

# Mathematical and software tools for implementation of method of assessing quality of piloting

Kashkovsky V. V.

Department of IS and protection of information  
Irkutsk state University of railway engineering,  
Irkutsk, Russian Federation  
kww542339@km.ru

Tihiy I. I.

Department of physics of mechanics and  
instrumentation  
Irkutsk state University of railway engineering,  
Irkutsk, Russian Federation

**Abstract**—the article describes the results of the development of a new criterion for assessing the quality of piloting. The difference between this criterion and the traditional one is that it is an integrated assessment of the quality of the execution of the landing maneuver and characterizes not only the error of the flight on the glide path, and the level of pilot training. Studies have shown that this criterion for assessing the quality of piloting can be used on all types of civil and military aircraft equipped with digital flight recorder (FR). The proposed criterion was the basis of the developed methodology for assessing the quality of piloting, which is of practical value for the introduction into practice of flight schools, aviation units and parts. The development of criteria for assessing the quality of piloting required formalization of the pilot model. To create it, both real flights and flights on a specialized flight simulator were studied. The peculiarity of the flight simulator developed for this purpose is the application of the discrete integration method, which allows to significantly reduce the cost of software development. To implement this method, an original form of Z - transformation for this problem is proposed and substantiated, which provides an adequate transition from p - to Z - region. In addition, the article deals with the practical use of digital filters to solve the problems of modeling dynamic systems.

**Keywords**—quality assessment of piloting, flight simulation, model pilot, aviation-training devices, z - transform.

## I. INTRODUCTION

Analysis of statistical data shows that of the total number of aviation incidents about 30% occurred for various reasons of a technical nature (including terrorist attacks). The remaining 70% of aviation incidents are caused by the action of the "human factor" – the mistakes of pilots, engineers and air traffic control specialists. For example, almost all of the accident and of the crash of Tu-154 was the fault of the person. In turn, about 80% of all aviation incidents during the flight occurred at the stages of take-off, pre-flight maneuver and landing. This so-called "eleven critical minutes of flight" (three minutes after take-off and eight before landing). Moreover, landing accounts for more than 50% of all air accidents.

Thus, the practice of flight operation of civil and military aircraft shows that the most common cause of the occurrence of an aviation accident is the error of piloting the aircraft by

the pilot at the landing stage. Therefore, one of the most important measures to prevent accidents can be the development and implementation of individual preventive measures for additional flight training of pilots based on the assessment of the quality of piloting the aircraft by the pilot during the flight on the glide path.

The task of assessing the quality of piloting at the landing stage is traditionally relevant for both civil and military aviation. Currently, the assessment of the quality of the piloting is done with a tolerable control of flight parameters in the control section: output to the landing pattern, the reduction in the landing pattern, the passage far of a drive beacon, the middle passage of the drive beacon, alignment and landing. This method allows you to identify piloting errors, but, in general, is not an assessment of the quality of piloting, because due to various circumstances, even an experienced pilot can deviate from the glide path. Therefore, an integrated assessment of piloting during the entire pre-flight maneuver is required. For its obtaining, it is necessary to carry out researches of dependence of psychophysical properties of the pilot on level of his training and to develop his dynamic model. In turn, to study the psycho-physical properties of the pilot and build his model requires a reconfigurable aircraft simulator (flight simulator) with a changeable system of differential equations describing the dynamics of the aircraft and the model of the atmosphere [1, 2]. The need for such a simulator is due to the fact that only the flight simulator can investigate the process of obtaining piloting skills unprepared pilot. In the existing standard simulators to solve this problem is not possible, because they do not provide the reconfiguration of flight dynamics parameters on different models of the aircraft, the replacement of the pilot model and the processing of the results of the flight on the computer [3, 4, 5, 6]. In addition, it was necessary to develop such a specialized aircraft simulator, which would provide an opportunity to work out on it various variants of the model of the pilot and the registration of flight parameters.

## II. THE MATHEMATICAL APPARATUS OF A FLIGHT SIMULATOR (FLIGHT SIMULATOR)

To develop a dynamic model of the pilot, it is necessary to create a flight simulator that allows you to significantly rebuild the dynamic model of the aircraft, as well as replace

the pilot with its model. In Fig. 1, the structural scheme of the longitudinal channel of the simulator that meets the requirements is shown. The implementation of the software for such a flight simulator was quite difficult, so it took a new mathematical apparatus to create it.

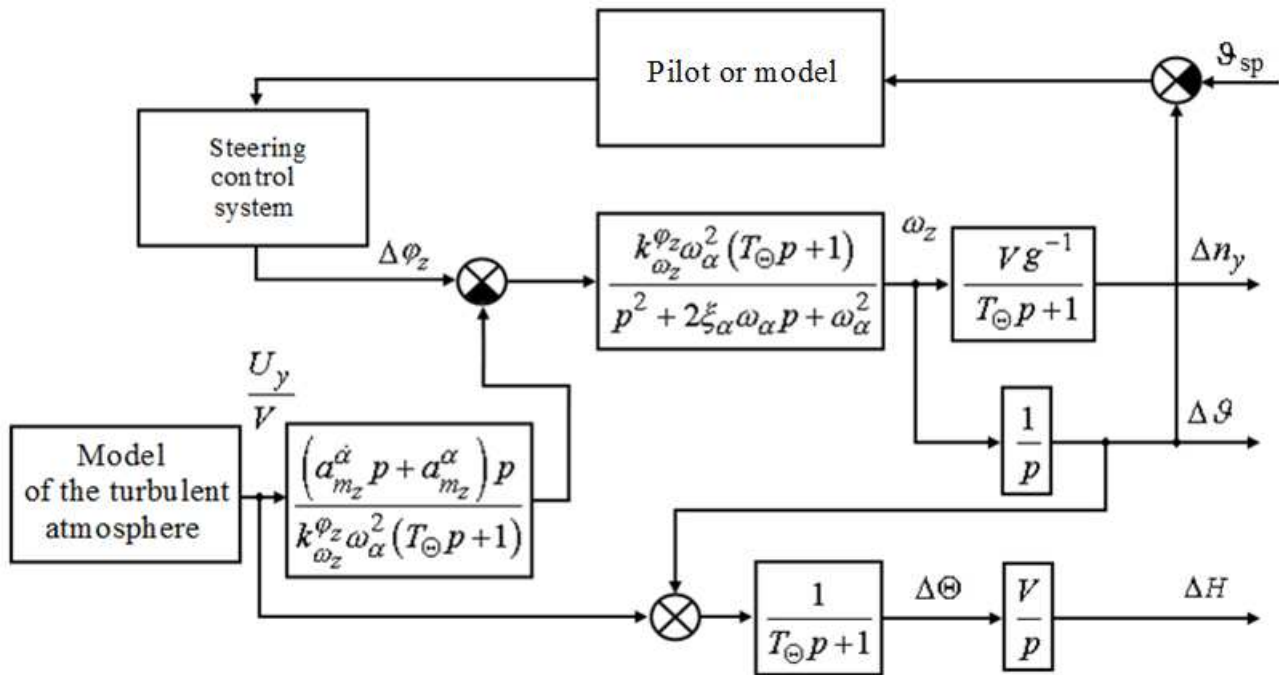


Fig. 1. A simplified block diagram of the longitudinal channel of the flight simulator.

In order to simplify the flight simulator software and reduce the cost of software development, it was proposed to use the method of discrete integration [7, 8, 9, 10] for solving differential equations describing the dynamics of the aircraft [11, 12]. This method belongs to the field of Z-transformations and is used in the design of recursive digital filters.

The essence of the method of discrete integration is as follows. Based on the requirements of the projected digital filter (DF) solve the approximation problem for its analog prototype (AP) and find the transfer function:

$$W(p) = \frac{Y(p)}{X(p)} = \frac{a_m p^m + a_{m-1} p^{m-1} + \dots + a_1 p + a_0}{b_n p^n + b_{n-1} p^{n-1} + \dots + b_1 p + b_0} \quad (1)$$

Further, by dividing the numerator and the denominator of  $W(p)$  on  $p^n$ , result (1) can be written as:

$$W(p) = \frac{\frac{a_0}{p^n} + \frac{a_1}{p^{n-1}} + \frac{a_2}{p^{n-2}} + \dots + \frac{a_m}{p^{n-m}}}{\frac{b_0}{p^n} + \frac{b_1}{p^{n-1}} + \frac{b_2}{p^{n-2}} + \dots + b_n} \quad (2)$$

By replacing the (2) integrator  $1/p$  with some discrete integrator, the transfer function  $H(z^{-1})$  of the required DF is obtained.

The nonlinearity of the transition from the p-plane to the z-plane caused numerous attempts of its approximate representation, which resulted in a variety of methods of z-transformation. In [3] the following type of discrete integrator providing stability and minimum distortions of DF frequency characteristics is proposed:

$$\frac{1}{p} = kT \frac{1+z^{-1}}{1-z^{-1}} \quad (3)$$

where  $k$  is an arbitrary nonzero coefficient;  $T$  is the sampling period.

The transformation in question provides the following connection between AP and DF frequencies:

$$\omega_a = \frac{1}{kT} \operatorname{tg} \frac{\omega_d T}{2}, \quad (4)$$

where  $\omega_a$  is the natural frequency of AP;  $\omega_d$  is the natural frequency of DF.

Transformation (3) at  $k > 0$  does not lead to loss of stability, at  $k < 0$  the integrator (1) is unstable.

Under the condition of the problem, it is necessary to observe equality  $\omega_d = \omega_a$ , for this arbitrary coefficient  $k$  is calculated for a given period of discretization.

If the circular sampling rate of the simulator  $\omega_{st}$ , the sampling period DF can be represented as:

$$T = \frac{2\pi}{\omega_{st}}. \quad (5)$$

Let us substitute (5) in (4) and under the condition  $\omega_d = \omega_a$  we get:

$$k = \frac{\omega_{rel}}{2\pi} \lg \frac{\pi}{\omega_{rel}}, \quad (6)$$

where  $\omega_{rel} = \frac{\omega_{st}}{\omega_d}$  is the relative sampling rate.

Thus, the value of the relative sampling rate  $\omega_{rel}$  determines the value of the coefficient  $k$ , and the degree of compliance of the frequency properties and transients AP and DF. Studies have shown that in order to ensure the required compliance of the transients of the digital model to the transients of the analog prototype, it is necessary to fulfill the condition:

$$10 \leq \omega_{rel} \leq 2000.$$

### III. THE APPLICATION OF THE METHOD OF DISCRETE MODELING FOR THE IMPLEMENTATION OF ELEMENTARY LINKS OF AUTOMATION

We consider a  $\Lambda_p^z$  transformation to solve a linear differential equation of 2nd order (oscillating link):

$$\ddot{x} + \xi\omega_a\dot{x} + \omega_a^2 = K\omega_a^2 y, \quad (7)$$

where  $\xi$  is the decay decrement.

The analog prototype of the oscillating link is described by the expression:

$$W(p) = \frac{Y(p)}{X(p)} = \frac{K\omega_a^2}{p^2 + 2\xi\omega_a p + \omega_a^2}. \quad (8)$$

Equation (8) can be written as:

$$\frac{K\omega_d^2 \frac{1}{p^2}}{1 + 2\xi\omega_d \frac{1}{p} + \omega_d^2 \frac{1}{p^2}}. \quad (9)$$

Let us substitute (3) instead of  $1/p$  in expression (9) and obtain the Z-transfer function for equation (7):

$$H(z^{-1}) = \frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{B_0 + B_1 z^{-1} + B_2 z^{-2}} \quad (10)$$

$$\begin{aligned} \text{where } A_0 = A_2 = K\omega_d^2 k^2 T^2; \quad A_1 = 2K\omega_d^2 k^2 T^2 = 2A_0; \\ B_0 = 1 + 2\xi\omega_d kT + \omega_d^2 k^2 T^2; \quad B_1 = 2(\omega_d^2 k^2 T^2 - 1); \\ B_2 = 1 - 2\xi\omega_d kT + \omega_d^2 k^2 T^2. \end{aligned}$$

Equation (10) can be written as:

$$H(z^{-1}) = \frac{A_0^* + A_1^* z^{-1} + A_2^* z^{-2}}{1 + B_1^* z^{-1} + B_2^* z^{-2}} \quad (11)$$

$$\text{where } A_0^* = A_2^* = \frac{A_0}{B_0} = \frac{K\omega_d^2 k^2 T^2}{1 + 2\xi\omega_d kT + \omega_d^2 k^2 T^2};$$

$$A_1^* = \frac{2A_0}{B_0} = \frac{2K\omega_d^2 k^2 T^2}{1 + 2\xi\omega_d kT + \omega_d^2 k^2 T^2};$$

$$B_1^* = \frac{B_1}{B_0} = 2 \frac{\omega_d^2 k^2 T^2 - 1}{1 + 2\xi\omega_d kT + \omega_d^2 k^2 T^2};$$

$$B_2^* = \frac{B_2}{B_0} = \frac{1 - 2\xi\omega_d kT + \omega_d^2 k^2 T^2}{1 + 2\xi\omega_d kT + \omega_d^2 k^2 T^2}.$$

In discrete form solution of equation (7) is defined by expression:

$$Y(z) = X(z)H(z^{-1}). \quad (12)$$

where  $Z^{-1}[Y(z)] = [y_i]$  is a sequence of discrete values of the output signal;  $Z^{-1}[X(z)] = [x_i]$  is a sequence of discrete values of the input signal with a sampling period  $T$ ;  $i = \overline{1, \infty}$  is the current reference point (measurement) of the discrete input signal  $x_i$  and the discrete output signal  $y_i$ .

From (11) gives a difference equation that can be implemented in a computer:

$$y_i = A_0^* x_i + A_1^* x_{i-1} + A_2^* x_{i-2} - B_1^* y_{i-1} - B_2^* y_{i-2}. \quad (13)$$

Thus, the digital model of the oscillating link presented by expressions (11) and (13) was obtained. This clearly shows the exceptional simplicity of the program implementation of the difference equation.

The difference equations of other elementary units of automatics are derived in the same way.

The transfer function of the aperiodic link is described by the expression:

$$W(p) = \frac{K}{1 + T_a p},$$

where  $T_a$  is the time constant of the aperiodic link.

Let us convert it to a form:

$$W(p) = \frac{K/p}{1/p + T_a}.$$

After substitution (3) and get:

$$H(p) = \frac{A_0(1+z^{-1})}{1+B_1z^{-1}},$$

$$\text{where } A_0 = \frac{KTk}{Tk+T_a}; B_1 = \frac{Tk-T_a}{Tk+T_a}; k=0.5.$$

From this there is the difference equation of the digital model of the aperiodic link:

$$y(n) = A_0[x(n) + x(n-1)] - B_1y(n-1).$$

The digital model of the permanent delay link is very simple:

$$H(z^{-1}) = z^{-m},$$

where  $m = \tau/T$ ;  $\tau$  is the lag constant.

The difference equation of a link of a constant delay has the form:

$$y(n) = x(n-m).$$

Digital model of an integrative link follows from (3):

$$H(z^{-1}) = Tk \frac{1+z^{-1}}{1-z^{-1}}.$$

From this expression there is the difference equation of the integrating link:

$$y(n) = A_0[x(n) + x(n-1)] + y(n-1),$$

where  $A_0 = 0.5T$ .

To verify the adequacy of transients and frequency characteristics of the AP and DF, studies of typical elementary units of automation were carried out. In the course of these studies, the transient  $h(t)$  and weight  $g(t)$  functions obtained by the inverse Laplace transform and calculated by the discrete integration method were compared. Thus it was established that the error of calculation of functions  $h(t)$  and  $g(t)$  depends on  $\omega_{rel}$  and in the range of  $100 \leq \omega_{rel} \leq 2000$  it can be neglected.

#### IV. MODEL OF THE PILOT AND EVALUATION OF THE QUALITY OF PILOTING

In a number of works of various authors, it is shown that one of the most informative parameters characterizing the quality of the maneuvering aircraft is the deviation of the aircraft control stick ( $\delta_{ACS}$ ). Handle control plane is designed to control the roll and pitch. Further, as a controlled parameter, we will consider only the deviation of ACS in pitch ( $\delta_{ACS_{PC}}$ ).

Currently, when analyzing psycho-physiological features of pilot and study the individual style of piloting can be divided into low and high frequency components.

The low-frequency component (trend) characterizes the trajectory control of the aircraft. High-frequency component, also called in the theory of flight safety performance model (PM) of the pilot, characterizes the individual manner of piloting the pilot.

Currently, when analyzing psycho-physiological features of pilot and study the individual style of piloting, it is customary to divide  $\delta_{ACS_{PC}}$  into low and high frequency components.

In this paper,  $\delta_{ACS_{PC}}$  is divided into trend and performance models ( $\delta_{ACS_{PC}}^0$ ) using a digital FIR filter [7, 8, 9, 10] with a tuning frequency of 0.5 Hz. The specified value of the FIR-filter setting corresponds to the frequency of natural oscillations of the research maneuvering aircraft at the angle of attack.

In order to build the model of the pilot, PM studies were carried out on the materials of 837 aircraft flights and a variety of flights on the developed simulator. Studies have shown that in all cases, the PM signal has a distribution density

$f_{EM}(\delta_{ACS_{PC}}^0)$ , which for research purposes can be represented as a sum of two distribution laws:

$$f_{EM}(\delta_{ACS_{PC}}^0) = P_1 f_1(\delta_{ACS_{PC}}^0) + P_2 f_2(\delta_{ACS_{PC}}^0),$$

where  $f_1(\delta_{ACS_{PC}}^0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\delta_{ACS_{PC}}^0)^2}{2\sigma^2}}$  is a normal distribution law with zero expectation and mean square deviation equal to  $\sigma$ ;  $f_2(\delta_{ACS_{PC}}^0) = 0.5\lambda e^{-\lambda|\delta_{ACS_{PC}}^0|}$  – double-sided exponential distribution (Laplace distribution [13]) with intensity  $\lambda$ ;  $P_1$  and  $P_2$  are the probability of deviation PM by law  $f_1$  and  $f_2$ , respectively,  $P_1 + P_2 = 1$ .

The typical form of the distribution density  $f_{PM}(\delta_{ACS_{PC}}^0)$  of a well-trained pilot, obtained by processing the flight information FR, is shown in fig. 2.

In Fig. 2 line 1 conditionally shows the boundary between the laws OF PM distribution. Above line 1, there is a fragment of symmetric exponential distribution, and below it – fragments of the normal distribution law.

Studies of flight simulator showed that the quality of piloting is determined by the ratio between the density of distribution  $f_1$  and  $f_2$ , so the quantitative evaluation of piloting can be expressed through the distribution parameters  $P_1$ ,  $P_2$ ,  $\sigma$  and  $\lambda$ . So, in particular, during the flights on the simulator it was found that the greater the probability  $P_1$ , the lower the overall level of training of the pilot. However, the practical evaluation of indicators  $P_1$  and  $P_2$  is difficult.

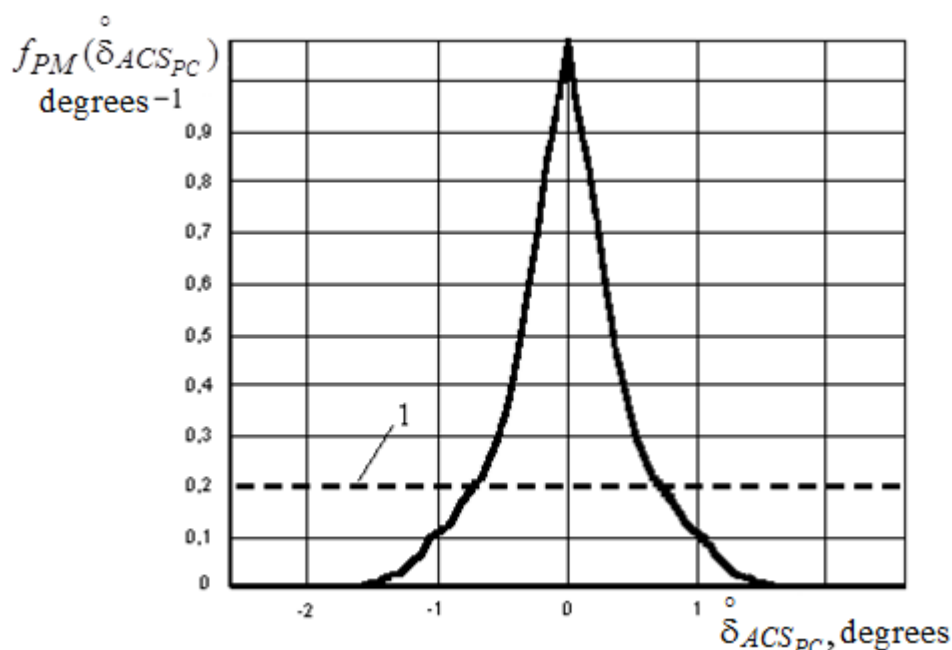


Fig. 2. A typical view of the density distribution of the PM

Apart from  $\sigma$  and  $\lambda$ , the distribution density is at fig. 2 can be characterized by two parameters: - the size of the extremum of the density distribution (M) and the width of the interval on which the density of the distribution takes values more than  $0.02^{-1}$  grad ( $\Delta L$ ). Further, the distribution indicators  $\sigma$ ,  $\lambda$ , M and  $\Delta L$  will be called preliminary criteria for assessing the quality of piloting.

A priori it was unknown which of the preliminary criteria for assessing the quality of piloting allows more efficient characterization of PM. Therefore, for comparative analysis of the effectiveness of preliminary criteria for assessing the quality of piloting and establishing their relationship with the level of training of pilots, flight information processing was performed for 428 personified flights of the aircraft, in each of which the pilot was known and his class qualification level. When analyzing each level of the classroom training was put into line a number (in points):

- class 1 - 1 point;
- class 2 - 2 points;
- class 3 - 3 points;
- "no classroom training" - 4 points.

Estimates of  $\sigma$ ,  $\lambda$ , M and  $\Delta L$  have been calculated for each of these 428 flights. Then a correlation analysis was performed to determine the degree of relationship between the indicator of the pilot's proficiency and the corresponding statistical parameters of the distribution of  $f_{EM}(\delta_{ACS_{PC}})$  in each particular flight. The results of the correlation analysis are presented in table. I.

TABLE I. THE RESULTS OF CORRELATION ANALYSIS

Name of parameter	Parameter value
The correlation coefficient between "Proficiency" and $\Delta L$	-0,648
The correlation coefficient between "Proficiency" and $\sigma$	-0,595
The correlation coefficient between "Proficiency" and M	0,563
The correlation coefficient between "Proficiency" and $\lambda$	0,334

As you can see from table I, the strongest relationship with the level of class qualification has criterion  $K_{PM}$ . Thus, it has been experimentally established that the value of  $\Delta L$  calculated according to the FR of a particular flight, the most correlated with the class qualification of the pilot and therefore can serve as an objective assessment of the quality of piloting.

Further, to determine the functional dependence between  $\Delta L$  and class qualification of the pilot, according to the results of processing 428 personified flights of the aircraft by the method of least squares, a regression line of the 2nd order was constructed, presented in fig. 3:

$$K_{PM}(\Delta L) = 0.054\Delta L^2 - 0.847\Delta L + 4.569. \quad (14)$$

According to the regression line (14), there is a calibration of the criteria to assess the quality of piloting:

- Class 1 -  $\Delta L$  more than 5,68 degrees.;



- Class 2 -  $\Delta L$  from 3.02 to 5.68 degrees;
- Class 3 -  $\Delta L$  from 1.39 to 3.02 degrees;
- "no classroom training" –  $\Delta L$  less than 1.39 deg.

The resulting criterion for assessing the quality of piloting  $K_{PM}(\Delta L)$  is quite easy to calculate ground processing devices for flight information records staff FR. Its main advantage is that it does not characterize errors in maintaining flight parameters (this task is important in itself and its

solution is not canceled), and the level of training of the pilot as a whole. The practice of applying the criterion  $K_{PM}(\Delta L)$  showed that it reacts not only to improve the pilot's skills, but even on such not quite obvious circumstances as the transition of the pilot from one aircraft modification to another.

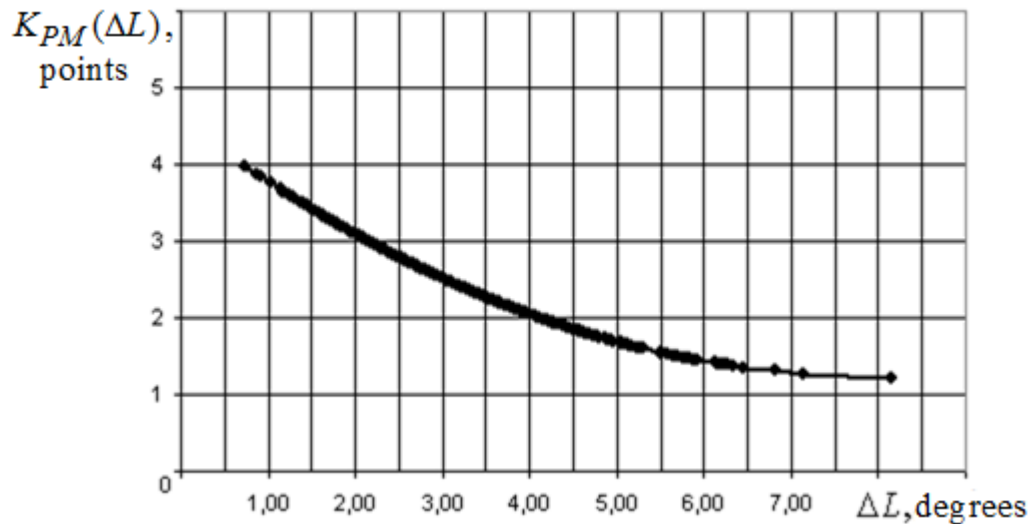


Fig. 3. Regression line  $K_{PM}(\Delta L)$  and  $\Delta L$  values depending on the class qualification of pilots based on the results of 428 flights

## V. CONCLUSION

The application of criterion  $K_{PM}(\Delta L)$  naturally fits into the practice of flight training and can serve as a visual information for the management of flight personnel in the organization of individual flight training. If the indicator  $\Delta L$  for the pilot is stable and exceeds the class qualification assigned to him, the head of flight training can allow him to more complex flight exercises and submit to the tests for a higher class qualification. Conversely, if the indicator  $\Delta L$  novice pilot can not stabilize his manner of piloting, it is advisable to recommend him additional flights on the simulator, the repeated execution of not credited flight exercises or flights with an instructor.

To build a model of an unskilled pilot, a flight simulator was developed, since such studies can not be conducted in real flight. A special feature of the flight simulator was the use of discrete integration method. In the construction of aircraft simulators, discrete integration method showed the following advantages:

1. Unlike step-by-step integration methods (for example, Runge-Kutta method), the discrete integration method does not have a cumulative integration error, even in the case of single-precision (32-bit floating-point) variable calculations.
2. One step of integration of the differential equation of the 2nd order by the Runge-Kutta method requires 2 division

operations, 22 multiplication operations and 18 addition operations. One cycle of calculation by discrete integration requires 5 multiplication operations and 4 addition operations. Computational costs (while maintaining accuracy) are reduced about 5 times.

3. The software implementation of the difference equation (13) is universal and allows modeling all elementary links of automatics of the first and second order. The difference between the obtained automatics links and their nonlinearity is provided by the value of the coefficients  $A_0^*$ ,  $A_1^*$ ,  $A_2^*$ ,  $B_1^*$  and  $B_2^*$ , calculated at each step of the discretization. This makes it possible to change the structural scheme of the aircraft simulator by simply replacing the coefficients  $A_0^*$ ,  $A_1^*$ ,  $A_2^*$ ,  $B_1^*$  and  $B_2^*$  in the difference equations.

4. Application of the method of discrete integration involves the use for the synthesis of dynamic models of the plane of the finished parts of control systems: aperiodic, oscillatory forcing, etc. This allows you to build aircraft simulator directly on the block diagram (Fig. 3) without bringing the problem to the Cauchy form. This approach significantly reduces software development costs.

The obtained results testify to the high reliability of solving flight dynamics problems by the method of discrete integration, and the resulting discrepancies in the calculation

of transients in the construction of aircraft simulators are not significant.

In general, the application of the discrete integration method for the construction of aircraft simulators can significantly accelerate software development, improve its reliability, significantly reduce the performance requirements of the used computers and makes it possible to quickly change not only the coefficients but also the structure of the studied models of flight dynamics.

### **References**

- [1] V.V. Kashkovsky, I.I. Tihiy, Yu.N. Shishkin, "Identification of the parameters of the dynamic model of the pilot according to the onboard recording device," *Proceedings of the Bulletin of Tomsk state University, Annex*, Vol. 14 (II), pp. 297 – 301, 2005.
- [2] V.V. Kashkovsky, I.I. Tihiy, S.P. Poluektov, "Assessment of the quality of the piloting in flight mode on final approach," *Scientific Bulletin of the Moscow state technical University of civil aviation*, No. 138(1), pp. 191-197, 2008.
- [3] A.A. Krasovskii, A.V. Kudinenko, *Navigation and complex simulators*. Moscow: Air force engineering Academy named after Professor N. Ye. Zhukovsky, 1984.
- [4] V.Ya. Kremlev, *Training and training facilities of the air force: the current state and problems of further perfection and development. Scientific and methodical materials*, Moscow: Air force engineering Academy named after Professor N. Ye. Zhukovsky, 1988.
- [5] A.A. Krasovsky, *Mathematical modeling and computer systems of training and coaching*, Moscow: Air force engineering Academy named after Professor N.Ye. Zhukovsky, 1989.
- [6] G.Sh. Meerovitsh, *Flight simulators and aviation safety*, Moscow: Air transport, 1990.
- [7] Yu.A. Kochetkov, *Fundamentals of automation of aviation equipment*. Moscow: Air force engineering Academy named after Professor N.Ye. Zhukovsky, 1995.
- [8] A.G. Ostapenko, A.B. Sushkov, V.V. Butenko, *Recursive filters on microprocessor*, Moscow: Radio and communication, 1988.
- [9] B. Gold, I. Raider, *Digital signal processing*, Moscow: Soviet radio, 1973.
- [10] V. Capellini, *Digital filters and their application* Moscow: Energoatomizdat, 1983.
- [11] G.S. Byushgens, R.V. Studnev, *The Dynamics of the aircraft. Spatial motion*. Moscow: Mechanical engineering, 1983.
- [12] G.S. Byushgens, R.V. Studnev, *Aerodynamics of aircraft: dynamics of longitudinal and lateral motion*, Moscow: Mechanical engineering, 1979.
- [13] M.L. Petrovich, M.I. Davidovich, *Statistical estimation and testing hypotheses on the computer*, Moscow: Finance and statistics, 1989.