $M_b$  with  $W^*$  and  $t^*$ . The ciphertext is passed to the adversary.

**Phase 2.** Phase 1 is repeated.

**Guess.** The adversary outputs a guess  $b'$  of  $b$ .

The advantage of an adversary  $A$  in this game is defined as  $\Pr[b' = b] - 1/2$ . We refer to the above game as an IND-FIBTKIE-CP-KI-CPA game. In the above game, it is mandated that the following conditions are simultaneously satisfied: (1)  $A$  is disallowed to issue key generation queries for the restricted identities; (2)  $A$  is disallowed to issue temporary private key queries for the restricted identities and the challenged time period  $t^*$ ; (3)  $A$  can only corrupt up to  $k - 1$  helper keys with respect to the restricted identities.

FIBTKIE-CP scheme is said to be secure against chosen plaintext attacks (CPA) in the sense of strong key-insulation if no probabilistic polynomial-time adversaries have non-negligible advantage in an IND-FIBTKIE-CP-SKI-CPA game. The IND-FIBTKIE-CP-SKI-CPA game is almost the same as the IND-FI&KI-CPA game except Phase 1.

**Phase 1.** The adversary adaptively issues a set of queries as below: (1) Key Generation Query  $\langle g \rangle$ : the same as the IND-FIBTKIE-CP-KI-CPA game; (2) Helper Key Query  $\langle g, i' \rangle$ : The same as the IND-FIBTKIE-CP-KI-CPA game.

The advantage of an adversary  $A$  in this game is defined as  $\Pr[b' = b] - 1/2$ . In the above game, it is mandated that the following condition is satisfied:  $A$  is disallowed to issue key generation queries for the restricted identities.

### Model of FIBTKIE-CP

#### Description of Our Scheme

Our proposed FIBTKIE-CP scheme is based on Cheung-Newport's construction [2]. Let  $G_1$  and  $G_2$  be two groups with prime order  $q$  of size  $k$ ,  $g$  be a random generator of  $G_1$ , and  $e$  be a bilinear map such that  $e : G_1 \times G_1 \rightarrow G_2$ . Let  $H$  be a collision-resistant hash function such that  $H : \{0, 1\}^* \rightarrow \{0, 1\}^{n_u}$ . We use a PRF family  $F$  such that given a  $k$ -bit seed (index)  $s$  and a  $k$ -bit argument (input)  $x$ , it outputs a  $k$ -bit string  $F_s(x)$ . An access structure on attributes is a rule  $W$  that returns either 0 or 1 given an identity  $S$  (a set of attributes). We say that  $S$  satisfies  $W$  (written  $S \models W$ ) if and only if  $W$  answers 1 on  $S$ . Let the set of attributes be  $N = \{1, \dots, n\}$  for some natural number  $n$ . We regard attributes  $i$  and their

negations  $\neg i$  as literals. We consider access structures that consist of a single AND gate whose inputs are literals. Let  $W = \bigwedge_{i \in I} \underline{l}_i$  where  $I \subseteq N$  and every  $\underline{l}_i$  is a literal (i.e.,  $i$  or  $\neg i$ ).

-Setup: The authority picks  $y, t_1, \dots, t_{3n} \in_{\mathbb{R}} \mathbb{Z}_p$ ,  $g_2, h_1 \in_{\mathbb{R}} G_1$ , sets  $Y = e(g, g)^y$  and  $T_k = g^{t_k}$  for each  $k \in \{1, \dots, 3n\}$ . We define  $H_w: \mathbb{Z}_p \rightarrow G_1$  to be the function  $H_w(x) = g_1^x h_1$ . The public key is  $pk = (G_1, G_2, e, g, g_1, Y, h_1, T_1, \dots, T_{3n}, H_w)$ . The master secret key is  $msk = (y, t_1, \dots, t_{3n})$ . As illustrated in Table 1, the public key elements  $T_i$ ,  $T_{n+i}$  and  $T_{2n+i}$  correspond to the three types of occurrences of  $i$ : positive, negative and *don't care*.

Table 1. Common Parameters

	1	2	3	...	$n$
positive	$T_1$	$T_2$	$T_3$	...	$T_n$
negative	$T_{n+1}$	$T_{n+1}$	$T_{n+3}$	...	$T_{2n}$
<i>Don't Care</i>	$T_{2n+1}$	$T_{2n+1}$	$T_{2n+3}$	...	$T_{3n}$

-KeyGen: To generate the helper key and the initial private key for identity  $S$ , the authority does as follows. Let  $S$  denote the input identity (attribute set). Every  $i \in S$  is implicitly considered a negative attribute. Pick  $r_i \in_{\mathbb{R}} \mathbb{Z}_p$  for every  $i \in N$  and set  $r = \sum_{i=1}^n r_i$ . Randomly choose a helper key  $HK_S \in_{\mathbb{R}} \{0, 1\}^k$ , compute  $k_{S,0} = F_{HK_S}(0)$ . Note that if the length of the input for  $F$  is less than  $k$ , we can add some "0"s as the prefix to meet the length requirement. Let  $\hat{D}'_{S,0} = g^{y-r} H_w(0)^{k_{S,0}}$ ,  $\hat{D}''_{S,0} = g^{k_{S,0}}$ . For each  $i \in N$ , let  $D_i =$  if  $i \in S$ ; otherwise, let  $D_i = g^{\frac{r_i}{t_{n+i}}}$ . Let  $F_i = g^{\frac{r_i}{t_{2n+i}}}$  for every  $i \in N$ . Pick  $b \in_{\mathbb{R}} \mathbb{Z}_p^*$  and set  $R = g^b$ , compute the initial private key  $TK_{S,0} = (R, -, \{D_i\}_{i \in N}, \{F_i\}_{i \in N})$ .

For each  $i \in w$  and each index  $i' \in \{1, \dots, k-1\}$ , pick  $l_{i,i'} \in_{\mathbb{R}} \mathbb{Z}_p^*$  and set the  $i'$ -th helper key to be

$$HK_{w,i'} = (\{HK_{i,i'}\}_{i \in w}) = (\{g_2^{l_{i,j}}\}_{i \in w}) \quad (1)$$

Let  $S' = \{0, 1, \dots, k-1\}$ . For each  $i \in w$  pick  $s_i \in_{\mathbb{R}} \mathbb{Z}_p^*$ . For each remaining index  $i' \in \{k, \dots, n\}$ , set the  $i'$ -th helper key to be

$$(\{ (g_2^{y-r-b})^{D_{i,S'}(0)} (\prod_{j=1}^{k-1} HK_{i,i'}^{D_{i,S'}(j)}) \}_{i \in w}) \quad (2)$$

-HelperUpt: Given a period index  $t$ , an identity  $w$  and his  $i'$ -th ( $1 \leq i' \leq n$ ) helper key  $HK_{w,i'}$ , this algorithm works as follows. Parse  $HK_{w,i'}$  as  $(\{HK_{i,i'}^{(1)}\}_{i \in w})$ . For each index  $i' \in \{1, \dots, n\}$ , pick  $u_{i'} \in_{\mathbb{R}} \mathbb{Z}_p^*$  and output user  $w$ 's  $i'$ -th key-update information share  $UI_{w,t,i'}$  for period  $t$  as

$$UI_{w,t,i'} = (\{HK_{i,i'} H_w(t)^{u_{i'}}\}_{i \in w}, g^{u_{i'}}) \\ = (\{g_2^{l_{i,j}} V(i)^{r_{i,i'}} H_w(t)^{u_{i'}}\}_{i \in w}, g^{u_{i'}})$$

-UserUpt: Given an identity  $w$ , a temporary private key  $TK_{w,t'}$  for period  $t'$ , and a set  $\{UI_{w,t,i'}\}_{i' \in S'}$  of key-update information shares for period  $t$ , where  $S' \subseteq \{1, \dots, n\}$  and  $|S'| \geq k$  (for convenience, we assume  $|S'| = k$ ), this algorithm works as follows. Parse  $TK_{w,t'}$  as  $(R, \hat{D}'_{S,t'}, \hat{D}''_{S,t'}, \{D_i\}_{i \in N}, \{F_i\}_{i \in N})$ ; Parse  $UI_{w,t,i'}$  as  $(UI_{i,t,i'}^{(1)}, UI_{i,t,i'}^{(2)})$ ; Set user  $w$ 's temporary private key  $TK_{w,t}$  for period  $t$  to be  $(R, (\prod_{i' \in S'} UI_{i,t,i'}^{(1)})^{D_{i,S'}(0)}, (\prod_{i' \in S'} UI_{i,t,i'}^{(2)})^{D_{i,S'}(0)}, \{D_i\}_{i \in N}, \{F_i\}_{i \in N})$ . Note that in time period  $t$ , if let  $u = \sum_{i' \in S'} \Delta_{0,S'}(i') \cdot u_{i'}$ , then  $TK_{w,t}$  is always set to be

$$(R, \hat{D}'_{S,t}, \hat{D}''_{S,t}, \{D_i\}_{i \in N}, \{F_i\}_{i \in N}) = (g^b, \{g^{y-r-b} H_w(t)^u\}_{i \in w}, g^u, \{D_i\}_{i \in N}, \{F_i\}_{i \in N}).$$

–Encryption: Given time period index  $t$ , a message  $M \in G_1$  and an AND gate  $W = \bigwedge_{i \in I} i$ , this algorithm does as follows. Pick  $s \in \mathbb{R}Z_p$ ; For each  $i \in I$ , let  $E_i = T_i^s$  if  $i = i$  and  $T_{n+i}^s$  if  $i = \neg i$ ; for each  $i \in N \setminus I$ , let  $E_i = T_{2n+i}^s$ . The ciphertext is  $(t, E) = (t, (W, E', E'', E''', \{E_i\}_{i \in N}))$

–Decryption: Suppose the input ciphertext is of the form  $(t, E) = (t, (W, E', E'', E''', \{E_i\}_{i \in N}))$ , where  $W = \bigwedge_{i \in I} i$ . Also, let  $w$  denote the identity used to generate the input secret key  $TK_{w,t} = (g^b, \{g^{y-r-b} H_w(t)^u\}_{i \in w}, g^u, \{D_i\}_{i \in N}, \{F_i\}_{i \in N})$ . For each  $i \in I$ , this algorithm computes the pairing  $e(C_i, D_i)$ .

If  $i = i$  and  $i \in w$ , then  $e(E_i, D_i) = e(g^{t_i \cdot s}, g^{\frac{r_i}{t_i}}) = e(g, g)^{r_i \cdot s}$ ; If  $i = \neg i$  and  $i \in w$ , then  $e(E_i, D_i) = e(g^{t_{n+i} \cdot s}, g^{\frac{r_i}{t_{n+i}}}) = e(g, g)^{r_i \cdot s}$ ; for each  $i \in I$ , this algorithm computes the pairing  $e(E_i, F_i) = e(g^{t_{2n+i} \cdot s}, g^{\frac{r_i}{t_{2n+i}}}) = e(g, g)^{r_i \cdot s}$ . Then, the ciphertext can be decrypted as

$$M = \frac{E' e(E''', \hat{D}_{S,t}'')}{e(E'', R \cdot \hat{D}_{S,t}') \prod_{i=1}^n e(g, g)^{r_i \cdot s}} = \frac{M \cdot Y^s e(H_w(t)^s, g^u)}{e(g^s, g^b g^{y-r-b} H_w(t)^u) e(g, g)^{r \cdot s}}$$

$$= \frac{M \cdot Y^s e(H_w(t)^s, g^u)}{e(g^s, g^{y-r}) e(g^s, H_w(t)^u) e(g, g)^{r \cdot s}} = \frac{M \cdot Y^s}{e(g, g)^{y \cdot s}} = \frac{M \cdot Y^s}{Y^s}$$

### Security

The proof of our proposed FIBTKIE-CP scheme is similar with that of Chen et al.'s FIBPKIE-CP[1].

### Conclusions

We introduce the notion of fuzzy identity-based key-insulated encryption with ciphertext policy (FIBTKIE-CP) and describe a construction that is provably secure.

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