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CONVERGING OF ANALYTICAL SOLUTION OF PILE UNDER HORIZONTAL STATIC LOAD WITH ANALYSIS THROUGH FINITE ELEMENT METHOD

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***Abstract* – It is often necessary to carry out an analysis of piles for horizontal loads in the process of designing buildings and constructions. There are various ways to perform such analysis. So, Professor B.N. Zhemochkin suggested calculating a pile as a rod in an elastic half-space. More details about the assumptions and principles of this analysis can be found in his relevant work. The given method assumes large computations for its time, which are not difficult to automate today. We will write a program, perform the analysis of a pile and compare the result with that obtained through the finite element method.**

Keywords–horizontal loads, designing buildings, elastic half-space, finite element method, stress-strain characteristics

I. INTRODUCTION

Let us suppose that we have a round reinforced concrete pile with the diameter of 500 mm and the length of 10.5 m, immersed in sandy soil to the depth of 10 m. Consequently, the length of the pile head will be 0.5 m. The static loads that are the horizontal force of 10 tf and the bending moment of 1 tf·m act upon the pile head.

We are writing the program in Mathcad that is the most common computer algebra system [12-14]. We successively enter the initial data: the depth of the pile immersion L_{sp} , the length of the pile head L_{hp} and the width (in this case, the pile diameter) of the pile b_p . The units in all the measurements are meters:

$$Lsp := 10 \qquad Lhp := 0.5 \qquad bp := 0.5$$

Let us enter the parameters characterizing the stiffness properties of the pile, in particular, the moment of inertia of the round cross-section of the pile I_p , m^4, and the modulus of elasticity of the pile material E_p , tf/m^2, conventionally accepted as that of heavy concrete of B25 class which equals 30000 MPa (around 3×106 t/m^2) according to the regulatory documents being in effect:

$$Ip := \frac{\pi \cdot bp^4}{64} \qquad Ep := 3 \cdot 10^6$$

The horizontal force P , tf, and the bending moment M , tf·m, acting on the pile head are:

$$P := 10 \qquad M := 1$$

The parameters characterizing the stress-strain characteristics of the soil are: the strain modulus E_0 , tf/m2, and the coefficient of transverse strains μ

(Poisson's ratio). They are accepted approximately, according to the standards [4, 5] as those of medium size sands of quaternary deposits with the coefficient of porosity of 0.55. Then the strain modulus will be 40 MPa (about 4,000 tf / m2), the Poisson's ratio will be 0.3:

$$E0 := 4000 \qquad \mu := 0.3$$

Let us introduce the number of the analysis sections n equal to an integer (for the purpose of convenience, we accept $n = 10$). On the basis of this, the lengths of the analysis sections c , m, are determined, as well as the dimension of the system of equations (indices k and i) are to be made up further:

$$n := 10 \qquad c := \frac{Lsp}{n}$$

$$k := 0..(n-1) \qquad i := 0..(n-1)$$

At this step, the input of the initial data to the program is completed. The formulas to be entered further need no correction.

The vectors with the values of the depths of the points under consideration from the surface a_k and from the middle of the upper section a_k^0 , m:

$$a_k := \frac{c}{2} + k \cdot c \qquad a0_k := k \cdot c$$

Let us introduce the auxiliary symbols suggested by B.N. Zhemochkin: coefficients $\xi_{k,i}$, $\zeta_{k,i}$, η , [1,7] and α [1, 8,9]:

Let us make the matrices with the values of the auxiliary functions $F_{k,i}^{\mathrm{I}}$, $F_{k,i}^{\mathrm{II}}$ and $F_{k,i}^{\mathrm{III}}$ [1, 16,17]. For the convenience of perception, we will split the function $F_{k,i}^{\mathrm{I}}$ into six summands taking into consideration that the elements of the matrix located at the main diagonal (at $k = i$) can be computed through another formula:

$$FI1_{k,i} := \begin{cases} \left(\frac{2\cdot\zeta_{k,i}+1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{2\cdot\zeta_{k,i}+1}\right)+\sqrt{\left(\frac{\eta}{2\cdot\zeta_{k,i}+1}\right)^2+1}\right] & \text{if } k \neq i \\ \ln\left(1+\sqrt{\eta^2+1}\right)+\frac{1}{\eta}\cdot\ln\left(\eta+\sqrt{\eta^2+1}\right)-\ln(\eta) & \text{otherwise} \end{cases}$$

$$FI2_{k,i} := \begin{cases} -\left(\frac{2\cdot\zeta_{k,i}-1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{2\cdot\zeta_{k,i}-1}\right)+\sqrt{\left(\frac{\eta}{2\cdot\zeta_{k,i}-1}\right)^2+1}\right] & \text{if } k \neq i \\ \ln\left|1+\sqrt{\eta^2+1}\right|+\frac{1}{\eta}\cdot\ln\left|\eta+\sqrt{\eta^2+1}\right|-\ln(\eta) & \text{otherwise} \end{cases}$$

$$FI3_{k,i} := \begin{cases} \ln\left[\dfrac{\left(\frac{2\cdot\zeta_{k,i}+1}{\eta}\right)+\sqrt{\left(\frac{2\cdot\zeta_{k,i}+1}{\eta}\right)^2+1}}{\left(\frac{2\cdot\zeta_{k,i}-1}{\eta}\right)+\sqrt{\left(\frac{2\cdot\zeta_{k,i}-1}{\eta}\right)^2+1}}\right] & \text{if } k \neq i \\ 0 & \text{otherwise} \end{cases}$$

$$FI4_{k,i} := \begin{cases} \left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}+1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{4\cdot\xi_i-2\cdot\zeta_{k,i}+1}\right)+\sqrt{\left(\frac{\eta}{4\cdot\xi_i-2\cdot\zeta_{k,i}+1}\right)^2+1}\right] & \text{if } k \neq i \\ \left(\frac{4\cdot\xi_i+1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{4\cdot\xi_i+1}\right)+\sqrt{\left(\frac{\eta}{4\cdot\xi_i+1}\right)^2+1}\right] & \text{otherwise} \end{cases}$$

$$FI5_{k,i} := \begin{cases} -\left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}-1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{4\cdot\xi_i-2\cdot\zeta_{k,i}-1}\right)+\sqrt{\left(\frac{\eta}{4\cdot\xi_i-2\cdot\zeta_{k,i}-1}\right)^2+1}\right] & \text{if } k \neq i \\ -\left(\frac{4\cdot\xi_i-1}{\eta}\right)\cdot\ln\left[\left(\frac{\eta}{4\cdot\xi_i-1}\right)+\sqrt{\left(\frac{\eta}{4\cdot\xi_i-1}\right)^2+1}\right] & \text{otherwise} \end{cases}$$

$$FI6_{k,i} := \begin{cases} \ln\left[\dfrac{\left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}+1}{\eta}\right)+\sqrt{\left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}+1}{\eta}\right)^2+1}}{\left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}-1}{\eta}\right)+\sqrt{\left(\frac{4\cdot\xi_i-2\cdot\zeta_{k,i}-1}{\eta}\right)^2+1}}\right] & \text{if } k \neq i \\ \ln\left[\dfrac{\left(\frac{4\cdot\xi_i+1}{\eta}\right)+\sqrt{\left(\frac{4\cdot\xi_i+1}{\eta}\right)^2+1}}{\left(\frac{4\cdot\xi_i-1}{\eta}\right)+\sqrt{\left(\frac{4\cdot\xi_i-1}{\eta}\right)^2+1}}\right] & \text{otherwise} \end{cases}$$

$$FI := FI1 + FI2 + FI3 + FI4 + FI5 + FI6$$

The functions $F_{k,i}^{\mathrm{II}}$ and $F_{k,i}^{\mathrm{III}}$ have a simpler form:

$$FII_{k,i} := \begin{cases} \dfrac{2\cdot\xi_i\cdot\left|\xi_i-\zeta_{k,i}\right|}{\left|2\cdot\xi_i-\zeta_{k,i}\right|^3} & \text{if } k \neq i \\ \dfrac{1}{4\cdot\xi_i} & \text{otherwise} \end{cases} \qquad FIII_{k,i} := \begin{cases} -\dfrac{2}{2\cdot\xi_i-\zeta_{k,i}} & \text{if } k \neq i \\ -\dfrac{1}{\xi_i} & \text{otherwise} \end{cases}$$

For a rod fixed at the point 0, the matrix $\overline{\omega}_{k,i}$ expressing the deflections of the k -th point due to the unit force applied at the i -th point is:

$$\omega1_{k,i} := \begin{cases} \left(\frac{a0_k}{c}\right)^2\cdot\left(3\cdot\frac{a0_i}{c}-\frac{a0_k}{c}\right) & \text{if } k \le i \\ \left(\frac{a0_i}{c}\right)^2\cdot\left(3\cdot\frac{a0_k}{c}-\frac{a0_i}{c}\right) & \text{otherwise} \end{cases}$$

For a rod in an elastic environment, the matrix $\delta_{k,i}$ expressing the resulting displacements of the k -th point due to the unit force applied at the i -th point is:

$$\delta_{k,i} := FI_{k,i} + \frac{1}{3-4\cdot\mu}\cdot FII_{k,i} + \frac{\mu\cdot(1-2\cdot\mu)}{3-4\cdot\mu}\cdot FIII_{k,i} + \alpha\cdot\omega 1_{k,i}$$

The matrix of the coefficients MA at the unknown forces X_k, the angles of rotation ϕ_0 and displacements u_0, and the vector of the external impacts VP are the following [6,10,11]:

$$\begin{array}{l} MA := \text{for } k \in 0..(n+1) \\ \quad \text{for } i \in 0..(n+1) \\ \quad\quad M_{k,i} \leftarrow \delta_{k,i} \text{ if } k \le (n-1) \wedge i \le (n-1) \\ \quad\quad M_{k,i} \leftarrow a0_k \text{ if } k \le (n-1) \wedge i = n \\ \quad\quad M_{k,i} \leftarrow a0_i \text{ if } k = n \wedge i \le (n-1) \\ \quad\quad M_{k,i} \leftarrow 1 \text{ if } k \le (n-1) \wedge i = (n+1) \\ \quad\quad M_{k,i} \leftarrow 1 \text{ if } k = (n+1) \wedge i \le (n-1) \\ \quad\quad M_{k,i} \leftarrow 0 \text{ otherwise} \end{array} \qquad \begin{array}{l} VP := \text{for } k \in 0..(n+1) \\ \quad VP_k \leftarrow P\cdot\left(\frac{c}{2} + Lhp\right) + M \text{ if } k = n \\ \quad VP_k \leftarrow -P \text{ if } k = (n+1) \\ \quad VP_k \leftarrow 0 \text{ otherwise} \end{array}$$

By means of multiplying the inverse matrix of the coefficients $-MA^{-1}$ by the vector of the external impacts VP, we obtain the vector of unknowns VX:

$$VX := -MA^{-1}\cdot VP$$

We separate the desired forces X_k, as well as the actual angle of rotation ϕ_0^d, rad, and the actual displacement u_k^d, m, at the point 0 from vector VX:

$$\begin{array}{l} X := \text{for } k \in 0..(n-1) \\ \quad X_k \leftarrow VX_k \end{array}$$

$$\phi 0 := \frac{(1+\mu)\cdot(3-4\cdot\mu)}{8\cdot\pi\cdot E0\cdot(1-\mu)\cdot c}\cdot VX_n \qquad \phi 0 = 2.189\times 10^{-3}$$

$$u0 := \frac{(1+\mu)\cdot(3-4\cdot\mu)}{8\cdot\pi\cdot E0\cdot(1-\mu)\cdot c}\cdot VX_{n+1} \qquad u0 = -2.479\times 10^{-3}$$

Knowing forces X_k, we can compute the actual displacements u_k^d:

$$\begin{array}{l} u := \text{for } k \in 0..(n-1) \\ \quad \text{for } i \in 0..(n-1) \\ \quad\quad u_k \leftarrow \frac{(1+\mu)\cdot(3-4\mu)}{8\cdot\pi\cdot E0\cdot(1-\mu)\cdot c}\cdot\sum_{i=0}^{n-1}\left[X_i\cdot\left[FI_{k,i} + \frac{1}{3-4\mu}\cdot FII_{k,i} + \frac{\mu\cdot(1-2\cdot\mu)}{3-4\mu}\cdot FIII_{k,i}\right]\right] \end{array}$$

The computation results of the displacements of the points through the analytical method u_k^d are given in table 1.

Let us perform a similar analysis of the pile through the Finite Element Method (FEM) applying LYRA 10.6 software package [6,19]. A rod (FE10) to which a load is applied is placed into a massive of soil with the dimensions of 20×20×20 m comprising FE36 solid finite elements.

The general view of the analytical model is given in figure 1. The computation results of the displacements of similar points through the Finite Element Method u_k^{FEM} are given in table 1.

Static loading

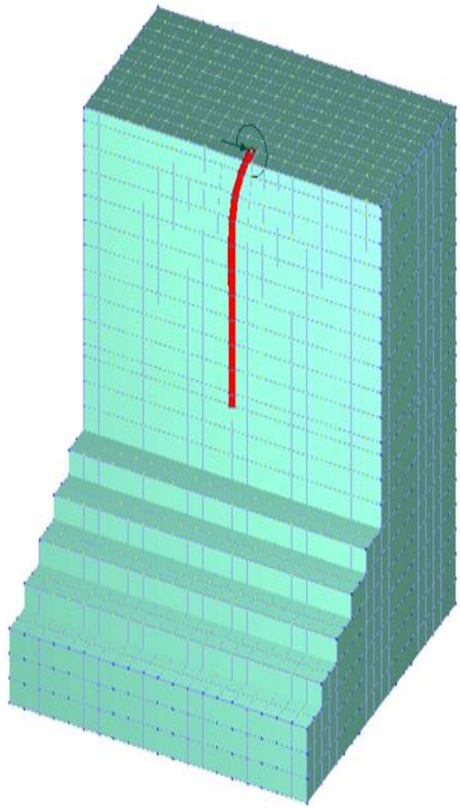

Fig. 1. A deformed analytical model according to the results of the analysis with Lyra 10.6 Software Package (a part of finite elements of soil is hidden) – Static loading [13,15,18].

TABLE I. COMPARISON OF THE COMPUTATION RESULTS OF DISPLACEMENTS

k	0	1	2	3	4	5	6	7	8	9
a_k, m	0,5	1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5	9,5
u_k^d, mm	2,479	0,847	0,131	−0,06	−0,04	0,010	0,052	0,070	0,073	0,069
u_k^{FEM}, mm	1,977	0,639	0,041	−0,12	−0,10	−0,05	−0,01	0,004	0,007	0,005

Comparing the results of the analyses through the analytical method and the finite element method, one can see that the displacements at the key points are quite equitable. The developed software program in Mathcad can be used for practical analysis for the preliminary determination of the section and the length of a pile carrying horizontal loads.

References

[1] B.N. Zhemochkin, Analysis of elastic fixing of rod: (Bending of rod in elastic half-space), Moscow: Stroyizdat, 1948.

[2] E.G. Makarov, Engineering analyses in Mathcad, Teaching course, Saint Petersburg: Piter, 2005.

[3] Building code SP 63.13330.2012, Concrete and reinforced concrete structures, Design requirements. Revised edition. Construction Standards and Regulations SNiP 52-01-2003. Moscow: FAU "FCC", 2012.

[4] Building code SP22.13330.2011, Foundations of buildings and constructions. Revised Edition. Construction Standards and Regulations SNiP 2.02.01-83. Moscow: OAO «CPP», 2011.

[5] Building code SP 24.13330.2011, Pile foundations. Revised Edition. Construction Standards and Regulations. SNiP 2.02.03-85. Moscow: OAO «CPP», 2011.

[6] O.A. Kovalchuk, A.V. Kolesnikov, E.M. Rusanova [et al., Introduction into Lyra 10.4 Software Package: teaching aid, Moscow: NRU MGSU, 2015.

[7] URL: http://lira-soft.com/download/metodpos/index.php .

[8] Valery N. Azarov, Dmitri P. Borovkov, Awadh M. Redhwan, Application of Swirling Flows in Aspiration Systems, International Review of Mechanical Engineering (IREME), Vol. 8, No 4, p. 750, 2014.

[9] O.K. Frelikh, Pressure distribution in soil, Moscow: Stroyizdat, 1933.

[10] B.N. Zhemochkin, A.P. Sinitsyn, Practical methods of analysis of foundation beams and slabs on elastic foundation bed. The second edition revised and enlarged, Moscow: Stroyizdat, 1962.

[11] S.N. Klepikov, Analysis of structures on elastic foundation bed, Kiev: Budivelnik, 1967.

[12] I.A. Simvulidi, Analysis of engineering structures on elastic foundation bed, Vysshaya Shkola, 1973.

[13] M.I. Gorbunov-Posadov, T.A. Malikova, Analysis of structures on elastic foundation bed, Moscow: Stroyizdat, 1973.

[14] S.I. Tsymbal, I.F. Potapenko, A.O. Oleynik, Analysis of pile foundations, Kiev: KCEI, 1990.

[15] S.B. Ukhov and others, Soil mechanics, foundation engineering, Moscow: ASV, 1994.

[16] B.I. Dalmatov, under the editorship, Foundation engineering. Part. 2. Fundamentals of geotechnical engineering, Moscow: ASV Publishing house; SPSUACE, 2002.

[17] M.V. Malyshev, G.G. Boldyrev, Soil mechanics. Foundation engineering (in questions and answers), Teaching aid. Moscow: Publishing house of Civil Engineering Universities Association. 2004.

[18] Z.G. Ter-Martirosyan, V.V. Sidorov, K.Z. Ter-Martirosyan, Creep and long-time load-bearing capacity of long pile immersed into clay soil massive, Vestnik MGSU, № 1, pp. 109—115, 2013.

[19] A.Z. Ter-Martirosyan and others, Settlement and load-bearing capacity of long piles, Vestnik MGSU, Vol. 5, pp. 52–61, 2015.

[20] Z.G. Ter-Martirosyan, Soil mechanics, Moscow: ASV Publishing house, 2009.