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Analysis of growth of elliptic delaminations in layered structural elements

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***Abstract* - Layered composite materials are an alternative to traditional materials for the manufacture of aviation components, details and structural elements. The delamination is a widespread destruction type of layered composite materials. The paper considered the characteristically destruction of defect growth by the example of a two-dimensional defect model of elliptic delamination type. The main stages of equilibrium parameters of Griffith's cracks are considered with account of the solution of the problem of post critical deformations. The only delamination of the elliptic form exists in the plate. The energy release rate is estimated. Generalized forces which advance delamination are determined. Generalized forces are found by numerical differentiation of the potential energy with respect to the size of the delamination with infinitesimal increments.**

Keywords - composite materials, inter-layer defects, delamination, nonlinear theory

I. INTRODUCTION

Layered composite materials are an alternative to traditional materials for the manufacture of aviation components, details and structural elements. Structural damage to composite materials is a complex problem. The engineering goals of fracture mechanics are to assess the dangers of various types of defects or voids. Defects can appear in the form of fiber ruptures, violations of fiber-matrix bonding, delamination and other types of destruction. The control of the strength of composites is not developed [1-4]. The delamination is a widespread destruction type of layered composite materials. The delamination is the determining factor when deciding on the use of composites in products. The model of linear destruction in the research of propagating cracks in the delamination form is used. The field of stress for a linearly elastic solid near the peak of the crack is expressed through external loads or stresses. The energy approach in the research is used. The energy approach originates from the classic Griffith's research [8-12].

II. MATERIALS AND METHODS

The main provisions of Griffith's energy concept are generalized [5-10].

The condition of equilibrium means that the total increment in the work of external forces and liberated potential energy of the solid is compensated by the work that falls to the growth of cracks:

$$-\delta U + \delta A - \delta \Phi \equiv \sum_{j=1}^{m} G_j \delta l_j ;$$

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$$G = \Gamma ,$$

where G – the parameter of energy liberation, Γ- the parameter of energy, that must be expended for a single increase of crack length. $\Gamma = 2\gamma$, where $\gamma[н/м]$ – the specific surface energy of destruction. The energy of destruction is the work necessary to form a unit of a new surface of a solid from this material.

The growth of the crack is accompanied by the formation of two defect edges. The energy 2γ is required to form the unit of the defect area. The value of the specific work of material γ destruction can be determined experimentally.

The research considered the characteristically destruction of defect growth using the example of a two-dimensional defect model of the elliptic delamination type. The main stages of equilibrium parameters of Griffith's cracks are considered taking into account the solution of the problem of post critical deformations [11]. Suppose that the only delamination of the elliptic form exists (Fig. 1). The semi-axes of the ellipse are *a* and *b*, and *h* – is the thickness of the defect. In addition, *h* satisfies the condition $h << a, b$. On the contour of the defect, the bases of correspond deformations are set $\varepsilon_x = \varepsilon_\circ$, $\varepsilon_y = \mu\varepsilon_\circ$, where μ - Poisson's ratio of the base. The material of the plate homogeneous, linearly elastic is considered. In the area of delamination, the plate consists of two parts (Fig. 1): an exfoliated layer (the upper part) and a layer below the delamination (the lower part is $H-h$). The main part of the laminated plate locates outside the defect. The thickness of the main part of the plate is H. When the loss of stability begins with the buckling of the exfoliated layer, local loss of stability is considered. A critical load is found from the solution of the linear problem.

The critical load is expressed through the deformations ε_{crx} and ε_{cry}, and the corresponding displacements $w_1(x,y)$, $u_2(x,y)$ and $v_2(x,y)$. The displacements of the points of the middle surface *u*, *v* play an important role in considering the nonlinear behavior of the delamination, when the value of the deflection *w* becomes comparable with the height of the delamination *h*.

The functions of displacements in this form are represented as $w(x,y)=w_0+\eta w_1$; $u(x,y)=u_0+\eta^2 u_2$; $v(x,y)=v_0+\eta^2 v_2$, where η – the parameter that depends on the loading condition of the plate. The plate passes into a new disturbed state adjacent to the initial plane.

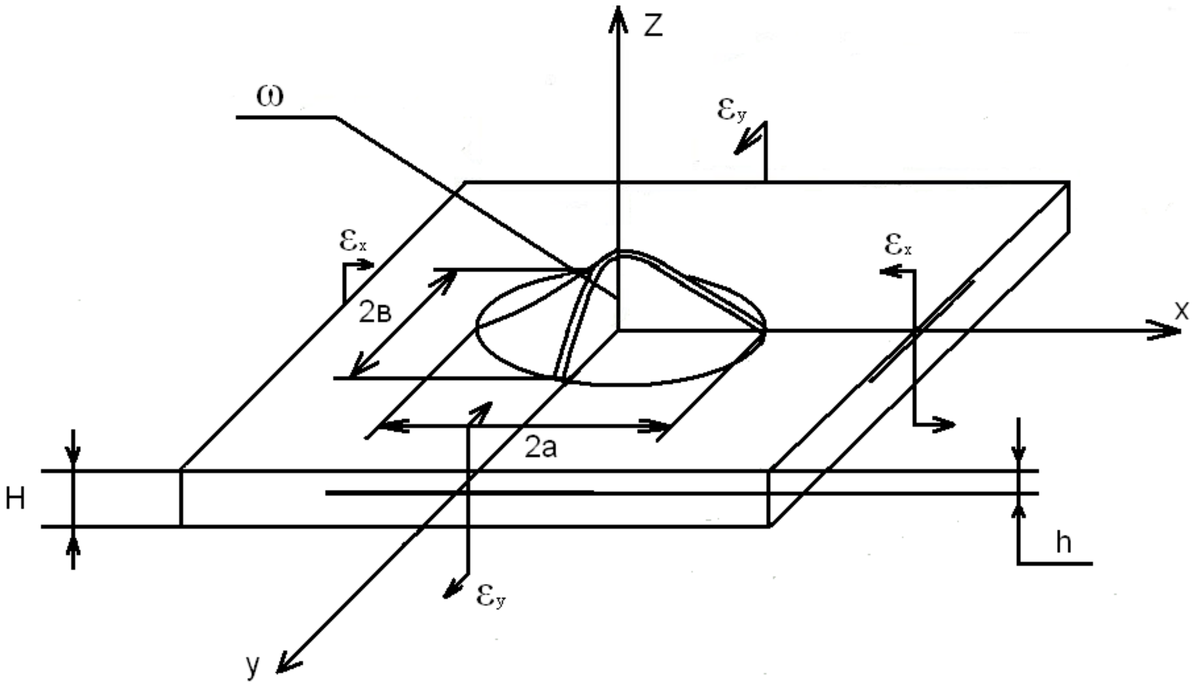

Fig. 1. The delamination of the elliptic form

The growth of the delamination is possible when $G \ge \Gamma$, G - the speed of energy liberation, Γ – the energy demanded to form a unit of a new surface. The elementary work of the resistance forces to advance the delamination front is equal to the product of the increment of the delamination area by the amount of work required to form a unit of a new surface $dA_\gamma = \pi(a\Delta в + в\Delta a + \Delta a\Delta в)2\gamma$. Quasi-fragile growth conditions of the defect are considered. The delamination and the main structural element are isotropic. The surface layers are deformed $\varepsilon_x > \varepsilon_{crx}$ and $\varepsilon_y > \varepsilon_{cry}$.

For the case of the structural element with the delamination, the potential deformation energy is determined:

$$U = U_c + U_o + V .$$

The compression deformations are determined by the critical deformations within the delamination. The potential energy of compression is:

$$U_c = \frac{Eh}{2(1-\mu^2)}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}} (\varepsilon_{crx}^2 + 2\mu\varepsilon_{crx}\varepsilon_{cry} + \varepsilon_{cry}^2)dydx ,$$

where $\varepsilon_{crx} = \varepsilon_{cr}$, $\varepsilon_{cry} = \mu\varepsilon_{crx} = \mu\varepsilon_{cr}$.

The expression for U_c can be simplified:

$$U_c = \frac{Eh}{2(1-\mu^2)}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}} \varepsilon_{cr}^2(1+3\mu^2)dydx .$$

The potential energy of the bending can be calculated by the formula:

$$V = \eta^2 \frac{D}{2}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}} \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)^2 + 2(1-\mu)\left[\left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 - \frac{\partial^4 w_1}{\partial x^2 \partial y^2}\right]dydx$$

.

The parameter is defined in [11]. We obtain the bending energy taking into account the analytic expression for ε_{cr} :

$$V = \frac{\mathrm{E}h}{2(1-\mu^2)} - \frac{F}{T}\left(\varepsilon_{cr}\varepsilon - \varepsilon_{cr}^2\right), \text{ where } F = [\oint_{S_1}(u_2 + \mu v_2)ds_1]^2$$

$$T = \frac{1}{4(1+\mu)^2}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}} \{[\frac{\partial u_2}{\partial x} + \frac{1}{2}\left(\frac{\partial w_1}{\partial x}\right)^2]^2 + [\frac{\partial v_2}{\partial y} +$$

$$+\frac{1}{2}\left(\frac{\partial w_1}{\partial y}\right)^2]^2+2\mu[\frac{\partial u_2}{\partial x}+\frac{1}{2}\left(\frac{\partial w_1}{\partial x}\right)^2]^2[\frac{\partial v_2}{\partial y}+\frac{1}{2}\left(\frac{\partial w_1}{\partial y}\right)^2]\}dydx$$

.

The potential energy of the main element is determined:

$$U=const-\frac{Eh}{2(1-\mu^2)}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}}(\varepsilon_x^2+2\mu\varepsilon_x\varepsilon_y+\varepsilon_y^2)dydx=$$

$$=const-\frac{Eh}{2(1-\mu^2)}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}}\varepsilon^2(1+3\mu^2)dydx,$$

where *const* is the potential energy of the defect plate, which does not depend on the size of the delamination. Thus, the final expression is obtained:

$$U=\frac{Eh}{2(1-\mu^2)}\int_0^a\int_0^{b\sqrt{1-(x/a)^2}}(1+3\mu^2)(\varepsilon_{cr}^2-\varepsilon^2)dydx+\frac{F}{T}(\varepsilon_{cr}\varepsilon-\varepsilon_{cr}^2)$$

.

The speed of energy liberation is estimated to determine the growth of the delamination. Generalized forces are defined that advance delaminations G_a, G_b.

Generalized forces are found by numerical differentiation of the potential energy with respect to the sizes of the delamination at infinitesimal increments $\Delta a, \Delta b$. The following notation are introduced: $dU=\frac{\partial U}{\partial a}da+\frac{\partial U}{\partial b}db$; $dA=\pi(adb+bda)$, where ΔA – the area of the delamination (Fig. 2).

The values are $G=\dfrac{G_a+G_b\dfrac{a}{b}\dfrac{db}{da}}{1+\dfrac{a}{b}\dfrac{db}{da}}$. The forces G_a, G_b advance the separation along the axes *a* and *в*. The forces are relations, respectively:

$$G_a=-\frac{\partial U}{\partial a};\quad G_b=-\frac{\partial U}{\partial b};$$

$$G_a=\frac{Eh}{2(1-\mu^2)}\frac{\partial}{\partial a}\left\{\frac{\pi ab(1+3\mu^2)}{4}\varepsilon^2-\left[\frac{\pi ab(1+3\mu^2)}{4}-\frac{F}{T}\right]\varepsilon_{cr}^2-\frac{F}{T}\varepsilon_{cr}\varepsilon\right\}$$

;

$$G_b=\frac{Eh}{2(1-\mu^2)}\frac{\partial}{\partial b}\left\{\frac{\pi ab(1+3\mu^2)}{4}\varepsilon^2-\left[\frac{\pi ab(1+3\mu^2)}{4}-\frac{F}{T}\right]\varepsilon_{cr}^2-\frac{F}{T}\varepsilon_{cr}\varepsilon\right\}$$

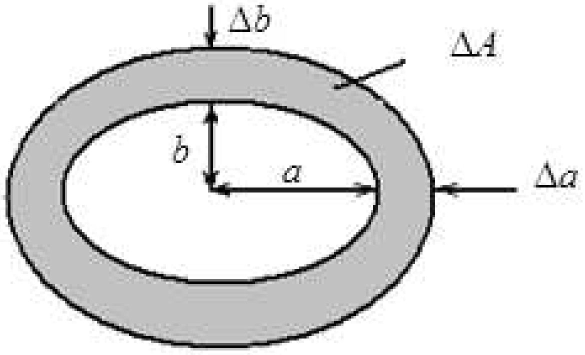

Fig. 2. The growth of the delamination

The change of the force G can be determined by one of the following options, depending on the value $\frac{db}{da}$. If the forces are $G_a>G_b$, then G is a maximum at $\frac{db}{da}=0$ and $G=G_a$. If $G_a<G_b$, then G is a maximum at $\frac{db}{da}\to\infty$ and $G=G_b$. If $G_a=G_b$, then G is not local maximum, and $G=G_a=G_b$. This analysis shows that the history of the possible growth of the delamination by the values of the functions is determined G_a, G_b (Fig. 3). At first, the curve G_a sharply increases and then monotonically decrease, monotonically G_b increases as the ratio a/b grows. The group of curves has a common value along the curve $G_a=G_b$. To the left of the curve, $G_a>G_b$. Only the value *a* can increase. The value *в* is fixed in $G=G_a$. Fig. 4 presents the results of the growth of elliptic delaminations.

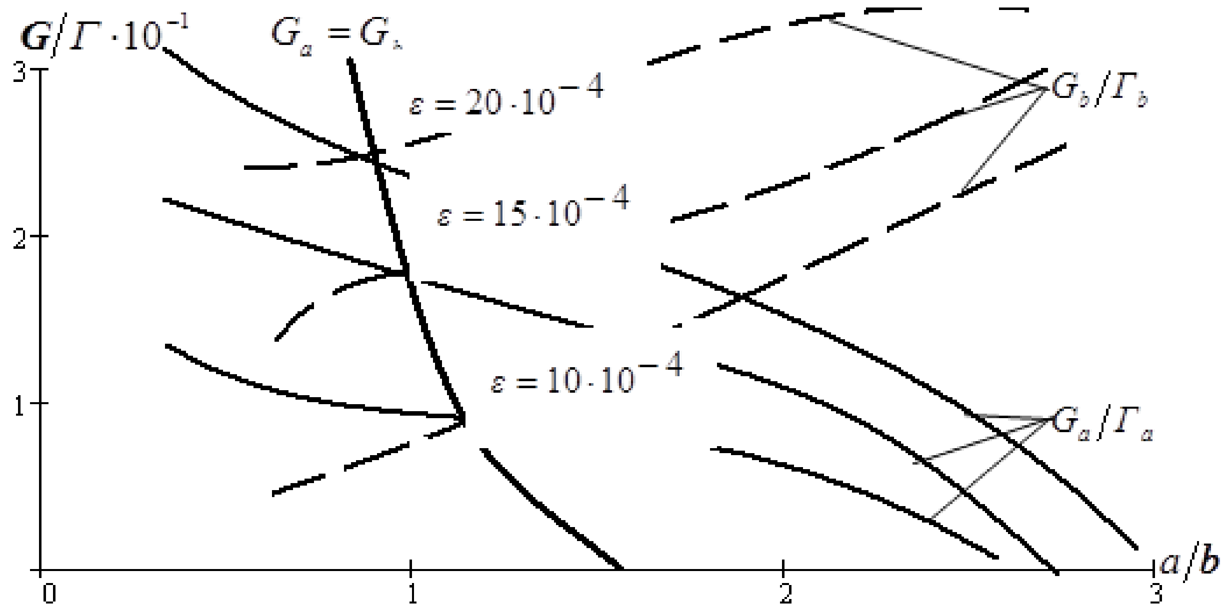

Fig. 3. The change of the growth force of elliptic delaminations

The characteristic features of defect growth are investigated by compressing the main plate with the following parameters: $\varepsilon_{ск}$ - the critical value of the loading parameters, ε_{γ} - the fracture curve. The load reaches point 1 (*a*=*в*), the delamination grows unstable along the axis «*a*» (to point 2), then the delamination increases steadily along the axis «*a*» and along the axis «*в*». The unstable growth of the delamination is possible by the axis «*a*», followed by a stop from point 3 to 4. The delamination grows steadily further along the axis «*a*». Unstable growth takes place along the axis «*в*» from point 5 to point 6. Steady growth follows the axes «*a*» and «*a*», and further.

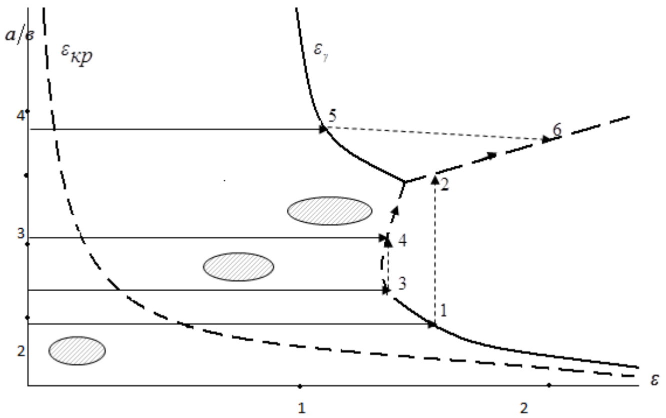

Fig. 4. The results of the growth of elliptic delamination

III. Conclusion

The problem of fracture is solved with the help of linear fracture mechanics using the example of a two-dimensional defect model of the elliptic delamination type. The characteristically destruction of defect growth is researched using the example of a two-dimensional defect model of the elliptic delamination type.

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References

[1] V.V. Bolotin, Z.Kh. Zebelyan and L.A. Kurzin, “Stability of compressed elements with defects of delamination type”, J. Problems of Strength, №7, pp. 3-8, 1980.

[2] V.V. Bolotin, “Defects of the delamination type in constructions made of composite materials”, J. Mechanics of composite materials, №2, pp. 239-256, 1984.

[3] A.S. Chermoshentseva, A.M. Pokrovskiy and L.A. Bokhoeva, “The behavior of delaminations in composite materials - experimental results”, IOP Conference Series: Materials Science and Engineering, 2015 [International Conference on Advanced Materials and New Technologies in Modern Materials Science, 2016].

[4] A.S. Chermoshentseva, L.A. Bokhoeva and V.E. Rogov, “Experimental research of models of thin-walled-compressed rings with defects type of delamination of layered composite materials”, Proceedings of IFOST-2016, pp.112-114, 2016 [11th International Forum on Strategic Technology (IFOST), 2016].

[5] L.A. Bokhoeva, V.E. Rogov, A.S. Chermoshentseva, D.V. Lobanov, “Stability and process of destruction of compressed plate of layered composite materials with defects”, Innovative Technologies in Engineering, URL: http://iopscience.iop.org/article/10.1088/1757-899X/142/1/012077/pdf, 2016 [VII International Scientific Practical Conference, 2016].

[6] V.V. Bolotin, “Mechanics of nucleation and initial development of fatigue cracks”, J. Physico-chemical mechanics of materials, №1, vol. 22, pp. 18-23, 1986.

[7] V.V. Bolotin, Yu.N. Novichkov, “Mechanics of multilayer structures”, J. Mechanical Engineering, pp. 375, 1980.

[8] V.V. Bolotin, “Multiparametric fracture mechanics”, J. Calculations on the strength, №25, pp. 12-33, 1984.

[9] V.E. Vildeman, Yu.V. Sokolkin, A.A. Tashkinov, “Mechanics of inelastic deformation and destruction of composite materials”, J. Science, pp. 288, 1997.

[10] V.V. Bolotin, A.A. Shipkov, “Forecasting the growth of fatigue cracks taking into account environmental factors”, J. Applied Mathematics and Mechanics, vol. 65, pp. 1033-1043, 2001.

[11] L.A. Bokhoeva, D. Ganbat, V.E. Rogov, A.S. Chermoshentseva, ”Study on stability of inter-layer defects of composite materials in the elliptic form regarding of nonlinear theories”, Proceedings of IFOST-2017, Part 1, pp. 197-201, 2017 [12th International Forum on Strategic Technology (IFOST), 2017].