Research on Attribute Reduction Algorithm Based on Extended Center Dominance Relation

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Abstract: The attributes reduction of incomplete interval-valued information system is a popular research topic in recent years. How to conduct efficient and quick attribute reduction is the target of many scholars. Based on the existing research work, with mean substitution, this paper tries to impute the incomplete interval-valued information system, easily distinguish out the dominance relation between two interval values by extended center dominance relation, and improve the classical rough set. The comparative analysis of examples shows that the extended center dominance relation and mean substitution can solve the problem in incomplete interval-valued information system with higher efficiency and more rapid calculation.

1. Introduction

Polish mathematician Pawlak first proposed the rough set in 1982. It was the ability to deal with some unclear phenomenon with incomplete information, or to classify data according to certain inaccurate results observed and measured. Recently, rough set has already achieved successful applications in information system analysis, artificial intelligence, decision support, knowledge discovery, pattern recognition, and fault detection. However, the classical rough set is only suitable for the complete information system, not suitable for the incomplete information system which contains a lot of noise in real life [1-2]. Later, Greco put forward the dominance relation of rough set and obtained the attribute reduction of rough set based on dominance relation [3]. So far, many domestic and foreign scholars have proposed the confidence dominance relation, α dominance relation, similarity dominance relation and so on. These methods perform attribute reduction of incomplete information system of rough set from different angles, but how to rapidly and efficiently solve attribute reduction is still a topic which is worthy of further research.

There are also many researches on incomplete interval-valued information systems in recent years. Reference [4] proposed a method of attribute reduction in rough set model of α dominance relation, however, the probability of this method of x better than y is too vague, inconsistent with the reality. Reference [5] put forward the method of limiting the dominance relation. The rules of dominance relation are too strict and easy to cause the object misclassification. Reference [6] proposed the attribute reduction of interval-valued information system based on α-β dominance relation. However, the determination of β in this article is too complicated, not suitable for the practical application. Reference [7] presented the attribute reduction of incomplete interval-valued information system, and performed attribute reduction with the extending probability dominance relation.

On the basis of above work, for the large computation and low efficiency in attribute reduction of incomplete interval-valued information system and other issues, this paper presents mean substitution, center dominance relation and attribute reduction based on discernible matrix. Mean substitution can convert the incomplete interval-valued information system into the complete interval-valued information system, and this method can effectively avoid the larger or smaller interval value led by traditional methods when substituting the incomplete interval. As two intervals exist in intersection, disjoint relation and properly including, the center dominance relation promotes the decision process of the dominance relation, and analyses and verifies with specific examples.

2. Basic Concept

Attribute values in the interval-valued information system exist in the form of interval numbers. Let R be a set of real numbers, a real range is called the interval number, abbreviated as $a=[a_l,a_h]$, where in $a_l,a_h \in R, a_l \le a_h$. If $a_l = a_h$, then $a=[a_l,a_h]$ degenerates into the certain real number. Interval number involves a lot of basic operations. Here are the formal definitions of interval-valued information system.

2.1. Definition 1 [8]

Suppose interval-valued information system $\xi=(U,AT,V,f)$, wherein domain $U=\{x_1,x_2 \cdot\cdot,x_n\}$ is a nonempty finite set of objects. $AT=\{a_1,a_2,\dots,a_m\}$ represents the set of all the attributes. $V=\bigcup_{a\in A}V_a$, V_a represents the range values of attribute *a*, the interval value of *x* under attribute *a*. For $\forall x\in U$, $\forall a\in AT$, there are $f(x,a)=\left[a^L(x),a^U(x)\right]$, $a^L(x),a^U(x)\in R$ and $a^L(x)\le a^U(x)$, $a^L(x)$ and $a^U(x)$ are the lower and upper limit of the interval number $f(x,a)$. Specially, when $a^L(x)=a^U(x)$, $f(x,a)$ degenerate into a real number. Therefore, it is considered that a single-valued information system is a special case of interval-valued information system.

In interval-valued information system, $V=\bigcup_{a\in A}V_a\bigcup\{*\}$, special symbol * represents unknown attribute values. Under the attribute *a*, there are three forms of unknown interval values of object *x*, $f(x,a)=\left[a^L(x),*\right]$, $f(x,a)=\left[*,a^U(x)\right]$, $f(x,a)=\left[*^L,*^U\right]$. Such interval-valued information system is called incomplete interval-valued information system [9].

The interval sequence is described as follows [10]. Suppose $C=[l,\ u]$ is the bounded closed interval. If $l,u\in R$, then $C=[l,\ u]$ is the interval number. The whole interval number of R on the real number set is denoted as I_R, then, $I_R=\{[l,u]\,|\,l<u,l,u\in R\}$ when $l=u$ then this interval degenerate into a real number. Record $m(C)=\frac{1}{2}(l+u),\omega(C)=\frac{1}{2}(u-l)$ are the center and radius of $[l,\ u]$. Then this interval sequence $[l,\ u]$ can be expressed as $C=\langle m(C),\omega(C)\rangle$. Therefore, the interval sequence is defined as follows.

2.2. Definition 2 [10]

Suppose $C,D\in I_R$, if $m(C)\le m(D)$ or when $m(C)=m(D)$, or $\omega(C)\ge\omega(D)$, shown in Equation (1).

$$C\preceq D \Leftrightarrow \begin{cases} m(C)\le m(D) \\ m(C)=m(D),\omega(C)\ge\omega(D) \end{cases} \tag{1}$$

The order relation $\preceq$ is the extension of real number sequence relation $\le$ in the interval numbers. However, the interval-valued information system always has a strong preference relation (represented by "$\succeq$") in the practical applications, so the pros and cons relation exists between objects. Thus, when extracting decision rules, it is necessary to consider the possible partial ordering relation between objects.

3. Center Dominance Relation in Interval-valued Information System

In a given interval-valued information system $\xi=(U,AT,V,f)$, $A\subseteq AT$, for any $x,y\in U$, $a\in A$, $a(x)=[a,b]$, $a(y)=[c,d]$, obviously $a<b$, $c\le d$, if $a=b,c=d$, then the interval degenerate into a point. Here provide $a<b,c<d$. According to Reference [18], if y is superior to x,then it must satisfies $m(y)\ge m(x)$.

3.1. Definition 3

In a given interval-valued information system $\xi=(U,AT,V,f)$, for any attribute subset $A\subseteq AT$, then the definitions of dominance relation $R_A^{\ge}$ on A and the dominance class $[x]_A^{\ge}$ are shown in Equation (2) and (3).

$$R_A^{\ge}=\{(y,x)\in U\times U \mid m_a(y)\ge m_a(x),\forall a\in A\} \tag{2}$$

$$[x]_A^{\ge}=\{y\in U \mid (y,x)\in R_A^{\ge}\} \tag{3}$$

3.2. Theorem1

In a given interval-valued information system $\xi=(U,AT,V,f)$, for $\forall A\subseteq AT$, then there is,

(1) $R_A^{\ge}=\bigcap_{a\in A} R_a^{\ge}$

(2) If $C\subseteq B\subseteq A$, then $R_A^{\ge}\subseteq R_B^{\ge}\subseteq R_C^{\ge}$

(3) If $C\subseteq B\subseteq A$, $\forall x\in U$, then $[x]_A^{\ge}\subseteq[x]_B^{\ge}\subseteq[x]_C^{\ge}$

(4) $R_A^{\ge}$ meets reflexivity, transitivity, but does not meet the symmetry.

3.3. Definition 4

Set interval-valued information system $\xi=(U,AT,V,f)$, any attribute subset $A\subseteq AT$, $\forall x\in U$, then called $\underline{R_A^{\ge}}(X)=\{x\in U \mid [x]_A^{\ge}\subseteq X\}$, $\overline{R_A^{\ge}}(X)=\{x\in U \mid [x]_A^{\ge}\cap X\ne\varnothing\}$ are the lower approximation and upper approximation of X on dominance relation $R_A^{\ge}$, the boundary region is denoted in Equation (4).

$$Bn_A^{\ge}(X)=\overline{R_A^{\ge}}(X)-\underline{R_A^{\ge}}(X) \tag{4}$$

4. Transform Incomplete Information System into Complete Information System.

Due to information loss, omission, measurement error, data noise and transmission medium malfunction, the incompleteness of information is inevitable, so it is necessary to study the incomplete information system. In fact, there are already many researches on incomplete information system. Some of the basic concepts of incomplete interval-valued information systems have already been described, and the incomplete information system is containing information system with parts of unknown attributes. These unknown attributes in this paper are considered missing but real. For unknown attributes, the set or interval of all possible values can be used in this attribute domain to indicate, which will convert the incomplete information system into the complete information system to process.

In view of the processing of incomplete information system, Reference [11] proposed that the upper limit dominance relation substitutes the incomplete interval system into the complete information system, and then performs attribute reduction. The improvements are shown in detail.

1) If $a(x)=\left[a^{L}(x),*\right]$, takes $*=\max\left\{a^{L}(x),\max\limits_{x\in U}\left\{a^{U}(x)\right\}\right\}$

2) If $a(x)=\left[*,a^{U}(x)\right]$, takes $*=\min\left\{a^{U}(x),\min\limits_{x\in U}\left\{a^{L}(x)\right\}\right\}$

3) If $a(x)=\left[*^{L},*^{U}\right]$, takes $*^{L}=\min\limits_{x\in U}\left\{a^{L}(x)\right\},*^{U}\max\limits_{x\in U}\left\{a^{U}(x)\right\}$

The proposed dominance relation is shown in Equation (5).

$$R_{A}^{\geq}=\left\{(x,y)\in U\times U\mid a^{U}(x)\geq a^{U}(y),\forall a\in A\right\} \tag{5}$$

Among them, $a^{L}(x),a^{U}(x)$ are the values of the lower limit value and the upper limit value of the interval.

This method can transform the incomplete interval into the complete interval, but it has a great defect. Because the value of ∗ is either the maximum value of the entire known interval attribute values, or the minimum value of the entire interval attribute values, or probably both of these two. This not only leads to the expansion of the interval, but also has a great impact on the classification decision, which directly leads to the greatly increasing possibility of unknown interval classification error. Furthermore, there is also problem in dominance relation proposed in Reference [11], which only compared the dominance relation between the upper limit values of two intervals and it will produce the classification error. For example, Reference [2,9] and [7,8] proposed the dominance relation based on Reference[11], [2,9] is superior to [7,8], but actually, [7,8] is superior to [2,9]. Therefore, this paper proposes the method of attribute means to avoid the rapid expansion of the interval values of unknown attributes, to reduce the classification errors and to improve the classification accuracy. The specific improvements are described.

1) If $a(x)=\left[a^{L}(x),*\right]$, takes $*=\max\left\{a^{L}(x),\frac{1}{N}*\sum_{i=1}^{N}a^{U}(x_i)\right\},N=n-\left|a^{U}(x)=*\right|$

2) If $a(x)=\left[*,a^{U}(x)\right]$, takes $*=\min\left\{a^{U}(x),\frac{1}{N}*\sum_{i=1}^{N}a^{L}(x_i)\right\},N=n-\left|a^{L}(x)=*\right|$

3) If $a(x)=\left[*^{L},*^{U}\right]$, takes $*^{L}=\frac{1}{N_1}\sum_{i=1}^{N_1}a^{L}(x_i),N_1=n-\left|a^{L}(x)=*\right|,$

$$*^{U}=\frac{1}{N_2}\sum_{i=1}^{N_2}a^{U}(x_i),N_2=n-\left|a^{U}(x)=*\right| \tag{6}$$

In Equation (6), $a^{L}(x),a^{U}(x)$ represent the lower limit value and the upper limit value of the interval. $|\bullet|$ represents cardinality of the set. Such algorithm can transform incomplete interval-valued information system into the complete interval-valued information system, which is called mean substitution of incomplete interval-valued information system in this paper and called mean substitution for short.

5. Attribute Values Reduction

Attribute reduction is to delete unimportant or irrelevant redundant attributes by keeping classification ability unchanged. In general, the attribute reduction of knowledge base is not unique. Calculating the optimal attribute reduction has already been proved to be a difficult NP problem. In practical applications, we can only seek near optimal algorithm to obtain second-best solution so that rules are more concise and more suitable.

Attribute values reduction is the reduction of decision rules, and the reduction of decision rules is to delete unnecessary condition of each decision rule in decision algorithm by using decision logic. It is not an overall reduction of attributes, but rather for each decision rule. It is not to perform reduction of attribute in the whole, but to delete redundant attribute values for each decision rule to further make the decision algorithm minimized.

5.1 Definition 6[12]

Set $\xi = (U, AT, V, f)$ is given interval-valued information system, arbitrary attribute subset $A \subseteq AT$, $R_{AT}^{\geq}$ is the dominance relation on attribute complete set *AT*, $R_{AT}^{\geq}$ is the dominance relation on attribute subset *A*. If $R_{AT}^{\geq} = R_{AT}^{\geq}, R_A^{\geq} \neq R_B^{\geq} (B \subset A)$, then called *A* as attribute reduction of interval-valued information system and represented by *red (AT)*.

The core of the attribute value is: for $\forall a \in AT$, if $R_{AT-\{a\}}^{\geq} = R_{AT}^{\geq}$, then the attribute *a* is unnecessary. If $R_{AT-\{a\}}^{\geq} \neq R_{AT}^{\geq}$, then *a* is necessary. All the attribute sets of the dominance relation $R_{AT}^{\geq}$ constitute the core. Visibly, core is the intersection of all attribute reduction, and denoted as $Core(AT) = \bigcap red(AT)$.

5.2 Definition 7[12]

In a given interval-valued information system $\xi = (U, AT, V, f)$, $\forall x, y \in U$, the discernible matrix of the given interval-valued information system is $D = [Dis_{AT}(x, y)]$, thereinto,

$$Dis_{AT}(x, y) = \begin{cases} \{a \in AT (x, y) \notin R_a^{\geq}\}, (x, y) \notin R_{AT}^{\geq} \\ \varnothing, else \end{cases} \tag{7}$$

Thus, $Dis_{AT}(x, y)$ is all different attributes sets of *x* and *y* on dominance relation $R_{AT}^{\geq}$.

5.3 Definition 8[12]

In a given interval-valued information system $\xi = (U, AT, V, f)$, $Dis_{AT}(x, y)$ is the discernible attribute set of dominance relation $R_{AT}^{\geq}$, then $M = \wedge\{\vee\{a \mid a \in Dis_{AT}(x, y)\} : x, y \in U\}$ is called the discernible function. So, the final result of attribute reduction is to obtain the minimal disjunctive normal form of *M*.

5.4 The Specific Calculation Steps

The specific calculation steps are detailed.

Step 1: By means of the method of mean substitution, the incomplete interval-valued information system is transformed into the complete interval-valued information system to obtain the results.

Step 2: Conclude dominance class of dominance relation $R_{AT}^{\geq}$.

Step 3: Deduce the discernible matrix according to the dominant class.

Step 4: With discernible function, solve the minimal disjunctive normal form, which is attribute reduction.

6. Case Analysis

6.1 Case

Table 1 gives an incomplete interval-valued information system, therein $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$, $AT = \{a_1, a_2, a_3, a_4, a_5\}$, then,

Table 1 Incomplete Interval-valued Information System

U	a_1	a_2	a_3	a_4	a_5
X_1	[2.2,2.9]	[2.5,*]	[5.3,7.2]	[3.2,4.0]	[2.5,3.1]
X_2	[3.4,4.9]	[3.4,4.9]	[7.2,10.5]	[4.0,5.8]	[3.2,4.7]
X_3	[1.8,2.7]	[1.8,3.0]	[7.2,10.3]	[3.0,4.1]	[2.1,2.8]
X_4	[1.3,2.2]	[1.4,2.1]	[*,3.9]	[1.9,2.6]	[1.7,2.3]
X_5	[*,*]	[3.4,5.1]	[6.3,10.3]	[3.8,5.7]	[*,5.7]
X_6	[2.3,3.4]	[2.4,3.3]	[6.7,8.8]	[*,*]	[3.0,3.8]
X_7	[2.2,3.0]	[2.2,2.9]	[4.3,*]	[2.7,3.7]	[2.4,3.2]
X_8	[2.5,4.0]	[2.5,4.1]	[7.1,11.3]	[4.4,6.9]	[3.1,4.7]
X_9	[1.2,*]	[1.4,1.9]	[3.8,4.3]	[2.1,3.0]	[1.7,2.3]
X_{10}	[1.0,1.7]	[1.1,1.8]	[3.6,5.7]	[1.7,2.5]	[1.1,1.8]

Step 1: First of all, by means of the method of mean substitution, transform the incomplete interval-valued information system into the complete interval -valued information system, shown in Table 2.

For elements x_5, there is Equation (8).

$$*^l = \frac{1}{9}(2.2+3.4+1.8+1.3+2.3+2.2+2.5+1.2+1.0) = 2.0 \tag{8}$$

$$*^U = \frac{1}{8}(2.9+4.9+2.7+2.2+3.4+3.0+4.0+1.7) = 3.1$$

For elements x_9, there is Equation (9).

$$*^U = \frac{1}{9}(2.9+4.9+2.7+2.2+3.1+3.4+3.0+4.0+1.7) = 3.1 \tag{9}$$

Similarly, incomplete interval-valued information system sequentially can be drawn to fill in the other property values, shown in Table 2.

Table 2 Complete Interval-valued Information System

U	a_1	a_2	a_3	a_4	a_5
x_1	[2.2,2.9]	[2.5,3.2]	[5.3,7.2]	[3.2,4.0]	[2.5,3.1]
x_2	[3.4,4.9]	[3.4,4.9]	[7.2,10.5]	[4.0,5.8]	[3.2,4.7]
x_3	[1.8,2.7]	[1.8,3.0]	[7.2,10.3]	[3.0,4.1]	[2.1,2.8]
x_4	[1.3,2.2]	[1.4,2.1]	[3.9,3.9]	[1.9,2.6]	[1.7,2.3]
x_5	[2.0,3.1]	[3.4,5.1]	[6.3,10.3]	[3.8,5.7]	[2.3,5.3]
x_6	[2.3,3.4]	[2.4,3.3]	[6.7,8.8]	[3.0,4.2]	[3.0,3.8]
x_7	[2.2,3.0]	[2.2,2.9]	[4.3,8.0]	[2.7,3.7]	[2.4,3.2]
x_8	[2.5,4.0]	[2.5,4.1]	[7.1,11.3]	[4.4,6.9]	[3.1,4.7]
x_9	[1.2,3.1]	[1.4,1.9]	[3.8,4.3]	[2.1,3.0]	[1.7,2.3]
x_{10}	[1.0,1.7]	[1.1,1.8]	[3.6,5.7]	[1.7,2.5]	[1.1,1.8]

Step 2: Conclude dominance class of dominance relation $R_{AT}^{\geq}$. For example, for the element x_1, according to,

$$R_{AT}^{\geq} = \{(y, x_1) \in U \times U \mid m_a(y) \geq m_a(x_1), \forall a \in A\}$$

$$[x_1]_{AT}^{\geq} = \{y \in U \mid (y, x_1) \in R_{AT}^{\geq}\}$$

Take attributes a_1 as an example, $[x_1]_{a_1}^{\geq} = \{x_1, x_2, x_5, x_6, x_7, x_8\}$

Then $[x_1]_{AT}^{\geq}=[x_1]_{a_1}^{\geq}\cap[x_1]_{a_2}^{\geq}\cap[x_1]_{a_3}^{\geq}\cap[x_1]_{a_4}^{\geq}\cap[x_1]_{a_5}^{\geq}$

So $[x_1]_{AT}^{\geq}=\{x_1,x_2,x_5,x_6,x_8\}$

Similarly, the dominance class of x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} can be obtained.

Step 3: Deduce the discernible matrix according to the dominant class, and obtain Figure1.

$$\begin{pmatrix}
\varnothing & \varnothing & a_1,a_2,a_4,a_5 & AT & \varnothing & \varnothing & a_2,a_3,a_4,a_5 & \varnothing & AT & AT \\
AT & \varnothing & AT & AT & a_1,a_2,a_4,a_5 & AT & AT & a_1,a_2,a_5 & AT & AT \\
a_3 & \varnothing & \varnothing & AT & a_3 & a_3 & a_3,a_4 & \varnothing & AT & AT \\
\varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & a_2 & a_1,a_2,a_4,a_5 \\
a_2,a_3,a_4,a_5 & a_2 & AT & AT & \varnothing & a_2,a_3,a_4,a_5 & a_2,a_3,a_4,a_5 & a_2 & AT & AT \\
a_1,a_3,a_5 & \varnothing & a_1,a_2,a_4,a_5 & AT & a_1 & \varnothing & AT & \varnothing & AT & AT \\
a_1,a_4 & \varnothing & a_1,a_2,a_5 & AT & a_1 & \varnothing & \varnothing & \varnothing & AT & AT \\
AT & a_3,a_4 & AT & AT & a_1,a_3,a_4,a_5 & AT & AT & \varnothing & AT & AT \\
\varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & a_1,a_2,a_4,a_5 \\
\varnothing & \varnothing & \varnothing & a_3 & \varnothing & \varnothing & \varnothing & \varnothing & a_3 & \varnothing
\end{pmatrix}$$

Figure 1 Discernible Matrix

Step 4: With discernible function, solve the minimal disjunctive normal form, which is attribute reduction.

$$\begin{aligned}
&M=AT\wedge(a_1\vee a_2\vee a_4\vee a_5)\wedge(a_2\vee a_3\vee a_4\vee a_5)\wedge\\
&(a_1\vee a_3\vee a_4\vee a_5)\wedge(a_1\vee a_2\vee a_5)\wedge a_3\wedge(a_3\vee a_4)\wedge\\
&a_2\wedge(a_1\vee a_3\vee a_5)\wedge a_1\wedge(a_1\vee a_4)\\
&=(a_1\vee a_2\vee a_3)
\end{aligned}$$

So the final result of attribute reduction for incomplete interval-valued information system is $\{a_1,a_2,a_3\}$.

The comparison of reduction results obtained in this paper and Reference [11] is shown in Table 3.

Table 3 Results Comparison

	Reference [11]	**this paper**
result	$\{a_1,a_2,a_3,a_4\}$	$\{a_1,a_2,a_3\}$

6.2 Result Analysis

For Reference [11], the defined upper limit dominance relation is too large and results in the classification error. This paper can avoid the shortcomings of Reference [11], and the obtained reduction results are more concise and redundancy of reduction is lower.

7. Conclusion

With the rapid development of computer network technology, the amount of data in various fields has increased dramatically. But due to technical limitations of data acquisition, transmission failures as well as some human factors and other reasons, the loss of data and defects of data frequently occur. In the real world, by the complexity of the environment and the impact of uncertainty, people will face the uncertain and incomplete information and the decision problems with decision-making preference information. Therefore, the study of incomplete ordered information system has been of great concern.

For the unknown attributes of the incomplete interval-valued information system, this paper proposes to complete the unknown attributes with the method of attribute mean, and transform the incomplete interval-valued information system into the complete interval-valued information system, calculates discernible matrix with the center dominance relation, and finally works out the final attribute reduction by the discernible function. This method is indeed feasible after verification of examples. By comparison with other methods of attribute reduction, this method is not only simple but also makes the reduction result more concise. This paper explores the attribute reduction by aiming at the unknown attribute of the incomplete interval-valued information system. The next research question is to extract the rules of incomplete interval-valued information system.

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