

Teaching Practice Technique in Teaching of Linear Algebra

Guo Yanfeng^{1, 2}

¹ School of Mathematics and Physics
China University of Geosciences Wuhan
Wuhan, Hubei, China

² School of Science
Guangxi University of Technology and Sciences,
Liuzhou, Guangxi, China

Guo Chunxiao*

School of Science
China University of Mining and Technology Beijing
Beijing, China

*Corresponding author

Mo Chunpeng

School of Science
Guangxi University of Technology and Sciences
Liuzhou, Guangxi, China

Xiong Weiling

School of Science
Guangxi University of Technology and Sciences
Liuzhou, Guangxi, China

Abstract—The teaching practice techniques of the teaching modes and teaching methods in the processes of linear algebra teaching are mainly considered in this paper. Teaching practices techniques are provided by two teaching modes and teaching methods including analogy methods, layered methods, extension methods, experimental methods. These teaching practices techniques provide a theoretical basis for the teaching developments and applications of linear algebra in college and university mathematics and promote students with interesting and enthusiasm in learning linear algebra.

Keywords—Linear Algebra; Techniques of Teaching; teaching modes; teaching methods

I. INTRODUCTION

Linear algebra is an important and indispensable course of mathematics for all science and engineering majors in the colleges and universities. It is a compulsory course in general education courses of various majors and a basic mathematics course to be applied in all majors. So, the teaching reforms of linear algebra have gradually become an important part of teaching in colleges and universities mathematics. As is well known, the knowledge of linear algebra is indispensable and applied more and more in the professional courses of various majors. Its ideas and methods have used in practical applications in all fields in today society. With the continuous developments of modern social economy, it is now applied in the era of big data. The processing, analysis and applications of big data are inseparable from the most basic knowledge in linear algebra. Therefore, the practical skills of the teaching methods of the linear algebra are urgent to continuously develop. It is more important to find the teaching methods suitable for the modern society and apply them to the teaching for making the modern college students quickly associate this course with their related professional and quickly grasping and applying the ideas and methods in linear algebra to the actual situation. At the same time, our goal is to inspire and promote

interesting and enthusiasm of students in learning linear algebra.

There are various teaching methods for the discussion of teaching methods of university mathematics courses[1-3]. However, the further researches for the teaching of linear algebra are needed. In recently, our teaching team has paid more attention to the improvement of quality and level of teaching in the basic courses of mathematics in colleges and universities, and has continuously carried out reforms and practices in teaching from various aspects, which is supported by the project of teaching reforms.

The teaching team of higher mathematics has continuously done hardworking to develop and establish excellent teaching teams of our university through lots of reforms and practices. The teaching of linear algebra has been greatly been practiced and reformed in various forms such as teaching modes, teaching methods, teaching methods, and teaching classifications, and so on. These behaviors have contributed to the development of higher mathematics in our school and laid a good foundation for improving the quality of science and engineering in our school.

We know that linear algebra is a very important basic course. Although the time of linear algebra is not as much as that of calculus, its applications in various professions are not less than the applications of calculus. Especially, the requirements for linear algebra will become more and more important in the era of big data. Then, the teaching reforms of linear algebra are also essential. Therefore, we have gradually formed a linear algebra teaching team, and published a textbook named linear algebra textbook[4], which is improved by teaching practice skills of this course. Furthermore, we also carry out the reforms of the teaching of linear algebra during the teaching of it to improve the overall quality of students.

II. TECHNIQUES OF TEACHING

According to the contents of the textbooks written by our team, the linear algebra is a course which is started after calculus. Through the teaching experience and practice of our teaching team, the knowledge of linear algebra can be learned after calculus which is familiar to students in the teaching processes. So, the some corresponding teaching techniques of calculus are also applied to the teaching processes of linear algebra. And some good results have achieved. The following are the practical skills during the teaching reforms of the course linear algebra, which are mainly given from two different aspects, teaching modes and teaching methods.

A. Teaching modes

It is mainly used two modes to teach the linear algebra according to our various experimental teaching modes, which are given in the following.

1) *The mode with combination of courseware and blackboard in teaching processes*

In order to improve the quality and quantity of teaching linear algebra, a corresponding courseware with our linear algebra textbook is done, which is combined with teaching experiments of our team and been used in our teaching plans.

The main contents and ideas of some concepts and logic theories are presented by computer. In classes, the main ideas for solving problems are gradually presented by computer, and students can be needed to understand and think, even answer some key problems in classes, in order to obtain inter-class effect and be interested in linear algebra. At the same time, the main processes of solving problems and deriving the main theories and principles have been given on the blackboard in order that students can understand the main ideas. However, due to the requirements of talents training programs of every major, we known that the plan time of linear algebra for each major are 40 hours or 32 hours. According to the teaching plans and teaching progresses of the linear algebra course, it is not more enough. But, it is necessary to have not only qualitative requirements but also a certain amount of requirements in the teaching processes. Although the lack of time for linear algebra has a certain degree of non-conformity with the requirements in the talents training programs, we have carried out the teaching according to every program. When teaching hours are limited, we adopt a teaching mode of combining courseware and blackboard on the condition of ensuring the quality and quantity requirements of the course in the talents training programs. After several semesters of teaching practices for linear algebra, this teaching mode can not only ensure students to understand the main mathematical ideas of linear algebra, but also guarantee to perform the programs. Both quality and quantity can ensure that students study with interesting and obtain a solid foundation for the application of the following professional courses in various professional training programs, which are on the basis of linear algebra.

2) *The mode of using the Internet to carry out the corresponding micro-course construction, Mooc and extracurricular of network question-and-answer (Q&A) mode.*

In order to enhance interesting and concentration of students in learning linear algebra courses, the Internet can be used in the teaching process to carry out the corresponding micro-course construction, Mooc and extracurricular network Q&A mode, which answer various questions of students online. The construction of micro-courses and Mooc are gradually being carried out. Through the construction of them, students can learn again on the side of the corresponding micro-courses and Mooc in the classroom, and can master the main contents of learning. In the way of answering questions in extracurricular exercises, we can answer the questions by face to face communication and Q&A with students. When they do not understand the knowledge in class, they can also directly give questions online by using the software of the Internet at certain time. The records of network Q&A can be used to allow students to repeatedly replay the network for understanding the knowledge, which provide students with a good network of repetitive learning. This kind of teaching mode for learning and answering counseling, which it combined with the continuous development and strengthening of the modern social Internet, are popular with students. Such a teaching mode not only allows students to learn online, but also provides students with interesting and concentration in learning, while it improve the quality of students to learn course of linear algebra and make a solid foundation for subsequent courses.

B. Teaching methods

From the aspects of teaching methods, we summarize several simple teaching methods and practices. On the basis of exploring and studying continuously, we improve the teaching methods and techniques. There are some teaching methods in linear algebra course including analogy methods, layered methods, extended methods and experimental methods.

1) *Analogy methods*

For example, when teaching the block matrix, in order to be able to know the purpose of the block, it can be considered to be zero, and reconsidered zero to it, which is main idea of mathematics to understand some concepts. It is analogy to the idea which is used in the definition of integrals in calculus. We can make students recall concepts of definition of definite integrals, which have already been learned in calculus. Trough remembering the ideas and methods of calculus, we can use the analogy method to link the ideas of the blocks with the definite integrals, So that students deeply understand the meaning of matrix partitions.

In order to have deeper understanding of this method, when one encounters various complex issues, one will use this idea to deal with some analogy concepts in the future. However, after one give the theoretical derivation of the properties of the block matrix, the similar properties of the block matrix can be easily obtained by analogy methods for the properties of the matrix itself. Then analogy methods can also be extended and applied to others courses when solving practical problems. That is to say, firstly, one can consider solving small problems. Secondly, one further becomes small problems into large problems. Finally, one solves it using the analogy methods.

Another example is to solve linear equations. When teaching the structure of the solution of homogeneous and inhomogeneous linear equations and the relationship between them, it can be used the analogy method with the idea in the ordinary differential equations course, which is the relationship between the solutions of the solutions of homogeneous differential equations and non-homogeneous differential equations. From this process, one can better understand the properties of linear equations and it is easier to remember them.

Through the analogy methods, we can understand the structure and relationship of solutions of linear equations in linear algebra. In order to deepen the understanding and memory of students, it can also give students a way of integrating ideas and methods among various disciplines. Then, it is possible to find ideas of solving problems in various subjects through analogy methods.

Therefore, the ideas of the analogy methods have many applications in practical phenomenon. From the teaching processes of mathematics knowledge, the students can learn to use this method to think some problems in practical applications.

2) Layered methods

When one gives a lecture about the solving solutions of linear equations, one can use the layered methods in teaching processes. Firstly, on the basis of elimination method to solve solutions, the second and three order determinants are introduced by the elimination method of the quadratic and cubic linear equations. Then equations with $n \geq 3$ unknown are further considered. So, the concepts of general determinants are introduced. Secondly, when solution is uniqueness, one can use the determinants to solve the n order linear equations. However, for the case of no solutions and infinite solutions, the determinants can not satisfy the requirements. So the properties of the matrix are further considered. The properties of the matrix can be used to solve the solutions of the linear system of equations. When one solves various solutions of the linear equations, it is necessary to study the various properties of the matrix. So various properties of matrix are important to solve the linear equations. Finally, by use of the various properties of the matrix, one can discuss and solve the solutions of the linear system of equations. We can discuss solutions of the linear system of equations, which are started from the general solutions of the simple homogeneous linear system of equations and the solutions of inhomogeneous linear system of equations. The methods are progressive layered methods.

Such progressive layered methods allow us to learn more knowledge linked before and after. We will emphasis to use progressive layered methods for cultivating thinking patterns of students and apply it to practical problems.

3) Extension methods

When some operational properties of matrices are given in classes, the special properties of unit matrices and zero matrices are considered, which be extended to identity matrix and zero matrix. The positions of them can be compared by the characteristics of units 1 and 0 in real numbers domains. However, it is important to consider the similarities and differences between them in their respective collection domains.

Then the numbers domains and matrix domains can be defined as different forms of sets. Under some conditions, they can extend to concepts of groups, rings and domains, which are important concepts in algebra. At the same time, we can use these algebra concepts to inspire students learning linear algebra and to make them realize that linear algebra is an important basis course for the following courses.

When the elementary transformation is transformed into a row stepped matrix and a row simplified matrix in the elementary row of the matrix in the lecture matrix, the ideas of transforming the matrix into the important matrix form can be extended to the application of the subsequent courses. For example, the simplex methods in the optimization theories are obtained by being applied to the elementary transformation of the elementary transformation column. Therefore, students are aware of the important applications and promotions of some ideas and methods of linear algebra in subsequent courses.

When we give the lectures for the character values of character vectors, the inner products of the vectors to the orthogonalization and the standard orthogonal basis, the inner products of character values and character vectors in the applied functional analysis are referred in the subsequent courses. The standard orthogonal basis is an extension of some properties of the vectors and has the same properties. These ideas and methods in linear algebra are the basis for the subsequent courses contents. So, it is more importance to be applied in various disciplines. From this, students can understand that linear algebra is not only a mathematics course, but also following essential contents in all subjects and researches.

Therefore, from the extension methods, some basic knowledge and applications in the following courses are mentioned, which stimulates students to learn linear algebra with stronger interesting and enthusiasm and achieve the better effects in improving the qualities and levels of linear algebra teaching.

4) Experimental methods

In the processes of teaching the determinant solutions and solving the linear system of equations, it is necessary to perform the elementary transformation for calculating and solving them. We have given the corresponding calculation processes in theory and specific methods. However, in examples of solving the determinant or linear equations in the textbook, the determinants and matrices are not very higher order determinants and matrices. So we can apply the corresponding calculation principles and methods to simple determinants and equations to illustrate the ideas of linear algebra and the original important ideas. However, when students really need to understand the application of these ideas and methods, they can use the mathematical software Matlab or software tools such as C++. It is useful to solve the higher order specific determinant or linear equations, which have more unknown in the teaching processes. The corresponding algorithm is given, and demonstrations are carried out during the teaching processes, in order to illustrate the importance and applicability of our ideas, methods and principles.

The experimental methods actually illustrate more extensive applications of the knowledge, which are learned

from the connection and combination between the classroom contents and the actual applications. In the views of this point, in our courses of colleges and universities, we can appropriately increase the teaching methods combined with teaching contents and experiments, which enhance the interesting and enthusiasm of students for learning linear algebra.

III. CONCLUSION

In short, some of the teaching methods mentioned above about teaching of linear algebra have already been reflected in the textbooks, which is written by our linear algebra teaching team and published by Fudan University Press. In the future, we will explore other teaching techniques in the teaching processes and give the certain teaching practices and applications further. The reforms of the linear algebra have been received good effects for students, which are found in the results of final exam tests. Although these teaching methods are already familiar to everyone, we also need to constantly explore and give a series of relatively perfect contents for the teaching techniques in the future. Indeed, practices of the reforms of teaching in linear algebra have provided some practical experiences. Our linear algebra teaching team will continue to explore and study better teaching techniques and practices of linear algebra, such as teaching modes and teaching methods. Therefore, the teaching and experiments of succeeding in teaching of linear algebra can be applied in

others mathematics courses, which improve the level of teaching quality of college and university mathematics courses.

ACKNOWLEDGMENT

This paper is supported by Guangxi Higher Education Undergraduate Teaching Reform Project(No. 2016JGB274), Guangxi University of Science and Technology Excellent Teaching Team Construction Project, and China University of Mining and Technology(Beijing) Education Reform Project, Fundamental Research Funds for the Central Universities, China University of Geosciences(Wuhan) (No.2018061).

REFERENCES

- [1] N. Ji, "The higher mathematics classroom teaching methods research and innovation," *Journal of Hebei Energy Institute of Vocation and Technology*, vol. 34, pp.86-88, 2009(in Chinese).
- [2] X.P. Jiang, "Research effective teaching design of advanced mathematics," *Journal of Capital Normal University (Natural Science Edition)*, Vol. 38, pp. 14-19, 2017(in Chinese).
- [3] S.L. Cao, "Research and practice of the "Teaching, Learning and Doing" integration of advanced mathematics based on the information technology," *Higher Vocational Education--Journal of Tianjin Professional College*, Vol. 1, pp. 52-55, 2018(in Chinese).
- [4] W.L. Xiong, *Linear Algebra*, 2nd ed., Fudan University Press, 2017(in Chinese).