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Graceful Labelling of Corona Product of Aster Flower Graph

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***Abstract*—There are many graph labelling that have been developed, one of which is a graceful labelling. A graceful labelling of a graph $G = (V, E)$ with E edges is an injective $f: V(G) \to \{0, 1, \dots, |E|\}$ that the resulting edge labels obtained by $|f(u) - f(v)|$ on each edges uv are pairwise distinct. An aster flower graph $(A_{(m,n)})$ is a graph which generated from a cycle graph $C_m, m \geq 3$ by connecting path graphs $P_{n+1}, n \geq 1$ at two adjacent vertices. A corona product of aster flower graph $(A_{(m,n)} \odot \overline{K_r})$ is a graph which generated from an aster graph $(A_{(m,n)}), m \geq 3, n \geq 1$ by adding r leaf vertices on each vertex. In this paper, we present graceful labelling of corona product of aster flower graph that is $A_{(3,1)} \odot \overline{K_r}$.**

***Keywords*—Graceful labelling, Corona product graph, Aster flower graph**

I. INTRODUCTION (*HEADING 1*)

Graph G is a set of sequential sets (V, E) where V is the set of non-empty vertices and E is the set of (possibly empty) edges. Elements V is called asvertices of graph$G$ that can written as$V(G)$ and elements$E$is called as edges of graph G that can written as$E(G)$[2]. Graceful labelling is defined as labelling the vertices of graph G that satisfies the injective function from the set to the set of non-negative integers $\{0, 1, 2, \dots, q\}$ such that each of the *xy* edges in *G* gets label $|f(x) - f(y)|$, then the label of each vertices will be distinct[1].

We choose corona product of aster flower graph $A_{(m,n)} \odot \overline{K_r}$ based on graceful labelling of cycle graph$C_n$by connecting some of $P_k$paths (chords) graphs at two adjacent vertices that generated aster flower graph$A_{(m,n)}$and operation that can be done between the graphs that is corona product operation. Corona product operation was introduced by Frucht and Harary in 1970. Let graph G and graph H with numbersof vertices aren$_1$andn$_2$. Corona product ofG $\odot$ H is defined as a graph generated from two graphs that are G and H by taking one copy from G and n$_1$ copy from H, and connected it by an edge from each vertices at copy i-th from H by i-th vertices from G [3].

Definition 1. An aster flower graph $(A_{(m,n)})$ is a graph generated from a cycle graph $C_m, m \geq 3$ by connecting path graphs $P_{n+1}, n \geq 1$ at two adjacent vertices.

Definition 2. A corona product of aster flower graph $(A_{(m,n)} \odot \overline{K_r})$ is a graph which generated from an aster graph $(A_{(m,n)}), m \geq 3, n \geq 1$ by adding r leaf vertices on each vertex.

In this paper, we present graceful labelling of a corona product of aster flower graph that is $A_{(3,1)} \odot \overline{K_r}$.

Main Result

Theorem 1. A corona product of aster flower graph$A_{(3,1)} \odot \overline{K_r}$ has graceful labelling.

Proof. Let the notation of vertices of a corona product of aster flower graph$A_{(3,1)} \odot \overline{K_r}$ is shown in Figure 1.

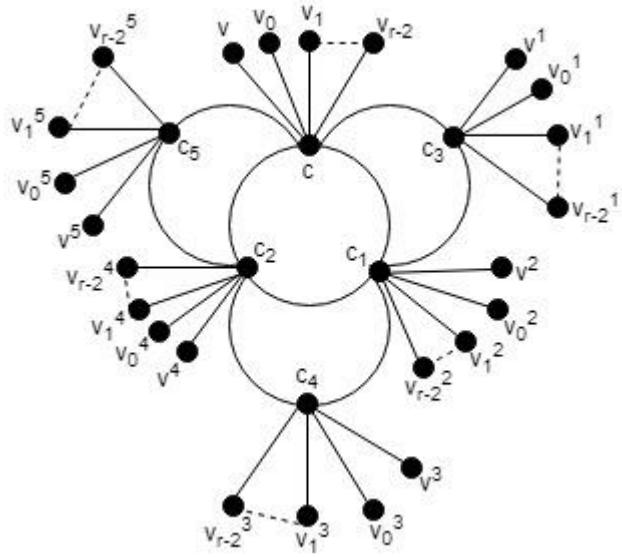

Figure 1: Vertices notation of corona product of aster flower graph $A_{(3,1)} \odot \overline{K_r}$

In Figure 1 is shown that a set of vertices $V(A_{(3,1)} \odot \overline{K_r})$= $\{c, c_1, c_2, c_3, c_4, c_5, v, v_0, v_1, \dots, v_{r-2},$
$v^1, v_0^1, v_1^1, \dots, v_{r-2}^1, v^2, v_0^2, v_1^2, \dots, v_{r-2}^2, v^3, v_0^3, v_1^3, \dots, v_{r-2}^3 v^4, v_0^4, v_1^4, \dots, v$
anda set of edges$E$ $(A_{(3,1)} \odot \overline{K_r}$= $\{cc_1, cc_2$,c
$c_3, cc_5, cv, cv_0, cv_1, \dots, v_{r-2}, c_1c_2, c_1c_3, c_1c_4, c_1v^2, c_1v_0^2,$
$c_1v_1^2, \dots, c_1v_{r-2}^2, c_2c_4, c_2c_5, c_2v^4, c_2v_0^4, c_2v_1^4, \dots, c_2v_{r-2}^4, c_3v^1, c_3v_0^1, c_3v_1^1,$

$\dots, c_4v^3_{r-2}, c_5v^5, c_5v^5_0, c_5v^5_1, \dots, c_5v^5_{r-2}\}$. The numbers of elements V and elements E are $6r+6$ and $6r+9$, can be written as $|V| = 6r+6$ and $|E| = 6r+9$.

Define function f labelling at vertices $A_{(3,1)} \odot \overline{K_r}$ as follows:

$$f(c) = 0. \quad (1)$$

$$f(c_i) = 6r+7+i, \text{for } i = 1 \text{ and } 2. \quad (2)$$

$$f(c_3) = 6r+3. \quad (3)$$

$$f(c_4) = 6r+5 \quad (4)$$

$$f(c_5) = 6r+7 \quad (5)$$

$$f(v) = 6r+6. \quad (6)$$

$$f(v_0) = f(v) - 10 = 6r-4. \quad (7)$$

$$f(v_i) = f(v_0) - 6i = 6r-4-6i, \text{for} i = 1,2,\dots,r-2. \quad (8)$$

$$f(v^1) = 1. \quad (9)$$

$$f(v^1_0) = 9. \quad (10)$$

$$f(v^1_i) = f(v^1_0) + 6i = 9+6i, \text{for} i = 1,2,\dots,r-2. \quad (11)$$

$$f(v^2) = 4. \quad (12)$$

$$f(v^2_0) = 7. \quad (13)$$

$$f(v^2_i) = f(v^2_0) + 6i = 7+6i, \text{for} i = 1,2,\dots,r-2. \quad (14)$$

$$f(v^3) = 5. \quad (15)$$

$$f(v^3_0) = 6. \quad (16)$$

$$f(v^3_i) = f(v^3_0) + 6i = 6+6i, \text{for} i = 1,2,\dots,r-2. \quad (17)$$

$$f(v^4) = 6r+2. \quad (18)$$

$$f(v^4_0) = f(v^4) - 3 = 6r-1. \quad (19)$$

$$f(v^4_i) = f(v^4_0) - 6i = 6r-1-6i, \text{for} i = 1,2,\dots,r-2. \quad (20)$$

$$f(v^5) = 2. \quad (21)$$

$$f(v^5_0) = 10. \quad (22)$$

$$f(v^5_i) = f(v^5_0) + 6i = 10+6i, \text{for} i = 1,2,\dots,r-2. \quad (23)$$

Based on function f labellings defined at (1 – 23), it can be seen that each vertex has distinct label that forming a set $\{0,1,2,\dots,|E|\}$, such function f labelling defined at (1 – 23) is injective function from V to set $\{0,1,2,\dots,|E|\}$.

Each edge $uv \in E$ has label assigned by function f' induced from vertices labelling from function f labelling by $f'(uv) = |f(u) - f(v)|$. Thus we have function f' labelling at edges $A_{(3,1)} \odot \overline{K_r}$ as follows:

$$f'(cc_i) = |f(c) - f(c_i)| = |(0) - (6r+7+i)| = 6r+7+i, \text{for} i = 1 \text{ and } 2. \quad (24)$$

$$f'(cc_3) = |f(c) - f(c_3)| = |(0) - (6r+3)| = 6r+3. \quad (25)$$

$$f'(cc_5) = |f(c) - f(c_5)| = |(0) - (6r+7)| = 6r+7. \quad (26)$$

$$f'(cv) = |f(c) - f(v)| = |(0) - (6r+6)| = 6r+6. \quad (27)$$

$$f'(cv_0) = |f(c) - f(v_0)| = |(0) - (6r-4)| = 6r-4. \quad (28)$$

$$f'(cv_i) = |f(c) - f(v_i)| = |(0) - (6r-4-6i)| = 6r-4-6i, \text{for} i = 1,2,\dots,r-2. \quad (29)$$

$$f'(c_1c_3) = |f(c_1) - f(c_3)| = |(6r+8) - (6r+3)| = 5. \quad (30)$$

$$f'(c_1c_4) = |f(c_1) - f(c_4)| = |(6r+8) - (6r+5)| = 3. \quad (31)$$

$$f'(c_1v^2) = |f(c_1) - f(v^2)| = |(6r+8) - (4)| = 6r+4. \quad (32)$$

$$f'(c_1v^2_0) = |f(c_1) - f(v^2_0)| = |(6r+8) - (7)| = 6r+1. \quad (33)$$

$$f'(c_1v^2_i) = |f(c_1) - f(v^2_i)| = |(6r+8) - (7+6i)| = 6r+1-6i, \text{for} i = 1,2,\dots,r-2. \quad (34)$$

$$f'(c_2c_1) = |f(c_2) - f(c_1)| = |(6r+9) - (6r+8)| = 1. \quad (35)$$

$$f'(c_2c_4) = |f(c_2) - f(c_4)| = |(6r+9) - (6r+5)| = 2. \quad (36)$$

$$f'(c_2c_5) = |f(c_2) - f(c_5)| = |(6r+9) - (6r+7)| = 4. \quad (37)$$

$$f'(c_2v^4) = |f(c_2) - f(v^1)| = |(6r+9) - (6r+2)| = 7. \quad (38)$$

$$f'(c_2v_0^4) = |f(c_2) - f(v_0^1)| = |(6r+9) - (6r-1)| = 9 - (-1) = 10. \quad (39)$$

$$f'(c_2v_i^4) = |f(c_2) - f(v_i^1)| = |(6r+9) - (6r-1-6i)| = 10 + 6i,$$

for $i = 1,2,\dots,r-2$. (40)

$$f'(c_3v^1) = |f(c_3) - f(v^1)| = |(6r+3) - (1)| = 6r+2. \quad (41)$$

$$f'(c_3v_0^1) = |f(c_3) - f(v_0^1)| = |(6r+3) - (9)| = 6r-6. \quad (42)$$

$$f'(c_3v_i^1) = |f(c_3) - f(v_i^1)| = |(6r+3) - (9+6i)| = 6r-6-6i,$$

for $i = 1,2,\dots,r-2$. (43)

$$f'(c_4v^3) = |f(c_4) - f(v^3)| = |(6r+5) - (5)| = 6r. \quad (44)$$

$$f'(c_4v_0^3) = |f(c_4) - f(v_0^3)| = |(6r+5) - (6)| = 6r-1. \quad (45)$$

$$f'(c_4v_i^3) = |f(c_4) - f(v_i^3)| = |(6r+5) - (6+6i)| = 6r-1-6i,$$

for $i = 1,2,\dots,r-2$. (46)

$$f'(c_5v^5) = |f(c_5) - f(v^5)| = |(6r+7) - (2)| = 6r+5. \quad (47)$$

$$f'(c_5v_0^5) = |f(c_5) - f(v_0^5)| = |(6r+7) - (10)| = 6r-3. \quad (48)$$

$$f'(c_5v_i^5) = |f(c_5) - f(v_i^5)| = |(6r+7) - (10+6i)| = 6r-3-6i,$$

for $i = 1,2,\dots,r-2$. (49)

Based on function f' edges labelling that defined at (24 – 49) , it can be seen that each edge has distinct label that resulting a set $\{1,2,\dots,|E|\}$. We can see that function f' labelling that induced by function f labelling provide distinct values at each edge that form a set $\{1,2,\dots,|E|\}$.

Hence the function f and the induced function f' provide distinct values for each edge and also the labelling for vertices set has distinct values in $\{0,1,2,\dots,|E|\}$. Thus a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_r}$ is graceful. ■

The following graph in Figure 2 is a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_1}$.

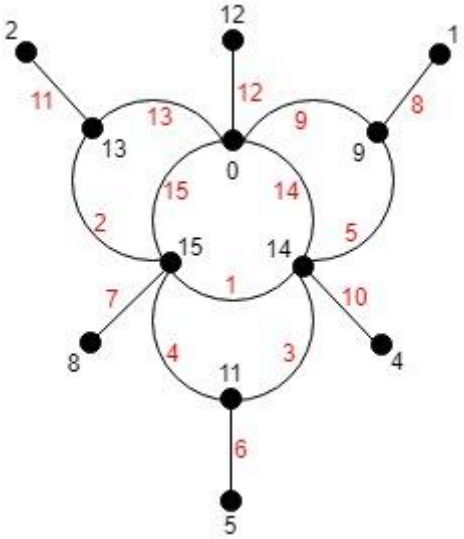

Figure 2: Graceful labelling of a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_1}$

The following graph in Figure 3 is a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_2}$.

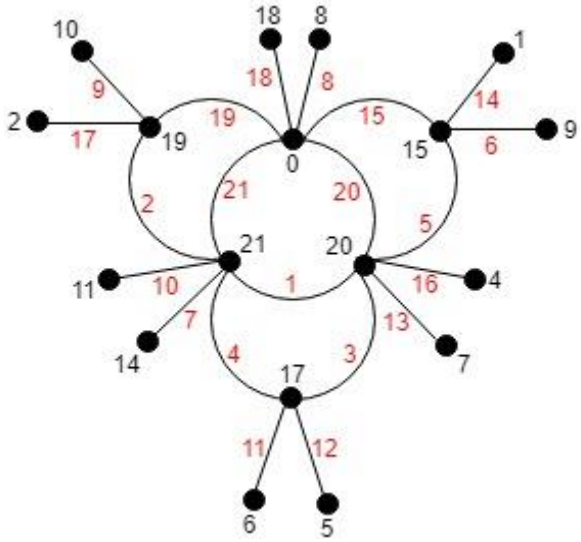

Figure 3: Graceful labelling of a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_2}$

The following graph in Figure 4 is a corona product of aster flower graph $A_{(3,1)} \odot \overline{K_3}$.

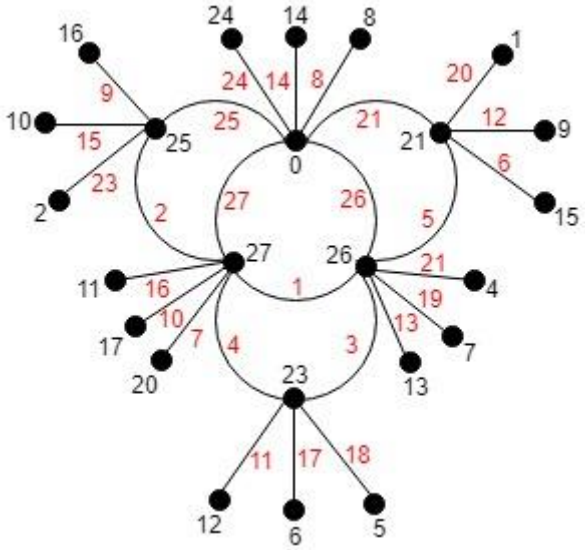

Figure 4:.Graceful labelling of a corona product of aster flower graph$A_{(3,1)} \odot \overline{K_3}$

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II. CONCLUSIONS

It is interesting to study graceful labelling of a corona product of aster flower graph that is$A_{(3,1)} \odot \overline{K_3}$. In the present work, we investigate graceful labelling of corona product of aster flower graph that is for $A_{(3,1)} \odot \overline{K_3}$for $m = 4$ and 5.

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