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STRESSED STATE OF HEAVY-DUTY HARMONIC GEAR DRIVE FLEXSPLINE

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***Abstract*— The topicality of research is determined by the progress trend in the development of the gearing technology consisting in the upgrading of power transmissions based on the multi-threaded power and kinematic principles of the rotation transmission. Maximum efficiency is characteristic of the harmonic gear drives that feature a wide range of gear ratios from 80 to 400 in one stage and low weight-and-dimensional characteristics. The solution of the problem of the stressed state elasticity of the most heavily loaded segment – flexspline of the heavy-duty harmonic gear drive improves the quality of design engineering works, enhances reliability, increases unit power and capacity as well as competitive properties of machines for mining complexes.**

The purpose of research is to increase the load capacity of heavy-duty harmonic gear drives through the improvement of the theoretical and experimental methods and procedures, evaluation of their power and strength properties, implementation of scientific results in the innovative design solutions.

Target of research – power factors, strains and stresses that are brought about in the flexspline when interacting with the wave generator and circular spline of the heavy-duty harmonic gear drive.

Keywords*— *flexspline, shell, gear ring, power factors, strains, stresses, disk, wave generator.

I. RESEARCH METHODS

Flexspline is considered as a cylindrical shell with the length of l, with the flexible gear ring of width b_1 at one end, and splines of width b_2 at the opposite end. The ratio of the shell thickness h to the circle diameter 2a dividing the shell wall in two, makes 0,012. The stressed state of the shell is considered as a field of perturbations in the consequence of boundary disturbance at the shell edge, placed on the field of tangential stresses at uniform distribution of tangential forces along the shell edge. The field of perturbations arising from boundary disturbance can be found using the semi-membrane shell theory by V.Z. Vlasov, under the following conditions:

1. Bending moments M_x and torques M in sections perpendicular to generatrix and shearing force in the same sections are ignored.

2. Shear strain of the middle surface is absent.

3. Poisson's ratio is set to zero (nu = 0).

II. RESEARCH FINDINGS

Mathematical model of the stress-strain state of the heavy-duty harmonic gear drive flexspline has been devised which, as opposed to the existing ones, makes allowance for the joint effect of the gear and splined rings connected through the

cylindrical shell, on the normal and tangential stresses as well as linear and shear strains of the flexspline that enables to refine strength design of the flexspline and to determine the following:

- maximum stresses connected with boundary disturbance, arising in proximity to the junction of the shell with the gear rim are proportional to the flexspline thickness;
- decrease in maximum values of normal bending stresses down to the level of maximum tangential stresses increases the load capacity of the heavy-duty harmonic gear drive by 20 … 25%.

III. INTRODUCTION

The flexspline strains form a multiple-tooth contact of the highest kinematic efficiency and load capacity [1–4]. The harmonic gear drives are produced with the cam wave generator [5] the operation of which differs from the same of the disk wave generator considerably varying the strains of the flexsplines.

The widest variety of theoretical and experimental research in the basic lines of geometry, kinematics, power analysis, oscillating processes, stress-strain state and strength of the flexspline as of the other load-carrying parts and units has been conducted preferably for rather small harmonic gear drives with cam wave generators [6-10].

Availability of the flexible segment changes not only the main principles and laws of the gearing operation but also provides the harmonic gear drives with the new positive properties, changes the level of qualitative and quantitative variables capable of adjusting technical characteristics of the harmonic gear drives, enhancing prospects for their priority development. Symmetric two-wave multi-threaded field of the gearing balances power structure of the wave gearing,

provides high torsional stiffness, manifold decreases loads on the teeth, secures high load capacity as well as consistency of the preset values of kinematic characteristics [11-14].

The available scale factor makes it difficult to carry scientific developments of small prototypes with the cam generator over to the heavy-duty harmonic gear drives with the disk generator. The development of the harmonic gear drives with due consideration of the specific features of the flexspline strains and stresses presents a topical problem of power transmission the solution of which would increase the unit power and capacity of the machines, reduce the dimensions and weight, improve the quality and increase competitive advantage of heavy engineering products. Key provisions of this paper consist in the development of mathematical model of the stress-strain state of the flexspline of the high-load harmonic gear drive with the disk wave generator and the determination of the flexspline field of stresses and strains required to optimize its design parameters.

IV. Setting and Solution of Elastic Boundary Value Problem

The shell features a part of the flexspline free of outer load and it transmits the torque [15, 16]. The field of perturbations caused by the nonuniform strain of the shell edge determines its strength [17, 18]. Nonuniform tangential strain is a critical one; it far exceeds the longitudinal strain. The assumptions taken are equivalent to the condition: $\frac{\partial^2 f}{\partial S_x^2} << \frac{\partial^2 f}{\partial S_y^2}$, where f – power and geometric factors;

S_x, S_y – elements of the length of coordinate lines in the axial and circumferential directions [19].

Geometric and power factors in the direction of the shell generatrix vary much more slowly than in the circumferential direction [20]. Let us set: $\xi^2 = \frac{h^2}{12a^2}$; $-\frac{h}{2} \le z \le +\frac{h}{2}$. We obtain tangential stresses Tau of outer load M

$$\tau = \frac{M(a+z)}{I_p} = \frac{M\left(1+\frac{z}{a}\right)}{2\pi a^2 h\left(1+3\xi^2\right)}. \quad (1)$$

Let us isolate the shell element by means of two planes passing through the shell axis, and of two orthogonal planes of this axis. Let us transfer power factors operating in the sections to the middle surface element and apply them along the coordinate lines of local coordinates *xyz*. The transfer of the stress tensor components to the coordinate lines of the middle surface (x=const; Theta = const) has determined power factors N_x, N_{Theta}, M_{Theta}, S, Q_{Theta}, referred to the unit of length of one of the coordinate lines (fig. 1).

We use the dimensionless coordinates: $x = \frac{X}{a}$ and Theta, where X is linear dimension of axis x. Then $\frac{\partial}{\partial X} = \frac{1}{a}\frac{\partial}{\partial x}$. Let us set up equilibrium equation for the element of the cylindrical shell relative to the local coordinate system x, y, z

$$\frac{\partial N_x}{\partial x} + \frac{\partial S}{\partial \theta} = 0, \quad \frac{\partial S}{\partial x} + \frac{\partial N_\theta}{\partial \theta} + Q_\theta = 0, \quad (2)$$

$$\frac{\partial Q_\theta}{\partial \theta} - N_\theta = 0, \quad -aQ_\theta + \frac{\partial M_\theta}{\partial \theta} = 0.$$

Power factors referred to the middle surface (N_x, N_{Theta}, Q_{Theta}, M_{Theta}) are associated with its strain by physical equations

$$N_x = Eh \cdot \text{Epsilon}_x, \qquad M_{\text{Theta}} = D \cdot \text{Chi}_{\text{Theta}}, \quad (3)$$

where Epsilon_x is unit strain in the direction of axis x;

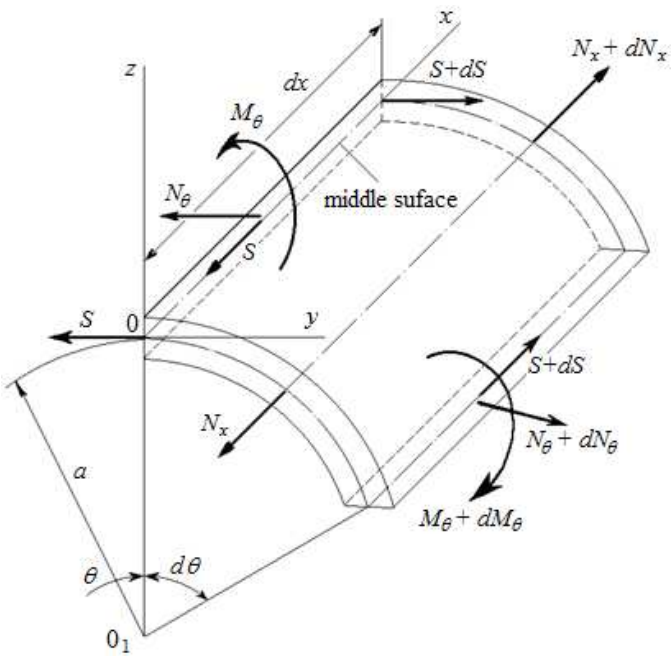

Fig. 1. Power factors applied to the shell element

$D = \frac{Eh^3}{12}$ – cylindrical stiffness of the shell with the proviso that Nu = 0.

Physical equations – consequence of Hooke's law and hypothesis of linear normals, will be obtained by summing up power factors and their moments in the direction of axis z.

Components of point displacement vector of the middle surface in the direction of coordinates *x, y, z* will be identified as *u, v, w*.

Unit strain in the direction of axis *x*,

$$\varepsilon_x = \frac{\partial u}{\partial X} = \frac{\partial u}{a\partial x}.$$

Total unit strain in the direction of axis y,

$$\varepsilon_\theta = \frac{1}{a}\left(\frac{\partial v}{\partial \theta} + w\right).$$

Total change of curvature of coordinate line *x = const* in the direction of change of angle Theta, i. e

$$\chi_\theta = -\frac{1}{a^2}\left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right).$$

Physical equations are obtained with due consideration of the assumption Nu= 0, let us substitute values Epsilon_x and $\text{Chi}_{\text{Theta}}$ into the formulae (3)

$$N_X = \frac{Eh}{a}\frac{\partial u}{\partial x},\quad M_\theta = D\chi_\theta = -\frac{D}{a^2}\left(-\frac{\partial v}{\partial \theta} + \frac{\partial^2 w}{\partial \theta^2}\right). \quad (4)$$

Let us express components of the displacement vector through the function of stresses Phi

$$u = \frac{1}{a}\frac{\partial \Phi}{\partial x},\quad v = -\frac{1}{a}\frac{\partial \Phi}{\partial \theta},\quad w = \frac{1}{a}\frac{\partial^2 \Phi}{\partial \theta^2}. \quad (5)$$

Let us insert expressions (5) into formulae (4)

$$N_X = \frac{Eh}{a^2}\frac{\partial^2 \Phi}{\partial x^2};\quad M_\theta = -\frac{D}{a^3}\left(\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^4 \Phi}{\partial \theta^4}\right). \quad (6)$$

From the forth equation of equilibrium (2) we deduce shearing force Q_{Theta}

$$Q_\theta = \frac{1}{a}\frac{\partial M_\theta}{\partial \theta} = -\frac{D}{a^4}\left(\frac{\partial^3 \Phi}{\partial \theta^3} + \frac{\partial^5 \Phi}{\partial \theta^5}\right).$$

(7)

From the third equation of equilibrium (2) we deduce force N_{Theta}

$$N_\theta = -\frac{D}{a^4}\left(\frac{\partial^4 \Phi}{\partial \theta^4} + \frac{\partial^6 \Phi}{\partial \theta^6}\right).$$

(8)

From the first equation of equilibrium (2) we deduce the derivative

$$\frac{\partial S}{\partial \theta} = -\frac{\partial N_x}{\partial x} = -\frac{Eh}{a^2}\frac{\partial^3 \Phi}{\partial x^3}. \quad (9)$$

From the second equation (2) we deduce the resolving equation for function Phi(*x*, Theta). We differentiate the mentioned equation on parameter Theta

$$\frac{\partial}{\partial x}\left(\frac{\partial S}{\partial \theta}\right) + \frac{\partial^2 N_\theta}{\partial \theta^2} + \frac{\partial Q_\theta}{\partial \theta} = 0. \quad (10)$$

In equation (10) we replace Q_{Theta}, N_{Theta}, $\frac{\partial S}{\partial \text{Theta}}$ using expressions (7 – 9)

$$\frac{\partial^4 \Phi}{\partial x^4} + \xi^2\left(\frac{\partial^8 \Phi}{\partial \theta^8} + 2\frac{\partial^6 \Phi}{\partial \theta^6} + \frac{\partial^4 \Phi}{\partial \theta^4}\right) = 0, \quad (11)$$

$$\frac{\partial}{\partial \theta^4}\left(\Phi_k(x)\cos k\theta\right) = k^4 \Phi_k(x)\cos k\theta;$$

$$\frac{\partial}{\partial \theta^6}\left(\Phi_k(x)\cos k\theta\right) = k^6 \Phi_k(x)\cos k\theta; \quad (12)$$

$$\frac{\partial}{\partial \theta^8}\left(\Phi_k(x)\cos k\theta\right) = k^8 \Phi_k(x)\cos k\theta.$$

The solution for function $\text{Phi}(x, \text{Theta})$ will be obtained as an expansion in series of cosines

$$\Phi(x,\theta) = \sum_{k=1}^{\infty} \Phi_k(x)\cdot\cos k\theta.$$

(13)

After substituting the values (12) and (13) into the equation (11), we will obtain

$$\sum_{k=1}^{\infty}\left[k^4\left(k^2-1\right)^2 \xi^2 \Phi_k(x) + \Phi_k^{IV}(x)\right]\cdot\cos k\theta = 0. \quad (14)$$

Since *cosk*Theta at k = 1;2… linearly independent orthogonal in the range [0; Pi] of function (14), the expressions in brackets should go to zero

$$\Phi_k^{(IV)}(x)+\xi^2 k^4\left(k^2-1\right)^2\Phi_k(x)=0,\ (k=1;2;3...)\,. \quad (15)$$

Characteristic equation for number k of terms of series for function Phi takes on the following form

$$\lambda^4+\xi^2 k^4\left(k^2-1\right)^2=0\,. \quad (16)$$

From formula (16) in view of Euler's formulae we obtain:

$$\lambda^4=-\xi^2 k^4\left(k^2-1\right)^2;\quad \lambda^2=\pm\xi\, k^2\left(k^2-1\right)\cdot i\,;$$

$$\lambda_{1,2}^2=k^2(k^2-1)e^{i\frac{\pi}{2}}\xi\,.$$

Roots of characteristic equation (16)

$$\lambda_{1,2,3,4}=\pm k\sqrt{\xi\left(\frac{k^2-1}{2}\right)}\cdot(1\pm i)\,. \quad (17)$$

According to the roots (17) of the equation (16), elementary linearly independent solutions of differential equation (15) will be functions:

$$\varphi_{1_k}(x)=ch\, m_k x\cos m_k x\,;$$

$$\varphi_{2_k}(x)=sh\, m_k x\cos m_k x\,; \quad (18)$$

$$\varphi_{3_k}(x)=ch\, m_k x\sin m_k x\,;$$

$$\varphi_{4_k}(x)=e^{-m_k x}\, sh\, m_k x\sin m_k x\,,$$

where $m_k=k\sqrt{\xi\left(\frac{k^2-1}{2}\right)}$,

$$ch\, m_k x=\frac{1}{2}\left(e^{m_k x}+e^{-m_k x}\right),$$

$$sh\, m_k x=\frac{1}{2}\left(e^{m_k x}-e^{-m_k x}\right).$$

Complete solution of differential equation (15) is expressed through arbitrary constants and fundamental functions

$$\begin{aligned}\Phi_k(x)=A_{1k}\varphi_{1k}(x)+A_{2k}\varphi_{2k}(x)+\\+A_{3k}\varphi_{3k}(x)+A_{4k}\varphi_{4k}(x).\end{aligned} \quad (19)$$

Functions $\mathrm{Phi}_{1\,k}$, $\mathrm{Phi}_{2\,k}$, $\mathrm{Phi}_{3\,k}$, $\mathrm{Phi}_{4\,k}$ are arcwise connected to Krylov's functions

$$K_1(m_k x)=\varphi_{1k}(x),\ K_2(m_k x)=0{,}5[\varphi_{2k}(x)+\varphi_{3k}(x)],$$

$$K_3(x)=0{,}5\varphi_{4k}(x), \quad (20)$$

$$K_4(x)=0{,}25\left[ch(m_k x)\cdot\sin(m_k x)-sh(m_k x)\cos(m_k x)\right].$$

System determinant (20) $\mathrm{Delta}\neq 0$, thus, complete solution of the equation (15) may be represented in terms of linear combination of Krylov's functions

$$\begin{aligned}\Phi_k(x)=C_{1k}K_1(m_k x)+C_{2k}K_2(m_k x)+\\+C_{3k}K_3(m_k x)+C_{4k}K_4(m_k x).\end{aligned} \quad (21)$$

where C_{1k}, C_{2k}, C_{3k}, C_{4k} – arbitrary constants.

Coefficients C_{1k}, C_{2k}, C_{3k}, C_{4k} in the formula (21) are determined from the boundary conditions. The shell edges:

$\left[(x{=}0),\ (x{=}q),\ q=\frac{\ell}{a}\right]$ are free, i.e.

$$N_x(x=0)=0;\quad N_x(x=q)=0. \quad (22)$$

The third condition is taken as inextensibility of the flexspline ring at $x=q$

$$V(x=q)=0. \quad (23)$$

The fourth boundary condition at x = 0 – equal displacements w of the gear ring and the shell. The gear ring model – a ring with the width of b_1 and thickness of h_1, equal to half-sum the gear ring tooth space and crest thickness, undergoes plane strain (fig. 2). Let us consider equilibrium of the ring element under the action of forces N^*_{Theta}, Q^*_{Theta}, bending moments M^*_{Theta}, force S and normal load q_n^*.

Power factors of the ring marked with asterisk N^*_{Theta}, Q^*_{Theta}, M^*_{Theta} are referred to the cross-section of the ring, S, q^*_n – linear loads referred to the unit of arc length. Equilibrium equations of the ring element in the local coordinate system $x\ y\ z$

$$\frac{\partial N^*_\theta}{\partial \theta} + Q^*_\theta + S \cdot a = 0, \tag{24}$$

$$\frac{\partial Q^*_\theta}{\partial \theta} - N^*_\theta + q^*_n a = 0, \quad \frac{\partial M^*_\theta}{\partial \theta} - Q^*_\theta a = 0.$$

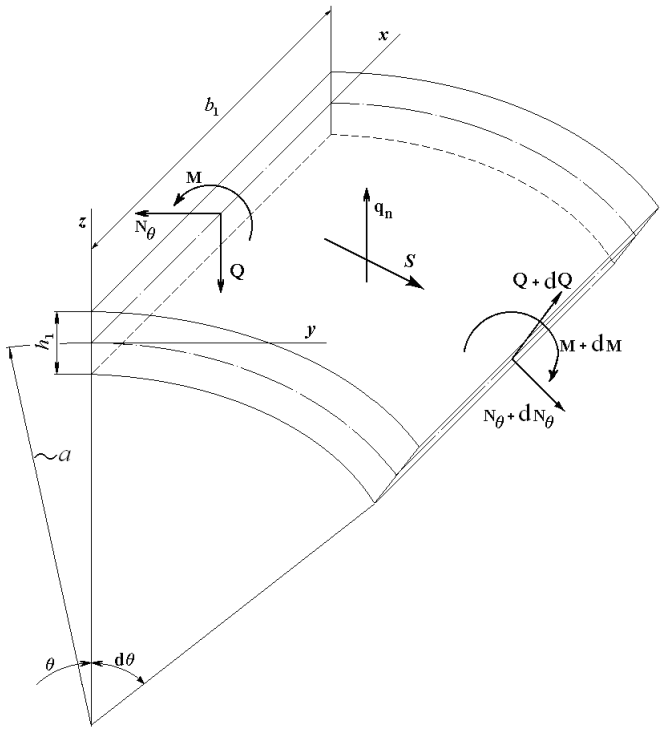

Fig. 2. Forces and moments applied to the gear ring element

Power factors in the equations (24) depend on Theta only, let us replace $\frac{\partial}{\partial \text{Theta}}$ by $\frac{d}{d\text{Theta}}$

$$Q^*_\theta = \frac{1}{a} \cdot \frac{dM^*_\theta}{d\theta}; \qquad N^*_\theta = \frac{1}{a} \cdot \frac{d^2 M^*_\theta}{d\theta^2} + q^*_n a; \tag{25}$$

$$\frac{1}{a} \cdot \frac{dN^*_\theta}{d\theta} + \frac{1}{a} Q^*_\theta + S = 0.$$

In the 3rd equation (25) we replace N^*_{Theta} and Q^*_{Theta} by the obtained values

$$\frac{d^3 M^*_\theta}{d\theta^3} + \frac{dM^*_\theta}{d\theta} + a^2 S = -a^2 \frac{dq^*_n}{d\theta}. \tag{26}$$

Let us use the second formula (4) of the general physical law, for the shell and the ring, b for the ring may be referred to the total width of the ring b_1 (24)

$$M^*_\theta = \frac{EJ_{x_0}}{a^2}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right), \tag{27}$$

where $\frac{b_1 h_1{}^3}{12} = J_{x_0}$ – moment of inertia of a cross-section of the ring about the axis x_0.

Let us assume that the middle surface of the ring is inextensible, i.e. $\text{Epsilon}_{\text{Theta}} = 0$.

Unit tangential strain of the coordinate line (x=const) depends on the component v of the displacement vector

$$\varepsilon'_\theta = \frac{dv}{dS_\theta} = \frac{dv}{a \cdot d\theta} = \frac{1}{a} \cdot \frac{dv}{d\theta}.$$

Fibers are elongated and at radial displacement w,

$$\varepsilon''_\theta = \frac{w d\theta}{dS_\theta} = \frac{w d\theta}{a \cdot d\theta} = \frac{w}{a}.$$

The ring strain $\text{Epsilon}_{\text{Theta}}$ is expressed through the displacement vector components $\text{Epsilon}'_{\text{Theta}}$ and $\text{Epsilon}''_{\text{Theta}}$

$$\varepsilon_\theta = \frac{1}{\alpha}\left(\frac{dv}{d\theta} + w\right), \text{ at } \varepsilon_\theta = 0 \quad \frac{dv}{d\theta} = -w. \tag{28}$$

Let us substitute values $\text{Epsilon}_{\text{Theta}}$ from formulae (28) into the expression (27)

$$M^*_\theta = -\frac{\text{E}J_{x_0}}{\alpha^2}\left(w + \frac{d^2 w}{d\theta^2}\right). \tag{29}$$

Insert value (29) into the formula (26)

$$S-\frac{EJ_{x_0}}{\alpha^4}\left(\frac{d^5w}{d\theta^5}+2\frac{d^3w}{d\theta^3}+\frac{dw}{d\theta}\right)=-\frac{dq_n}{d\theta}. \qquad (30)$$

Let us replace distributed load q_n^* by two opposing

concentrated forces P with the help of Delta - Dirac delta function

$$q_n^*=\frac{P}{\alpha}\left[\delta(\theta)+\delta(\theta+\pi)\right], \qquad (31)$$

where $\text{Delta(Theta)}=\left|\begin{matrix}0 \ npu \ \text{Theta}\neq 0\\ \infty \ npu \ \text{Theta}=0\end{matrix}\right.$ – Dirac delta function corresponds to the equation $\int_{-\pi}^{+\pi}\delta(\theta)d\theta=1$; P – resultant of the distributed load q_n^*. Supposing that $ds=ad\theta$, we find

$$\int_{-\pi}^{+\pi} q_n^*\cdot\cos\theta\, ds_\theta=\frac{P}{\alpha}\int_{-\pi}^{+\pi}\delta(\theta)\cos\theta\cdot ad\theta=P\cdot\cos 0=P.$$

The distributed load resultant equals to concentrated force P. Delta – function will be formally expanded in Fourier series

$$\delta(\theta)=\alpha_0+\sum_{k=1}^{\infty}\alpha_k\cos k\theta.$$

We will find the Fourier coefficients a_0, a_k and obtain values Delta(Theta) и Delta(Theta + Pi)

$$\int_{-\pi}^{+\pi}\delta(\theta)d\theta=1,\quad \alpha_0=\frac{1}{2\pi},\quad a_k=\frac{1}{\pi},$$

$$\int_{-\pi}^{+\pi}\delta(\theta)\cos k\theta\, d\theta=1=a_k\int_{-\pi}^{+\pi}\cos^2\theta\, d\theta=\pi\cdot a_k,$$

$$\delta(\theta)=\frac{1}{2\pi}\left(1+2\sum_{\kappa=1}^{\infty}\cos k\theta\right), \qquad (32)$$

$$\delta(\theta+\pi)=\frac{1}{2\pi}\left(1+2\sum_{\kappa=1}^{\infty}\cos(k\theta+\pi k)\right)=$$
$$=\frac{1}{2\pi}\left(1+2\sum_{\kappa=1}^{\infty}(-1)^k\cos(k\theta)\right). \qquad (33)$$

Substitute values (32) and (33) into the formula (31), find q_n^* and the derivative

$$q_n^*=\frac{P}{\pi\alpha}\left(1+2\sum_{k=2,4,6,...}^{\infty}\cos k\theta\right).$$

$$\frac{dq_n^*}{d\theta}=-\frac{2p}{\pi\alpha}\cdot\sum_{k=2,4,...}^{\infty}k\cdot\sin k\theta. \qquad (34)$$

The series in the second of the formulae (34) diverges, and we substitute it into the differential equation (30); when integrating it we will obtain the convergent series

$$S-\frac{EJ_{x_0}}{\alpha^4}\left(\frac{d^5w}{d\theta^5}+2\frac{d^3w}{d\theta^3}+\frac{dw}{d\theta}\right)_{x=0}=$$
$$=\frac{2P}{\pi a}\sum_{k=2,4,...}^{\infty}k\sin k\theta. \qquad (35)$$

The expression (35) is the fourth boundary condition for section x = 0, where the inflexions and forces S are common for the ring and the shell. The conditions (22), (23), (35) are sufficient to determine coefficients C_{1k}, C_{2k}, C_{3k}, C_{4k} in representation (21) for the function $\text{Phi}_k(x)$. From the expressions (13) and (21) let us represent the stress function

$$\Phi(x,\theta)=\sum_{k=2,4,...}^{\infty}\left[C_{1k}K_1(m_kx)+C_{2k}K_2(m_kx)+\right.$$
$$\left.+C_{3k}K_3(m_kx)+C_{4k}K_4(m_kx)\right]\cos k\theta. \qquad (36)$$

According to the expression (38), with the odd k, coefficients C_{1k} C_{2k}, C_{3k}, C_{4k} in the formula (36) equal to zero.

Under the conditions of (22), according to the first of the formulae (4) and formulae (5) we obtain

$$N_X=\frac{Eh}{a}\frac{\partial u}{\partial x}=\frac{Eh}{a^2}\frac{\partial^2\Phi}{\partial x^2}=$$
$$=\sum_{k=2,4,6,...}^{\infty}\left[C_{1k}\frac{d^2K_1(m_kx)}{dx^2}+C_{2k}\frac{d^2K_2(m_kx)}{dx^2}+\right.$$
$$\left.+C_{3k}\frac{d^2K_3(m_kx)}{dx^2}+C_{4k}\frac{d^2K_4(m_kx)}{dx^2}\right]\cos k\theta. \qquad (37)$$

Let us use the properties of Krylov's functions

$$\frac{d^2K_1(m_kx)}{dx^2}=-4m_kK_3(m_kx);$$

$$\frac{d^2K_2(m_k x)}{dx^2} = -4m_k^2 K_4(m_k x);$$

$$\frac{d^2K_3(m_k x)}{dx^2} = m_k^2 K_1(m_k x); \tag{38}$$

$$\frac{d^2K_4(m_k x)}{dx^2} = m_k^2 K_2(m_k x).$$

Substitute values (38) into the formula (37)

$$N_X = \frac{Eh}{a^2}\sum_{k=1}^{\infty} m_k^2\big[-4C_{1k}K_3(m_k x) - 4C_{2k}K_4(m_k x) +$$

$$+C_{3k}K_1(m_k x) + C_{4k}K_2(m_k x)\big]\cos k\theta. \tag{39}$$

Considering that

$$K_1(0) = 1;\ K_2(0) = K_3(0) = K_4(0) = 0;\ C_{3k} = 0; \tag{40}$$

$$-4C_{1k}K_3(m_k q) - 4C_{2k}K_4(m_k q) + C_{4k}K_2(m_k q) = 0\,. \tag{41}$$

Using the equation (40) we will transform the expression (39)

$$N_X = \frac{Eh}{a^2}\sum_{k=2,4,6,...}^{\infty} m_k^2\,\big[-4C_{1k}K_3(m_k x) -$$
$$-4C_{2k}K_4(m_k x) + C_{4k}K_2(m_k x)\big]\cos k\theta. \tag{42}$$

From the second formula (5), with consideration for (36), (20), (40), (23), we will obtain

$$V = -\frac{1}{a}\sum_{k=2,4,6,...}^{\infty} k\,\big[C_{1k}K_1(m_k x) + C_{2k}K_2(m_k x) +$$
$$+C_{4k}K_4(m_k x)\big]\sin k\theta. \quad At\ x = q, \quad V = 0.$$

By the solution of the ring and the shell elastic boundary value problem the power factors required for calculating the stressed state of the flexspline have been obtained [21] (fig. 3).

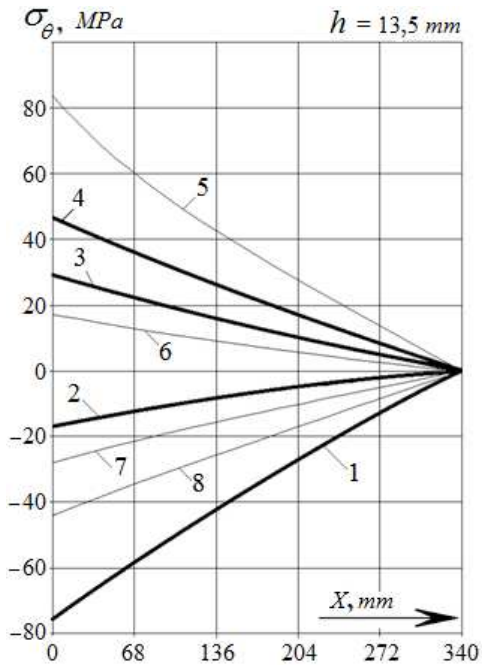

Fig. 3. Normal tangential stresses $Sigma_{Theta1}$: 1 – 4 and $Sigma_{Theta3}$: 5 – 8 on the inner and outer surfaces of the flexspline shell; 1 and 5 - at Theta = 0°; 2 and 6 - at Theta = 30°; 3 and 7 - at Theta = 60°; 4 and 8 - at Theta = 90°

The strength design has been performed for the flexspline with the wall thickness h = 13,5*mm* at load M_{2max}=5·10^5*Nm* of the harmonic drive unit for relining MGR5500×7500 ore-pulverizing mill with the useful capacity of 160 m^3, weight of ore to be loaded amounting to 220 tonne. Normal tangential stresses acting on the inner and outer surfaces of the shell are designated as $Sigma_{Theta1}$ and $Sigma_{Theta3}$ *MPa* depending on the distance X to the gear ring, at fixed values of the angle of deflection Theta from the major axis of the wave generator.

V. CONCLUSION

The disks form elastic wave-induced strain of the flexible gear ring the rotation component of which is taken off by the shell to the output shaft. The stress-strain state of the flexspline strained by the wave generator disks is worked out through the combined solution of the ring and the shell. External load on the gear ring is transformed using the Dirac delta function. Based on the research findings, determining are the normal tangential stresses on the flexspline inner and outer surfaces $Sigma_{Theta1}$ and $Sigma_{Theta3}$, reaching maximum values at the joint of the shell with the gear ring where tensile stresses $(Sigma_{Theta3})_{max}$ = +84 *MPa* on the outer surface of the flexspline, maximum compression stresses $(Sigma_{Theta1})_{max}$ = -77 *MPa* on the inner surface. Mathematical model evaluation has been made by experiment, employing the strain-gauging method. Divergence of the obtained results of the comparative analysis does not exceed 8 %. The accuracy of solution of the boundary value problem of the flexspline elasticity has been increased by 25… 30 %.

The results of the solution of the boundary value elasticity problem of the flexspline subject to permanent strain by the disk wave generator have made it possible to develop heavy-

duty waveform gear reduction units meeting the up-to-date requirements imposed on heavy engineering products. The mentioned results have been also used when working out calculation and design technical materials for the design and production of waveform gear reduction units for heavy engineering and have been applied to the main drives of mining equipment and other machinery intended for heavy industry.

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