

# Optimal Design of Integral Steering Linkage

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**Abstract.** The integral steering linkage is the key part on vehicle, which can ensure the Ackerman steer angle when steering. in this paper, The mathematical model of the Ackerman steering linkage is established. The author introduces the optimal design program for the Ackerman steering linkage with MATLAB optimtoolbox. With the program, users can obtain optimized calculating results by inputting basic structural parameters. Actual output angle and expected input angle curves variation with input angle is automatically drawn so as to facilitate users for analysis, comparison and selection. The proposed method is accurate and efficient to design the splitting Ackerman steering linkage.

## Introduction

One of the important works to design the steering trapezoid body is to determine the best parameters of steering linkage. The work ensures that each tire is in the state of pure rolling on vehicle turning so as to reduce the additional resistance and tire abrasion. Generally, technical staffs use the modern design theory methods such as complex method and penalty function method to optimize the design with the application of programming language such as FORTRAN and VISUALC++. However, it's difficult for laypeople to program the procedure. In this paper, the pertinent function from the Optimization Toolbox of Matlab is used for implementing the optimal design of the steering linkage. Compared with the traditional methods, the proposed method is easier to be applied.

## Mathematical Model of Steering Mechanism

### Mathematical Model

The analytical model of two-axle vehicle steering problem is set up with ignore the tire cornering effect. On this condition, the vehicle moves along a curved path with the radii originating from a common center, which is shown in Fig. 1. In order to ensure that all wheels keep pure rolling without slip on vehicle turning, the steer angle of the inside front wheel  $q_i$  and that of the outside front wheel  $q_o$  should satisfy the following relationship:

$$\cot q_o - \cot q_i = \frac{K}{L} \quad (1)$$

where  $K$  and  $L$  are the track and wheelbase of the vehicle, respectively.

If the variable angle is  $q_o$ , the expectations of depending variable angle  $q_i$  is:

$$q_i = f(q_o) = \arccot \left( \cot q_o - \frac{K}{L} \right) \quad (2)$$

Steering linkage of existing institutions can only satisfy the similar relationship mentioned above. Taking Figure 1-- post-steering linkage for example, we can get the actual depending variable angle  $q_i'$  of steering linkage through cosine theorem

$$q_i' = g - \arcsin(C/A) - \arccos(B/A) \quad (3)$$

$$\text{where } A = \sqrt{\left(\frac{K}{m}\right)^2 + 1 - 2\frac{K}{m}\cos(g + q_o)}$$

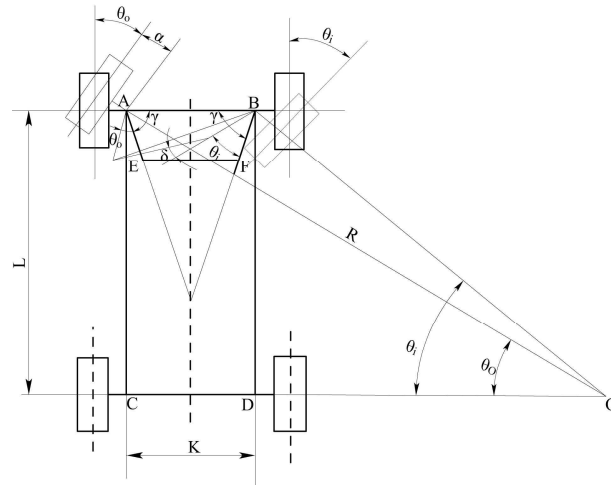


Fig.1. Diagram of ideal relationship between the steer angle of the inside and that of the outside

$$B = \frac{K}{m} [2 \cos g - \cos(g + q_o) - \cos 2g]$$

$$C = \sin(g + q_o)$$

$m$  is the trapezoidal arm length;  $g$  is the trapezoidal bottom corner.

The actual depending variable angle  $q_i'$  of steering linkage should approach the theoretical Expectation  $q_i$  as close as possible. In most cases, vehicles move at the small corner. So the deviation of  $q_i'$  from  $q_i$  should be as small as possible near the central location. On the contrary, as the vehicle do not turn at low speed frequently, the requirements on the deviation can be reduced advisably. By introducing weights factor  $w(q_{oi})$ , we set up the objective function  $f(x)$ , which can evaluate the design

$$f(x) = \sum_{q_{oi}=1}^{q_{oi\max}} w(q_{oi}) \left[ \frac{q_i'(q_{oi}) - q_i(q_{oi})}{q_i(q_{oi})} \right] \times 100\% \quad (4)$$

Substitute the Eq. (2), Eq. (3) into the Eq. (4), and we get

$$f(x) = \sum_{q_{oi}=1}^{q_{oi\max}} w(q_{oi}) \left| \frac{g - \arcsin \frac{\sin(g + q_{oi})}{\sqrt{\left(\frac{K}{m}\right)^2 + 1 - 2\frac{K}{m} \cos(g + q_{oi})}}}{\arccot \left( \cot \left( \cot q_{oi} - \frac{K}{L} \right) \right)} - \frac{\frac{K}{m} [2 \cos g - \cos(g + q_{oi})] - \cos 2g}{\arccos \frac{\sqrt{\left(\frac{K}{m}\right)^2 + 1 - 2\frac{K}{m} \cos(g + q_{oi})}}}{\arccot \left( \cot q_{oi} - \frac{K}{L} \right)}} \right| \times 100\% \quad (5)$$

in the equation,  $x$  is design variables,  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} g \\ m \end{bmatrix}$ ;  $q_{oi\max}$  is the largest steer angle outside front wheel,

from Figure 1 we get

$$q_{oi\max} = \arcsin \frac{L}{\frac{D_{\min}}{2} - a}$$

in the equation,  $D_{\min}$  is the smallest turning diameter,  $a$  is kingpin offset distance. Considering the  
Considering the difference of turning occurrences frequency, we choose

$$w(q_o) = \begin{cases} 1.5 & 0^\circ < q_o \leq 10^\circ \\ 1.0 & 10^\circ < q_o \leq 20^\circ \\ 0.5 & 20^\circ < q_o \leq q_{o\max} \end{cases} \quad (6)$$

### Constraint Conditions

The constraint conditions are formed by the range of the design variables is as follow:

$$m - m_{\min} \geq 0 \quad (7)$$

$$m_{\max} - m \geq 0 \quad (8)$$

$$g - g_{\min} \geq 0 \quad (9)$$

Usually, trapezoidal arm length  $m$  and trapezoidal bottom corner  $g$  should be:  $m_{\min} = 0.11K$ ,  $m_{\max} = 0.15K$ ,  $g_{\min} = 70^\circ$ . According to the mechanical principle, the transmission angle  $d$  of four-bar linkage should not be too small, normally  $d \geq d_{\min} = 40^\circ$ . As is shown in Fig.1, when the vehicle turns right to the limits,  $d$  reaches the minimum, the constraint should be:  $d \geq d_{\min}$ . We use auxiliary dotted lines made on the plan and the cosine theorem to deduce the constraint conditions of minimum transmission angle:

$$\frac{\cos d_{\min} - 2 \cos g + \cos(g + q_{o\max})}{(\cos d_{\min} - \cos g) \cos g} - \frac{2m}{K} \geq 0 \quad (10)$$

in the equation,  $d_{\min}$  is the minimum transmission angle.

### MATLAB programming cruces

After setting up the objective function and the constraints, two functions ynh1.m and ynh2.m need to be established under the environment of Matlab. By calling the "constr" function of MATLAB's Optimization Toolbox, we can achieve optimal design about the steering linkage.

### Examples of calculation and the analysis of results

Taking a certain vehicle type's steering linkage as an example, the optimal parameters of steering linkage are calculated with different initial values, and the optimization results are shown in Table 1.

**Table 1. The Different optimization results from different initial values**

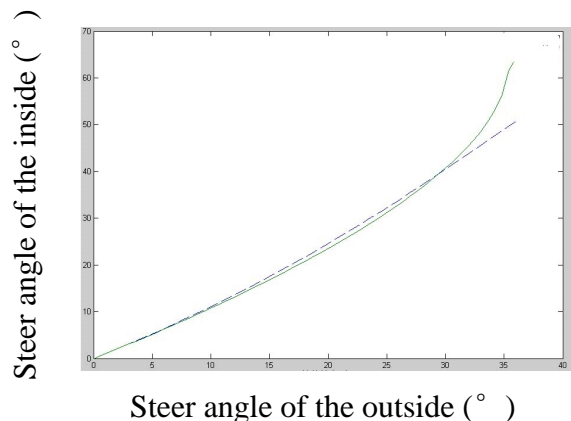
Initial values		Optimal calculation results	
Trapezoidal arm length(mm)	Trapezoidal angle(°)	Trapezoidal arm length(mm)	Trapezoidal angle(°)
255	70	254.9987	70.0128
280	75	280.0224	70.0000
315	80	315.0191	70.0000
345	80	345.00064	70.4750

As shown in Fig.2 ~ Fig.5, the optimization results are used as input data to get the curves of actual angle and expectation angle with varying of the input angle. In each chart, the solid line is delegated to the actual angle, and the dotted line is delegated to the expected angle.

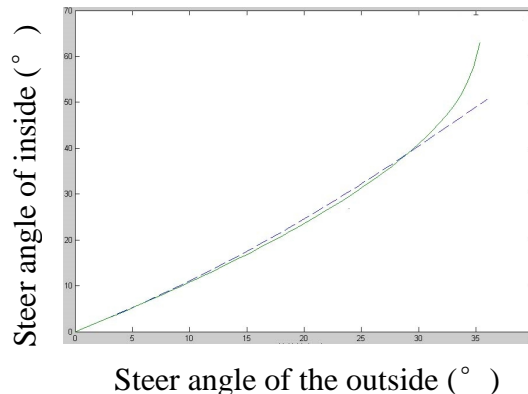
The charts of optimization results show that the actual angle line (225, 70) is the best in agreement with the expectation, and in this case the actual angle is able to change as required, while other actual angles are unable in some extent. So it is the best choice to select 255mm trapezoidal arm and 70 degrees trapezoidal angle as initial values.

## Conclusion

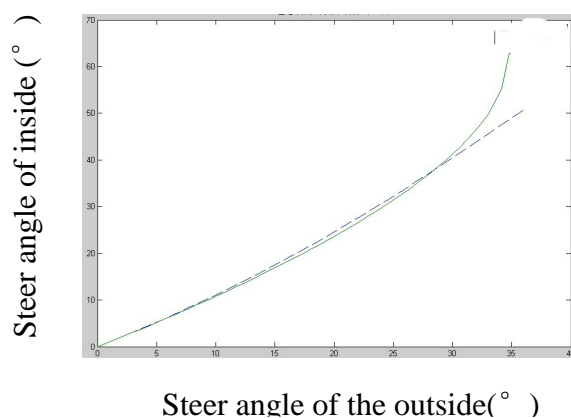
This paper deals with the vehicle steering linkage optimal design based on programming language MATLAB. The method, which is simple, efficient and practical, is a good reference for the design of vehicle components.



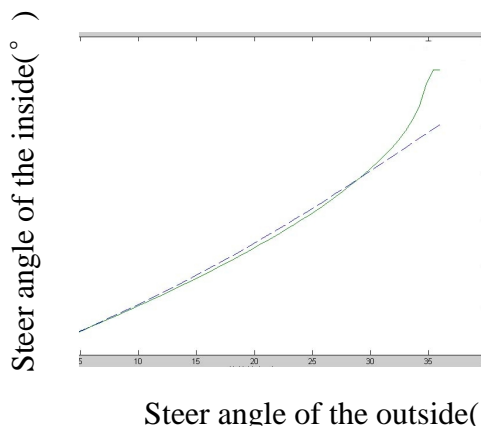
**Figure2. Curve of initial values is (255, 70)**



**Figure3. Curve of initial values is (280, 75)**



**Figure4. Curve of initial values is (315, 80)**



**Figure5. Curve of initial values is (345, 85)**

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