Advances in Social Science, Education and Humanities Research, volume 217
2nd International Conference on Social, economic, and academic leadership (ICSEAL 2018)

# Achieving academic leadership through teaching mathematical methods in economics and finance

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**Abstract** This paper is devoted to ensuring the achievement of academic leadership on the market of educational services and for the sustainable development of higher education in the future. It is quite common that various optimization problems with linear constraints on variables often arise in economics and finance.
We present an educational method (in fact, a blueprint of a course) which explains financial and economic problems taking into account modern achievements in mathematical computer modeling. As an example, we outline an educational approach which teaches portfolio investments theory using mathematical optimization methods. The educational approach proposed and formulated in this paper allows us to provide the market with professionals, who possess fluent knowledge of the methods used for investing and construction of an optimal securities portfolio, and, most importantly, are capable of further self-development of their skills and knowledge in this field.

## 1 Introduction

One would probably agree that in order to achieve market leadership in educational services, it is necessary to develop educational programs corresponding and meeting the needs of the modern labor market (Gulicheva and Osipova 2017; Naushad 2018; or Bordean and Sonea 2018)). Currently, there is a growing demand in Russia for qualified specialists who know how to evaluate investments both in securities and assets, which are not traded on financial markets (e.g. direct investments in production companies). One of the goals of the new state educational standards is to teach methods and principles of making investment decisions based on assessment of the economic conditions and situation on the financial markets. It is impossible to properly evaluate those factors without modern mathematical methods, including mathematical optimization methods, econometrics, Big Data and other computer technologies. A modern specialist in economics and finance should have an idea of how those methods work in order to correctly assess and apply the results they produce (Lisin et al. 2014). Thus, it is necessary to teach the specialists to set and solve optimization problems at least on the simplest examples which arise in economics and finance. This can be achieved either by including related topics into the “Mathematics for economists and financiers” course, or by creating a new separate course. In this paper, we consider a mathematical approach to present the portfolio investment theory. The goal of the paper is to provide the know-how on how to obtain the ability to formalize the tasks arising in their professional activity by studying mathematical formalizations and solutions of the specific optimization problems contained in the course.

## 2 Literature review

The new state educational standard adapted in Russia requires graduates of economic universities to have certain competences, including professional ones, and the ability to solve practical problems and assess various situations arising at work (Sukhorukova and Chistyakova 2018a; 2018b; 2018c). The proposed method would provide potential students with skills and competencies for making optimal financial decisions in order to achieve functional and corporate strategy.

By applying high professional standards and skills combined with appropriate methods while constructing an investment portfolio, financial analyst ensures growth in portfolio value for the benefit of the investors. Mathematical reasoning allows students to establish cause-effect relationships in various areas of academic and professional activities, contributes to scientific mindset development and achievement of the necessary general cultural level (see e.g. Strielkowski 2018). Similar educational programs in European countries, USA and Canada are very versatile due to many years of practice and include such topics as direct investments in assets, which are not traded on the financial market, investments in securities (shares, bonds, etc.) and their derivatives (Dodd et al. 2008; Siegel and Shim 2009; Brown and Zima 2011; Carlin and Soskice 2014; Hill, 2015; or Gottesman 2016). In Russian educational institutions, the lecturer is tasked with development of a course exploring the principles and methods of investment valuation. (Akhmadeev and Manakhov 2015; Grishina and Zvonova 2016) The main goal of this paper is to provide theoretical knowledge and skills on investments using mathematical and computer modeling methods (see Pignataro 2013; Minashkin 2014; Sydsaeter 2016; Bobrik and Bobrik 2017; Prado 2018; or Chistyakova and Sukhorukova 2018a; 2018b). It appears important for the students tos understand economic and financial nature mathematical substance of various types of investments taking into account different risks arising from uncertainty in various conditions. The course has both a theoretical and practical component and allows student to apply the knowledge and skills gained to evaluate financial and production assets.

## 3 Subject, objectives and the method of research

Obtaining knowledge and skills to choose possible investments, navigate through methods and mechanisms of investment activity, its regulation, funding and lending to it, is relevant for employees of various corporate economic departments, commercial banks, investors, contractors, and for entrepreneurs. Thus, it is important for any higher educational institution to have a modern educational course on investments, especially in order to achieve a leading position on education market for economic specialties. The task becomes even more complicated given the shortage of textbooks and workbooks on investments and portfolio management which takes into account specifics of investing in Russia (Sukhorukova and Chistyakova 2018b; 2018c). It is difficult to build the course based only on foreign teaching materials and textbooks. The research topic becomes especially relevant taking into account new market specifics such as sanctions imposed by EU countries and USA and other upcoming events. Based on the above the main objective of our study is to improve the methodology of educational programs for investors. The core part of the investor's work is to optimize financial risks associated with the probabilistic outcome of the upcoming events. In this paper we present a theoretical approach of teaching investments courses. This approach allows students majoring in economy to understand basic methods and principles of investments and provides them with the practical guide for achieving the optimal risk-to-return value for the investor. The set of securities chosen for investment is designed to minimize the risk of losses to the investors, while maximizing their income. Low yield of one security can be compensated by high yield of another, while high risk associated with high-yield securities can be compensated by low risk associated with low-yield securities. Simultaneous risk reduction and profitability increase is achieved via diversification of the portfolio, by including securities with zero or negative correlations of returns in the portfolio.

## 4 Results and discussion

In this section we will give the detailed overview of the developed educational program (course), in which classical portfolio management problems are analyzed and solved using modern mathematical and computer modeling methods. The proposed course is developed in accordance with the current Russian legislation, regulatory and other documents applicable to investment activities. The course is constructed to ensure that students, having received knowledge on the topics studied, will be able to independently study various investment processes their future work.

First of all, students need to understand the key term of the course being studied – "investments". While studying the investment process and its circuit, special attention should be paid to its relationship with the capital circuit. It is necessary to gain understanding of the reasons why little funds are invested long-term in Russia, how the government influences investments and conducts its investment policy, what instruments are used by the government to provide with and guarantee investor's rights (especially foreign investors). It is also necessary to study different types of securities. While studying all the topics above, Civil Code of the Russian Federation and other domestic legislative acts should be used as the primary reference point. Strong emphasis should be made on shares and bonds analysis as those are principal securities for long-term investment. Assessment of security's investment qualities should be accompanied with overview of methods used to analyze financial condition of an issuer. It should be well understood that the speculative component has a large influence on the value of a particular financial asset (shares, etc.) traded on the market. Students need to understand the formulas used for evaluation of investment quality and risks associated with investment in securities. Solid mathematical background is required to successfully learn the topics above. It is highly recommended to carefully go through mathematical tools and framework before explaining criteria and methods used for evaluation of investment projects, as students often experience difficulties studying this topic.

Optimization problems with linear constraints on variables often arise in economics and finance. Commonly, functions to be minimized and set of possible variables are convex. Such extremal problems are called convex programming problems. Thus, we include brief overview of the convex optimization theory and theory of portfolio investments in the first part of our course. The second part is dedicated to the mathematical formulation and solution of standard problems from the portfolio investment theory. A portfolio is a set of different investment instruments, which are put together to achieve a specific investment goal. Securities, on the one hand, act as investments themselves, and simultaneously, on the other hand, as instruments or means of raising funds for other investments. In this paper we consider risky securities, with each of them being characterized by the expected return and the average quadratic deviation of the return as the latter is usually considered a quantitative measure of risk. The main objective of the portfolio construction process is to achieve the optimal risk-return ratio for the investors, i.e. the corresponding set of securities is chosen to minimize the risk of losses to the investors, while maximizing their income. Low yield of one security can be compensated by high yield of another, while high risk associated with high-yield securities can be compensated by low risk associated with low-yield securities. Simultaneous risk reduction and profitability increase is achieved via diversification of the portfolio, by including securities with zero or negative correlations of returns in the portfolio.

Let $\overline{r} = (r_1, r_2, \ldots, r_n)$ be a vector of mathematical expectations of securities' returns and $\mathbf{\Lambda}$ be a covariance matrix of those returns

$$\mathbf{\Lambda} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots\cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots\cdots & \sigma_{2n} \\ \vdots & \vdots & \cdots\cdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots\cdots & \sigma_{nn} \end{pmatrix}$$

Here

$\sigma_{ii} = \sigma_i^2$ is the variance of the return of the i-th security,

$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$ is the covariance between the returns of i-th and j-th securities,

$\rho_{ij}$ is the correlation between the returns of i-th and j-th securities.

The portfolio will be determined by the vector of weights corresponding to the securities included in the portfolio: $\overline{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)$. I.e. if the value of the entire portfolio is equal to $\Pi$, then the value of all securities of type i in the portfolio is equal to $\omega_i \cdot \Pi$. We will assume that securities can be bought on the market in any quantities (fractional) and in any size (amount of money). Weights of the securities must satisfy the following criteria

$$\omega_1 + \omega_2 + \cdots + \omega_n = 1, \ \omega_j \geq 0, \ j = 1, \ldots, n.$$

Portfolios meeting those criteria are called standard portfolios, which means that an investor buys assets for the purpose of further sale, or, that he enters into a long position in securities. If so-called short sales (or positions) are allowed, then the conditions of non-negativity are abolished.

We can calculate the variance of the entire portfolio return using the formula below:

$$\sigma^2(\overline{\omega}) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij}\omega_i\omega_j = \sum_{i=1}^{n} \sigma_i^2 \omega_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \sigma_i \sigma_j \rho_{ij} \omega_i \omega_j = \boldsymbol{\omega}^{\mathbf{T}} \mathbf{\Lambda} \boldsymbol{\omega} \cdot$$

Portfolio optimization problem is to find the weights of the securities $\overline{\omega}$, which ensure expected income and risk level in line with investors' objectives.

A risk averse investor would also be interested to know the lowest level of risk for a given set of portfolios. Mathematically, this corresponds to the following convex programming problem (in case short positions are not allowed):

$$\sigma^2(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} \to \min,$$
$$\boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1,\ \omega_j \geq 0,\ j = 1,\ldots,n.$$

If the following notation is used

$$\varphi(\overline{\omega}) = \boldsymbol{\omega}^T\boldsymbol{\Lambda}\boldsymbol{\omega},\quad f_i(\overline{\omega}) = -\omega_i,\ i = 1,\ldots,n;\quad f_{n+1}(\overline{\omega}) == \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} - 1,$$

the problem can be rewritten as follows:

$$\varphi(\overline{\omega}) \to \min,$$
$$f_i(\overline{\omega}) \leq 0,\ i = 1,\ldots,n;$$
$$f_{n+1}(\overline{\omega}) = 0.$$

Optimal solution can be found by solving the system of equations

$$\begin{cases} 2\boldsymbol{\Lambda}^{\mathbf{1}}\boldsymbol{\omega} - \mu_1 - \mu_{n+1}r_1 + \mu_{n+2} = 0, \\ 2\boldsymbol{\Lambda}^{\mathbf{2}}\boldsymbol{\omega} - \mu_2 - \mu_{n+1}r_2 + \mu_{n+2} = 0, \\ \vdots \\ 2\boldsymbol{\Lambda}^{\mathbf{n}}\boldsymbol{\omega} - \mu_n - \mu_{n+1}r_n + \mu_{n+2} = 0, \\ \mathbf{r}^T\boldsymbol{\omega} \geq r,\ \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1, \\ \mu_i\omega_i = 0,\quad i = 1,\ldots,n,\ \mu_{n+1}(r - \mathbf{r}^T\boldsymbol{\omega}) = 0, \\ \omega_j \geq 0,\ j = 1,\ldots,n,\ \mu_i \geq 0,\ i = 1,\ldots,n+1 \end{cases}$$

where $\boldsymbol{\Lambda}^{\mathbf{1}},\boldsymbol{\Lambda}^{\mathbf{2}},\ldots,\boldsymbol{\Lambda}^{\mathbf{n}}$ are the rows of the covariance matrix $\boldsymbol{\Lambda}$.

The nonlinear part of the system is solved by considering a number of cases, where variables from a chosen subset are set at zero. In each such case number of variables in the linear part of the system equal the number of equations. If the covariance matrix is invertible, then the matrix corresponding to the resulting linear system is also invertible and thus the system of linear equations has the solution (only one solution). If the solution found satisfy other criteria (outlined by inequalities), then the optimal solution to the initial problem is found, if not, other case should be analyzed.

Now, let's consider the problem of minimizing the risk of the portfolio for a given level of profitability (the Markowitz model). This problem is a convex programming problem with a quadratic objective function and linear constraints:

$$\sigma(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} \to \min,$$
$$\boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1,\ \omega_j \geq 0,\ j = 1,\ldots,n,$$
$$r(\overline{\omega}) == \mathbf{r}^T\boldsymbol{\omega} \geq r,$$

where r is given and represent minimal profitability requirement.

To find the optimal solution of the problem it is necessary and sufficient to solve the system of equations

$$\begin{cases} 2\boldsymbol{\Lambda}^{\mathbf{1}}\boldsymbol{\omega} - \mu_1 - \mu_{n+1}r_1 + \mu_{n+2} = 0, \\ 2\boldsymbol{\Lambda}^{\mathbf{2}}\boldsymbol{\omega} - \mu_2 - \mu_{n+1}r_2 + \mu_{n+2} = 0, \\ \vdots \\ 2\boldsymbol{\Lambda}^{\mathbf{n}}\boldsymbol{\omega} - \mu_n - \mu_{n+1}r_n + \mu_{n+2} = 0, \\ \mathbf{r}^T\boldsymbol{\omega} \geq r,\ \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1, \\ \mu_i\omega_i = 0,\quad i = 1,\ldots,n,\ \mu_{n+1}(r - \mathbf{r}^T\boldsymbol{\omega}) = 0, \\ \omega_j \geq 0,\ j = 1,\ldots,n,\ \mu_i \geq 0,\ i = 1,\ldots,n+1 \end{cases}$$

The next problem being considered is maximization of portfolio profitability for a given level of risk (the Tobin model). Corresponding mathematical problem is a convex programming problem with a linear objective function and quadratic constraints:

$$r(\overline{\omega}) = \mathbf{r}^{\mathbf{T}}\boldsymbol{\omega} \to \max,$$
$$\boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1,\quad \omega_j \geq 0,\ j = 1,\ldots,n,$$
$$\sigma^2(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} \leq \sigma^2,$$

where $\sigma^2$ is given and represents maximal risk level that can be accepted.

If the following notation is used

$$\varphi(\overline{\omega}) = -\mathbf{r}^{\mathbf{T}}\boldsymbol{\omega},\quad f_i(\overline{\omega}) = -\omega_i,\ i = 1,\ldots,m;$$

$$f_{n+1}(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} - \sigma^2, \quad f_{n+2}(\overline{\omega}) = \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} - 1.$$

the problem can be rewritten as follows:

$$\varphi(\overline{\omega}) \to \min,$$
$$f_i(\overline{\omega}) \le 0,\ i = 1,\dots,m;$$
$$f_{n+1}(\overline{\omega}) \le 0;$$
$$f_{n+2}(\overline{\omega}) = 0.$$

To find the optimal solution of the problem it is necessary and sufficient to solve the system of equations:

$$\begin{cases} 2\mu_{n+1}\boldsymbol{\Lambda}^{\mathbf{1}}\boldsymbol{\omega} - \mu_1 - r_1 + \mu_{n+2} = 0, \\ 2\mu_{n+1}\boldsymbol{\Lambda}^{\mathbf{2}}\boldsymbol{\omega} - \mu_2 - r_2 + \mu_{n+2} = 0, \\ \\ 2\mu_{n+1}\boldsymbol{\Lambda}^{\mathbf{n}}\boldsymbol{\omega} - \mu_n - r_n + \mu_{n+2} = 0, \\ \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} - \sigma^2 \le 0,\ \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} - 1 = 0, \\ \mu_i\omega_i = 0,\ i = 1,\dots,n,\ \mu_{n+1}(\boldsymbol{\omega}^{\mathbf{T}}\Lambda\boldsymbol{\omega} - \sigma^2) = 0, \\ \omega_j \ge 0,\ j = 1,\dots,n,\ \mu_i \ge 0,\ i = 1,\dots,n+1. \end{cases}$$

Also, one of the most relevant problems is to construct the effective border for a multitude of investment opportunities.

Let's put each portfolio $\overline{\omega}$ from a set of feasible portfolios $V$ on the “risk-return” plane with coordinates $(\sigma(\overline{\omega}), r(\overline{\omega}))$. The set of all such points is called the set of investment opportunities and is denoted as $\mathrm{M}(V)$. If such set corresponds to a standard portfolio, then it is bounded and:

$$\min_{i=1,\dots n} r_i \le r \le \max_{i=1,\dots n} r_i \cdot$$

Portfolios that minimize risk for the given profitability are called effective portfolios. The set of all such portfolios is called the effective set. The return of such portfolios must satisfy criteria:

$$r \ge r^*(V),$$

where $r^*(V)$ represents the return of the portfolio with the lowest risk among the given feasible portfolios. On the “risk-return” plane each effective set of portfolios defines the effective border $\Gamma(V)$ of the set of investment opportunities $\mathrm{M}(V)$, which consists of the points $(\sigma_{\min}(r); r)$ where

$$r \in S(V) = \{r \mid ((\sigma; r) \in \mathrm{M}(V); r \ge r^*(V)\}.$$

The parametric convex programming problem needs to be solved in order to find the effective border of the set of investment opportunities (for the portfolios with prohibited short positions):

$$\sigma(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega} \to \min,$$
$$\boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} = 1,\ \omega_j \ge 0,\ j = 1,\dots,n,$$
$$r(\overline{\omega}) = \mathbf{r}^{\mathbf{T}}\boldsymbol{\omega} = r \ ,$$
$$r \in S(\Omega^+) \cdot$$

If the following notation is used:

$$\varphi(\overline{\omega}) = \boldsymbol{\omega}^{\mathbf{T}}\boldsymbol{\Lambda}\boldsymbol{\omega},\ f_i(\overline{\omega}) = -\omega_i,\ i = 1,\dots,m,$$
$$f_{n+1}(\overline{\omega}) = \boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\omega} - 1,\ f_{n+2}(\overline{\omega}) = \mathbf{r}^{\mathbf{T}}\boldsymbol{\omega} - r \cdot$$

the problem can be rewritten as follows:

$$\varphi(\overline{\omega}) \to \min,$$
$$f_i(\overline{\omega}) \le 0,\ i = 1,\dots,m,$$
$$f_{n+1}(\overline{\omega}) = 0,$$

$$f_{n+2}(\overline{\omega}) = 0.$$

This problem satisfies the conditions of the global minimum criteria for a convex function and conditions of the Kuhn-Tucker theorem. Thus, we'll use the Kuhn-Tucker theorem to find the solution. Lagrange function corresponding to the problem is composed:

$$L(\overline{\omega}, \overline{\mu}) = \varphi(\overline{\omega}) + \sum_{i=1}^{n+2} \mu_i f_i(\overline{\omega}) =$$
$$= \boldsymbol{\omega}^{\mathbf{T}} \boldsymbol{\Lambda} \boldsymbol{\omega} - \sum_{i=1}^{n} \mu_i \omega_i + \mu_{n+1}(\boldsymbol{\varepsilon}^{\mathbf{T}} \boldsymbol{\omega} - 1) + \mu_{n+2}(\mathbf{r}^T \boldsymbol{\omega} - r).$$

In order to find the optimal solution of the problem it is necessary and sufficient to solve the system of equations:

$$\begin{cases} 2\boldsymbol{\Lambda}^{\mathbf{1}}\boldsymbol{\omega} - \mu_1 + \mu_{n+1} + \mu_{n+2} r_1 = 0, \\ 2\boldsymbol{\Lambda}^{\mathbf{2}}\boldsymbol{\omega} - \mu_2 + \mu_{n+1} + \mu_{n+2} r_2 = 0, \\ \\ 2\boldsymbol{\Lambda}^{\mathbf{n}}\boldsymbol{\omega} - \mu_n + \mu_{n+1} + \mu_{n+2} r_n = 0, \\ \omega_1 + \omega_2 + \cdots + \omega_n = 1, \ \mathbf{r}^{\mathbf{T}}\boldsymbol{\omega} = r, \\ \mu_i \omega_i = 0, \ i = 1, \ldots, n, \ \omega_j \geq 0, \ j = 1, \ldots, n, \ \mu_i \geq 0, \ i = 1, \ldots, n. \end{cases} \tag{1}$$

In the case short positions are allowed, the system of equations (1) will have the form

$$\begin{cases} 2\sum_{j=1}^{n} \sigma_{1j}\omega_{j_1} + \mu_{n+1} + \mu_{n+2} r_1 = 0, \\ 2\sum_{j=1}^{n} \sigma_{2j}\omega_{j_2} + \mu_{n+1} + \mu_{n+2} r_2 = 0, \\ \vdots \\ 2\sum_{j=1}^{n} \sigma_{nj}\omega_{j} + \mu_{n+1} + \mu_{n+2} r_n = 0, \\ \omega_1 + \omega_2 + \cdots + \omega_n = 1, \\ \sum_{j=1}^{n} \omega_j r_j = r. \end{cases} \tag{2}$$

The number of variables is equal to the number of equations, and thus, if the covariance matrix is invertible, and if at least two returns do not match each other, then the system has the solution (at only one). Let $\mathbf{A_0}$ denote the matrix corresponding to the system (2), and $\mathbf{X_0}$ represent the vector of variables. Then $\mathbf{X_0} = \mathbf{A_0^{-1}B}$, where $\mathbf{B} = (0 \quad \ldots \quad 0 \quad 1 \quad r)^{\mathbf{T}}$ is the right-hand side of system (2). We represent the vector $B$ as

$$\mathbf{B} = \mathbf{B_1} + \mathbf{B_2} r, \ \mathbf{B_1} = (0 \quad 0 \quad 0 \quad 1 \quad 0)^{\mathbf{T}}, \mathbf{B_2} = (0 \quad 0 \quad 0 \quad 1 \quad 0)^{\mathbf{T}}.$$

Then:

$$\mathbf{X_0} = \mathbf{A_0^{-1}B_1} + \mathbf{A_0^{-1}B_2} r = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ a_{\mu(n+1)} \\ a_{\mu(n+2)} \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \\ b_{\mu(n+1)} \\ b_{\mu(n+2)} \end{pmatrix} r.$$

The latter means that if short positions are allowed then the effective set of portfolios can be described as:

$$\overline{\omega}(r) = (a_1 + b_1 r, \ a_2 + b_2 r, \ldots, a_n + b_n r),$$
$$r \geq r^*(\Omega_n) \tag{3}$$

and the numbers $a_i, b_i, \ i = 1, \ldots, n+2,$ do not depend on r (are constant for each r).

Then the effective boundary will be defined by the equation

$$\sigma = (Ar^2 + Br + C)^{1/2},$$
$$r \geq r^*(\Omega_n),$$

where

$$A = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}b_i b_j = \mathbf{b}^{\mathbf{T}}\mathbf{\Lambda b},$$
$$B = 2\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}a_i b_j = 2\cdot\mathbf{a}^{\mathbf{T}}\mathbf{\Lambda b} = 2\cdot\mathbf{b}^{\mathbf{T}}\mathbf{\Lambda a},$$
$$C = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}a_i a_j = \mathbf{a}^{\mathbf{T}}\mathbf{\Lambda a},$$
$$\mathbf{a}^T = \begin{pmatrix} a_1 & a_2 & \dots & a_n \end{pmatrix},$$
$$\mathbf{b}^{\mathbf{T}} = \begin{pmatrix} b_1 & b_2 & \dots & b_n \end{pmatrix}. \tag{4}$$

With prohibited short positions and in case $\mu_1 = 0$ , $\mu_2 = 0$ ..., $\mu_n = 0$ the matrix of the linear part of system (1) equals the matrix of system (2) for the problem with allowed short positions. Therefore, effective portfolios have the form (3), but the non-negativity conditions and restriction on profitability must be satisfied:

$$a_1 + b_1 r \geq 0,\ a_2 + b_2 r \geq 0, \dots,\ a_n + b_n r \geq 0,$$
$$\min_{i=1,\dots n} r_i \leq r \leq \max_{i=1,\dots n} r_i,$$
$$r \geq r^*(\Omega_n^+).$$

Let's consider the case $\omega_1 = 0$ , $\mu_2 = 0$ , ..., $\mu_n = 0$ . In this case, the linear part of the system (4) takes the following form:

$$\begin{cases} 2\sum_{j=2}^{n}\sigma_{1j}\omega_j - \mu_1 + \mu_{n+1} + \mu_{n+2}r_1 = 0, \\ 2\sum_{j=2}^{n}\sigma_{2j}\omega_j + \mu_{n+1} + \mu_{n+2}r_2 = 0, \\ \vdots \\ 2\sum_{j=2}^{n}\sigma_{nj}\omega_j + \mu_{n+1} + \mu_{n+2}r_n = 0, \\ \omega_1 + \omega_2 + \dots + \omega_n = 1, \\ \sum_{j=1}^{n}\omega_j r_j = r. \end{cases} \tag{5}$$

Let $\mathbf{A_1}$ denote the matrix corresponding to the system (5), and $\mathbf{X_1}$ represent a vector of variables. Then:

$$\mathbf{X_1} = \mathbf{A_1^{-1}B} = \mathbf{A_1^{-1}}\begin{pmatrix}0\\0\\\vdots\\0\\1\\0\end{pmatrix} + \mathbf{A_1^{-1}}\begin{pmatrix}0\\0\\\vdots\\0\\0\\1\end{pmatrix} r = \begin{pmatrix} a_2^{(1)} \\ a_3^{(1)} \\ \vdots \\ a_n^{(1)} \\ a_{\mu 1}^{(1)} \\ a_{\mu(n+1)}^{(1)} \\ a_{\mu(n+2)}^{(1)} \end{pmatrix} + \begin{pmatrix} b_2^{(1)} \\ b_3^{(1)} \\ \vdots \\ b_n^{(1)} \\ b_{\mu 1}^{(1)} \\ b_{\mu(n+1)}^{(1)} \\ b_{\mu(n+2)})^{(1)} \end{pmatrix} r, \tag{6}$$

$$\overline{\omega}^{(1)} = (0,\ a_2^{(1)} + b_2^{(1)}r,\ \dots, a_n^{(1)} + b_n^{(1)}r),\quad r \in S_1,$$
$$S_1 = \left\{ r \left| \begin{array}{l} a_{\mu 1}^{(1)} + b_{\mu 1}^{(1)}r \geq 0,\ a_2^{(1)} + b_2^{(1)}r \geq 0, \dots,\ a_n^{(1)} + b_n r^{(1)} \geq 0, \\ \min_{i=1,\dots n} r_i \leq r \leq \max_{i=1,\dots n} r,\ \ r \geq r^*(\Omega_n^+) \end{array} \right. \right\}$$

Having considered all the cases, one finds that:

$$S(\Omega_n^+) = S_0 \cup S_1 \cup \dots \cup S_k,$$

Where the non-empty sets from the equation above have at most one intersection point and each non-empty set corresponds to respective part of the effective boundary defined by equation:

$$\sigma = \left(A^{(i)} r^2 + B^{(i)} r + C^{(i)}\right)^{1/2}, \quad (7)$$
$$r \in S_i,$$

where $A^{(i)}$, $B^{(i)}$, $C^{(i)}$ are defined similarly to (4):

$$\overline{\omega}^{(i)} = (a_1{}^{(i)} + b_1{}^{(i)} r, \ldots, a_{i-1}{}^{(i)} + b_{i-1}{}^{(i)} r, 0, \ldots, a_n{}^{(i)} + b_n{}^{(i)} r). \quad (8)$$

## 5 Conclusions

Overall, it seems that the skills, knowledge and concrete methods of evaluation and optimization of the portfolio of securities learned by students during the course will definitely be useful in working practice of employees of various corporate economic departments, commercial banks, investors, contractors, and for risk-managers, when making management decisions. The proposed mathematical and computer modeling approach of solving investment problems allows students to build skills needed to construct portfolio of securities.

The proposed approach of teaching investments courses provides students majoring in economy with the practical guide to achieve the optimal risk-to-return ratio for an investor. The set of securities chosen for investment is designed to minimize the risk of losses for the investors, while maximizing their income.

The practical examples and problems used in the course allow students to receive a broad understanding of the methods and principles used in investments. Low yield of one security can be compensated by high yield of another, while high risk associated with high-yield securities can be compensated by low risk associated with low-yield securities. Simultaneous risk reduction and profitability increase is achieved via diversification of the portfolio, by including securities with zero or negative correlations of returns in the portfolio.

The teaching approach proposed and elaborated in this paper and the base of practical problems developed for the course allow us to carefully select and to saturate the market with professionals who have fluent knowledge of the methods used in investments and portfolio management and are capable of further self-development in this field.

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