

# Statistical Model of Centimeter Range Signals That Have Passed The Surface and Space Propagation Path

Alexey S. Anikin

Radio Systems Department, Tomsk State University of Control Systems and Radioelectronics,  
Tomsk, Russia  
rbk@sibmail.com

**Abstract**—This article describes the correlation function and the angular energy spectrum of scattered waves for terrestrial paths to distribute radio waves of space propagation path and diffraction. The angular energy spectrum of the scattered waves on such paths is multimodal and causes the periodic nonstationarity of the signals at the outputs of the spaced antennas as the angular position of the narrowly directed antenna of the radio emission source changes. These features take into account the Gaussian model of signals under consideration on the basis of the multimodal angular energy spectrum of scattered waves. Analytical relations, defining conditions for appearance of abnormally large errors in case of receiving signals from radio source with scanning antenna by a direction finder along terrestrial paths, are obtained. It is theoretically proved that it is necessary to compare the phase difference measurement at various bases of the direction finder with a certain threshold in order to detect bearings with abnormally large errors.

**Keywords**—correlation function; angular energy spectrum; Gaussian model; quadrature; antenna; radio source

## I. INTRODUCTION

It is known that the output signals of antennas on the tropospheric and space propagation paths of radio waves can be expressed as the sum of the radio emission source signal and the scattered component due to medium inhomogeneities [1-5]. While scattering of radio waves by small-scale inhomogeneities in the troposphere, the scattered component is described by a stationary normal random process, when the observation interval of the signal does not exceed several seconds and the transmitting points are fixed [3, 5]. The angular energy spectrum of scattered waves  $G(\alpha)$  is usually described by a continuous function with a maximum, in general, not coinciding with the angular position of the radio emission source [5].

Radio waves are scattered by unevenness of the underlying surface, the edge of forest massifs or reflected from other terrain elements on the surface paths of propagation of radio waves of sight line or diffraction with fixed transmitting points [5]. For such surface paths of radio wave propagation, the

angular energy spectrum of scattered waves is multimode (Fig. 1).

These features of surface propagation paths are not taken into account in known statistical models of received signals [3, 5, 6].

An objective of the report is to determine the spatial correlation function of the signals at the outputs of receiving antennas that have passed the ground path from a radio source with a narrowly oriented and arbitrarily oriented antenna.

The angular energy spectrum of scattered waves becomes asymmetric when the main lobe is irradiated by the radiation pattern of the radio source of local objects located away from the direction to the direction finder.

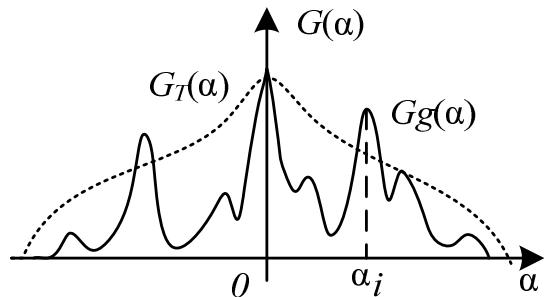


Fig. 1. Angular energy spectrum characteristic of the tropospheric propagation path  $G_T(\alpha)$  and ground distribution path  $G_g(\alpha)$

In this case, the direction finding of the radio emission source by traditional algorithms is accompanied by anomalously large errors [7, 8]. The results of this work can be used to develop new algorithms for finding direction sources of radio emission with a narrowly directional antenna excluding the occurrence of anomalously large errors.

Phenomenological approach based on the representation of the scattered field by the additive sum of waves from taken into account objects of the surface propagation path is used when considering the statistical model.

Experimental results for ground paths attest probably relation between root-mean-square error (RMSE) of phase difference for the signals and their amplitude ratio in two- or multichannel systems [2, 3]. In the specific conditions, the statistical characteristics of phase difference for some signals relate on their amplitude ratio. These relations could be utilized for increasing the accuracy of phase difference estimation on paths with local terrain features and objects scattering.

## II. MODEL OF THE SIGNAL AT THE ANTENNA OUTPUT

Let us suppose that fixed source emits a narrowband signal and has a fixed angular position of the transmitting antenna. The model of the output signal of the fixed receiving antenna is represented by the sum of the direct signal and the reflected signals by the terrain elements:

$$\dot{U}_p(x_r, \theta) = \sum_{i=1}^N \dot{F}^t[\theta - \theta_i] \dot{U}_{scatt.i}(x_r), \quad (1)$$

where

$i$  – number of the signal reflected from the object on the ground,  $i = 1 \dots N$ ;

$\dot{U}_{scatt.i}(x_r)$  – complex amplitude of the reflected signal from the  $i$ -th object;

$\dot{F}^t(\theta)$  – radio source direction pattern multiplier (RES);

$\theta_i$  – direction to the  $i$ -th object of the terrain with respect to OY axis counted from the RES.

Let us suppose that the reflected signal from the terrain element is a stationary normal random process with uncorrelated quadratures, zero mean and the same variances. For a statistical description of the received signals, we apply the two-dimensional normal distribution of the quadratures of the received signals with the known average vector and the spatial correlation matrix [3, 5, 6].

Let's consider the spatial coefficient of scattered signals correlation when radio waves are re-emitted by elements of the terrain of a surface crossed path.

In conditions of signals reception on surface paths from a directional radiation source, the parameters of the spatial correlation function of the scattered signals (the spatial correlation interval, the shape) depend on the angular position of the antenna of the radio source  $\theta$ :

$$\dot{R}_p(\Delta l; \theta) = M[\dot{U}_p(l, \theta) \dot{U}_p(l + \Delta l, \theta)], \quad (2)$$

where the averaging is performed over the ensemble of successive implementations of the scattered signals.

The assumption of statistical independence of scattered signals from various terrain objects is physically justified. Assuming the statistical homogeneity of the field, we

---

The reported study was funded by RFBR, according to research project No. 16-38-60091 mol\_a\_dk

represent (2) as:

$$\begin{aligned} \dot{R}_p(\Delta l, \theta) &= \\ &= 2 \sum_{i=1}^N [F^t(\theta - \theta_i)]^2 \dot{R}_{scatt.i}(\Delta l, \theta), \end{aligned} \quad (3)$$

where

$\dot{R}_{scatt.i}(\Delta l, \theta) = R_{scatt.i}(\Delta l, \theta) e^{j\gamma(\Delta l, \theta)}$  – spatial correlation function for the  $i$ -th reflected signal;

$R_{scatt.i}(\Delta l, \theta) = \sqrt{r_i^2(\Delta l, \theta) + s_i^2(\Delta l, \theta)}$  – spatial correlation coefficient module for the  $i$ -th reflected signal;

$\gamma_{scatt.i}(\Delta l, \theta) = \arctg \{s_i(\Delta l, \theta) / r_i(\Delta l, \theta)\}$  – phase of the spatial correlation function of the  $i$ -th reflected signal,  $r_i(\Delta l, \theta)$  и  $s_i(\Delta l, \theta)$  – cosine and sine quadrature component of the spatial correlation function.

Let us suppose that the number points of reflecting objects on the surface propagation paths is large and their angular dimensions are small, thus, in the first approximation, we consider the angular energy spectrum of the  $i$ -th object of the terrain as wide  $\Delta \alpha_{scatt.i}$  and symmetric with respect to the mean angle of incoming  $\alpha_{scatt.i}$ .

Under these conditions, the spatial correlation coefficient of the scattered signal can be written in the following form [3, 5]:

$$\begin{aligned} \dot{R}_p(\Delta l, \theta) &= \\ &= 2 \sigma_{scatt.i}^2 \sum_{i=1}^N \frac{[F^t(\theta - \theta_i)]^2}{e^{\frac{\Delta l}{l_0^i} - j2\pi \frac{\Delta l}{\lambda} \sin(\alpha_{scatt.i})}}, \end{aligned} \quad (4)$$

where

$\sigma_{scatt.i}^2$  – variance of fluctuations of the  $i$ -th reflected signal;

$l_0^i$  – interval of spatial correlation for the  $i$ -th reflected signal fluctuations inversely proportional to the width of the angular spectrum  $l_0^i = \lambda / 2 \sin(\Delta \alpha_{scatt.i})$ .

## III. SPATIAL CORRELATION FUNCTION OF THE SIGNALS AT THE OUTPUTS OF THE RECEIVING ANTENNAS

Time fluctuations of the partial reflected signals at the receiving site lead to corresponding fluctuations of the initial phases. Let's assume the distribution of the initial phases fluctuations for the partial scattered signals at the receiving point uniform in the interval  $[0 \dots 2\pi]$ . In this case, any partial scattered signal and direct signal are uncorrelated at the receiving site. Thus, the spatial correlation coefficient of the received signals at points  $-\Delta l/2$  and  $\Delta l/2$  by means of (4), we can write as follows:

$$\dot{R}_y(\Delta l, \theta) = R_y(\Delta l, \theta) e^{j\gamma_y(\Delta l, \theta)}, \quad (5)$$

where

$R_y(\Delta l, \theta)$  – module of the spatial correlation coefficient of the sum of the direct signal and the scattered component;  
 $\gamma_y(\Delta l, \theta)$  – phase of the spatial correlation coefficient of the sum of the direct signal and the scattered component.

The phase of the spatial correlation coefficient (5) corresponds to the mean angle of arrival of the scattered signal equal to the center of gravity of the angular energy spectrum  $\alpha_c$ :

$$\gamma_y(\Delta l, \theta) = (2\pi\Delta l/\lambda) \sin[\alpha_c(\theta)]. \quad (6)$$

The phase of the spatial correlation coefficient corresponds to the average phase difference between the received signals of spatially spaced antennas. We show that the dependence of the phase of the spatial correlation function (6) on the angular position of the radio source antenna for a fixed separation of the receiving antennas ( $\Delta l$ ) complies with the available experimental data.

#### IV. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

The experimental data are the dependences of the received signal levels on the outputs of the receiving antennas and the phase difference between them from the angular position of the radio emission source antenna in the azimuth plane.

Experimental data were obtained for ground paths from 16 to 30 km long. Scattered (reflected) radio waves are caused mainly by terrain features: underlying surface, natural and artificial objects, forest edge etc. The transmitting point apparatus (radio location system) and receiving-measurement apparatus (receiving point) utilized in the experiments were developed by workers of Institute of Radio electronic System of TUCSR. A full description of digital receiving and measurement apparatus is given in [3 - 7]. Devices of emitter source and receiving point were synchronized. During the experiment, digital samples for received signals within a specific time period were saved into PC memory. Pulse signal amplitudes and their phase differences were defined using quadrature components obtained relatively to pulse start [4, 11].

The introduction of new radio frequency bands for use in the forest requires preliminary studies. Modern trends is the increase in operating radio frequency. Also, there is a necessity for locating radio sources.

The purpose article is to study the characteristics and properties of the pulse signals, which have passed a short forest track near surface. With the objective of evaluating the direction finding accuracy of radiation sources.

The studies were conducted between September and October 2012 in the vicinity of the city of Tomsk. Measurements were obtained for three routes. Length of the route is from 130 to 400 meters. Trails differ in greenery density.

Radiation source was a portable radar, a pulse signal duration of 275 ns and a frequency of 9.2 GHz without intra-modulation, antenna gain 23 dB and a peak power of 10 kW.

Reception and recording equipment consisted of an antenna system comprising two dipole antennas located at 45° to the horizontal and the two-channel receiver with quadrature receive channels. The antennas formed a direction finder with base length 20λ, recording equipment can record the quadrature components of the signal with a sampling frequency of 90 MHz.

A receiver with a dynamic range of 71 dB has a sensitivity of -100 dB / mW (3.2 μV). The transmission bandwidth of the receiver is 25 MHz, so linear distortion of signals due to the influence of the receiving path can be neglected. Before the analog-to-digital conversion, the signal is transferred to the video frequency by a quadrature demodulator (detector). The envelopes and the phase difference were determined by quadratures over the duration of the plane vertex of the pulse, using the formulas given in [6].

Obstacles on the line "transmitter-receiver" are presented for a group of trails with the length of:

- 16.8 km and 19.0 km in the form of separate rare forest tracts; the transfer positions were located in a field covered with meadow grass, up to 0.25 m high;

- 23.0 km in the form of frequently encountered forest massifs; and the transmitting positions were located in a meadow overgrown with meadow grass, with a height of 0, 5 m, and shrubs;

- 28.0 km in the form of dense forest areas; the transferring positions were located on a clearing, the grass on which was mown, surrounded by a thick forest.

The firing position was located on the right bank of the river. Tom is over 80 m above the surrounding terrain. The right bank is covered with forest of medium density, then along the routes there is a river valley, an island with extensive mowing, a populated village of rural type with suburban areas of loosely built up. The remaining sections of the road are a plain with a slight upward slope to an absolute 140 m mark, covered with meadow vegetation, mowing, arable land, medium-thick forest, and rare-wood.

In Fig. 3 we present experimentally founded conditional RMSE (3.a, dashed line with circles) and mean phase difference between signals received on spaced antennas (with spacing distance of  $d = 30\lambda$ ) in a relation to their amplitude ratio for 6 ground paths in a logarithmic scale. The results were obtained by processing more than 500 thousands of RF pulse signals (with signal-to-noise ratio more than 15-20 dB). For every phase difference, RMSE value the confidence interval at significant level of  $\alpha_e = 0,05$  [7-10] is denoted.

The characteristic normalized radiation patterns of the radio emission antenna source and the dependence of the phase difference between the output signals of the receiving antennas with the spatial separation of  $10\lambda$  from the angular position of the transmitter antenna are shown in Fig. 2, 3.

Experimental results were obtained on the surface road which represents a flat terrain with small irregularities.

Let's compare the experimental dependence of the signal level on the antenna outputs on the angle of rotation for antenna RES with similar calculated dependences and the experimental phase difference with the phase of the correlation function.

In Fig. 4, 5 shows the calculation results for the radio source antenna radiation pattern [5, 7, 9-14] and the phase correlation coefficient (6) for random mean angles  $\alpha_{scatt,1} \dots \alpha_{scatt,N}$  from the following conditions:

- wavelength  $\lambda = 0,03$  m;
- directional pattern of the transmitter is 3 degrees wide in zeros and has the form  $\sin(\theta)/\theta$ ;
- number of reflected signals  $N = 5$ ;
- average angles of incoming  $\alpha_{scatt,1} \dots \alpha_{scatt,N}$ , width of angular spectra  $\Delta\alpha_{scatt,1} \dots \Delta\alpha_{scatt,N}$  and the dispersion of the reflected waves amplitudes were random and distributed according to a uniform law within  $\pm \alpha_{scatt} = 20$  degrees и  $\Delta\alpha_{scatt} = 5$  degrees respectively.

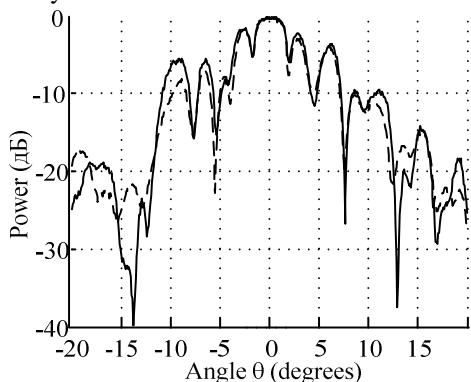


Fig. 2. Experimental normalized directivity patterns of RES antenna at the outputs of the receiving antennas,  $\Delta l = 10\lambda$

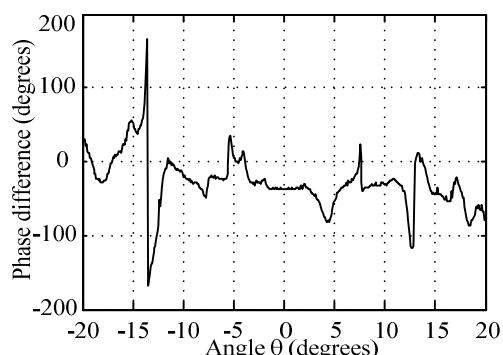


Fig. 3. Experimental phase difference between the signals at the outputs of the receiving antennas,  $\Delta l = 10\lambda$

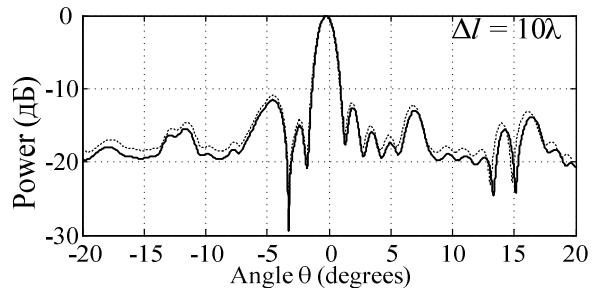


Fig. 4. Calculated normalized radiation patterns of antenna RES of the receiving antenna signals,  $\Delta l = 10\lambda$

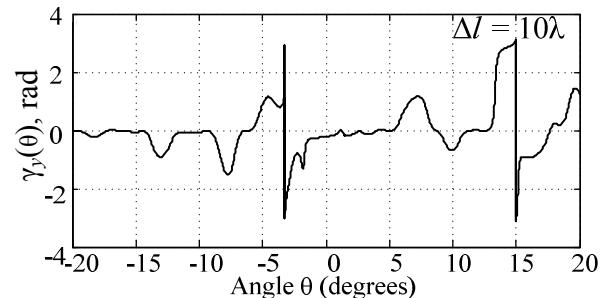


Fig. 5. Calculated the argument of correlation coefficient of the receiving antenna signals,  $\Delta l = 10\lambda$

An analysis of the experimental and theoretical results shows that the phase difference and the phase of the spatial correlation coefficient vary from the orientation of the transmitter antenna, the angular position of the reflected wave sources on the surface propagation path of radio waves. We note that there is a qualitative similarity between the phase of the spatial correlation coefficient and the phase difference of the signals at the output of spatially separated antennas depending on the angular position of antenna RES. Sharp deviations of the phase difference reaching the boundaries of a single measurement are observed in the area of some minima of the "apparent" radiation pattern of the transmitter antenna  $\pm\pi$  which are qualitatively similar to the analogous bipolar spikes of the phase of the spatial correlation coefficient at  $\Delta l = \text{const}$ . This causes the periodic nonstationarity of the field at the receiving site.

The spatial correlation coefficient (5) can be used in the statistical description of signals at the outputs of spatially-spaced antennas within the normal model of the scattered signal.

During the comparison of test results by two different methods, we discovered a fine match between model and experimental relations of RMSE and received signals mean phase difference to their amplitude ratio.

#### V. STATISTICAL MODEL OF RECEIVED SIGNALS

Let us write in general form the two-dimensional distribution density of the quadrature components of the output signals  $U_{y,c1}, U_{y,s1}$  first and  $U_{y,c2}, U_{y,s2}$  second antennas with spatial separation  $\Delta l$  [1, 3, 5]:

$$W_U = \frac{e^{-\frac{1}{2}[\mathbf{U}_y - \mathbf{M}_U]^T \mathbf{B}_U^{-1} [\mathbf{U}_y - \mathbf{M}_U]}}{(2\pi)^2 \sqrt{\det \mathbf{B}_U}}, \quad (7)$$

Where vector of the quadrature components of the output signals of the spatially separated receiving antennas  $U_y$  and  $M_U$ , the vector of their corresponding mean values can be written in the following form:

$$\mathbf{U}_y = [U_{c1} \ U_{s1} \ U_{c2} \ U_{s2}]^T,$$

$$\mathbf{M}_U = [U_{n,c1} \ U_{n,s1} \ U_{n,c2} \ U_{n,s2}]^T,$$

and the correlation matrix of the scattered component is described as:

$$\mathbf{B}_U = \sigma_{scatt}^2 \begin{bmatrix} 1 & 0 & r' & s' \\ 0 & 1 & -s' & r' \\ r' & -s' & 1 & 0 \\ s' & r' & 0 & 1 \end{bmatrix},$$

which elements are the normalized quadrature components of the correlation function (3).

The presented model allows us to describe the statistical characteristics of the received signals at such angular positions of the radio source antenna when anomalously large direction finding errors are observed. From the previous results, it follows that bipolar phase-space phase correlation throws corresponding to anomalously large direction finding errors are observed at angular positions  $\theta$  of the radio source antenna that the following conditions are satisfied for spatial separation  $\Delta l$ :

- power of the scattered and direct signal is commensurable;
- mean angle of arrival of scattered waves  $\alpha_c$  is commensurate with or exceeds the sector of single-sided direction finding of the source of radio emission for a given spatial spacing of the receiving antennas.

Physically, the reason for the appearance of anomalously large errors is caused by the interference of the signal from the radiation pattern with the waves reflected by the terrain elements. An analysis of experimental estimates of impulse responses has shown [8] that on ground crossed paths a limited number of reflected signals is observed. This causes the multimode energy spectrum of the scattered waves. Because of the limited number of reflected signals, the conditions of the central limit theorem may not be fulfilled, and the distribution of the quadrature components is different from the normal law.

In the developed statistical model, the correlation function of scattered signals corresponds to the multimode angular spectrum of scattered waves. Each mode in the angular spectrum is characterized by the angular position, the dispersion of the fluctuations in the amplitude of the wave, and the width of the angular spectrum. This provides a qualitative similarity between the results of calculating the phase of the spatial correlation function from the angular position of the radiation pattern antenna and the experimental

data: there are anomalously large errors in the direction finding of IRI in the region of the minima of its "apparent" radiation pattern.

Relations are determined, for which the statistical model of the received signals corresponds to the conditions for observing anomalously large errors in the direction finding of the radiation pattern.

## VI. CONCLUSIONS

Proposed results allow making the following conclusions.

1. Statistical model of signals that have passed the surface propagation path from a centimeter-wide RES with a narrow antenna is applicable under the following conditions:

- radio signal reflected by the terrain element is described by a stationary random process;
- there is no correlation between the signal from the radio source and the reflected signals as well as between the reflected signals;
- angular dimensions of the re-reflective objects are small in comparison with the width of the main lobe of the radiation pattern of the radio source antenna.

2. In the developed statistical model, the correlation function of scattered signals corresponds to the multimode angular spectrum of scattered waves. Each mode of the angular spectrum is characterized by the angular position, the variance of the fluctuations in the amplitude of the wave, and the width of the angular spectrum.

## Acknowledgment

The reported study was funded by RFBR, according to research project No. 16-38-60091 mol\_a\_dk, and performed within the scope of State Assignment No. 8.7348.2017/8.9 of the Ministry of Education and Science in chapter of the experimental results.

## References

- [1] Kazemipour A. "Fast and stable signal deconvolution via compressible state-space models" //IEEE Transactions on Biomedical Engineering, 2018, vol. 65, № 1, pp. 74-86.
- [2] Liu, Hai "Quantitative Stability Analysis of Ground Penetrating Radar Systems", IEEE Geoscience and Remote Sensing Letters (2018), vol. 15, № 4, pp. 522-526.
- [3] Guo S. "An Imaging Dictionary Based Multipath Suppression Algorithm for Through-Wall Radar Imaging" //IEEE Transactions on Aerospace and Electronic Systems, 2018, vol. 54, № 1, pp. 269-283.
- [4] Jiu B. "Wideband cognitive radar waveform optimization for joint target radar signature estimation and target detection" //IEEE Transactions on Aerospace and Electronic Systems, 2015, vol. 51, № 2, pp. 1530-1546.
- [5] Zilz D. P., Bell M. R. Statistical "Modeling of Wireless Communications Interference and Its Effects on Adaptive-Threshold Radar Detection" //IEEE Transactions on Aerospace and Electronic Systems, 2017, vol. 54, № 2, pp. 890-911.
- [6] Setlur P., Qureshi T., "Rangaswamy M. Random and localized random projections for radar: Statistical and performance analysis" //Signals,

- Systems, and Computers, 2017 51st Asilomar Conference on, IEEE, 2017, pp. 392-397.
- [7] Ju Y. "Enhancing direction-finding accuracy for shortwave fixed stations" //Wireless Communications and Signal Processing (WCSP), 2017 9th International Conference on, IEEE, 2017, pp. 1-5.
- [8] Park D. "Adaptive beamforming for low-angle target tracking under multipath interference" //IEEE Transactions on Aerospace and Electronic Systems, 2014, vol. 50, №. 4, pp. 2564-2577.
- [9] Berngardt O. I. "First Joint Observations of Radio Aurora by the VHF and HF Radars of the ISTP SB RAS" //Radiophysics and Quantum Electronics, 2018, pp. 1-22.
- [10] Ma J. "Angle Estimation of Extended Targets in Main-Lobe Interference With Polarization Filtering" //IEEE Transactions on Aerospace and Electronic Systems, 2017, vol. 53, №. 1, pp. 169-189.
- [11] Rytov S.M. "Principles of Statistical Radiophysics", vol. 3, Wave Propagation Through Random Media. – Springer-Verlag, 1989. – 464 p.
- [12] Anikin A. S., Denisov V. P. "Estimation of the small sized radio direction finder errors in case of scattered signals" //Micro/Nanotechnologies and Electron Devices (EDM), 2016 17th International Conference of Young Specialists on, IEEE, 2016, pp. 61-63.
- [13] Xu J., Ma M., Law C. L. "Cooperative angle-of-arrival position localization //Measurement", 2015, vol. 59, pp. 302-313.
- [14] A. Ishimaru "Wave Propagation and Scattering in Random Media", Vol. 2, Academic Press, New York (2013), p. 339.