Stability of Mooring System Based on Linear Complementarity System Theory

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***Abstract*—In order to determine if a given mooring system is globally asymptotically stable, the mooring system is analyzed based on the hybrid system theory. The stability of linear complementarity system is investigated by using the common Lyapunov function approach. A method to find the common Lyapunov function is presented. The linear complementarity model of a typical mooring system is constructed, and its asymptotic stability is verified by the common Lyapunov function approach. The results show that the linear complementarity model is suitable to describe the mooring systems, and the method proposed in the paper is feasible to analyze the stability of a given system.**

Keywords- Mooring system, Hybrid system, Stability, Linear complementarity system, Common Lyapunov function approach

I. INTRODUCTION

Ship mooring is a common practice which is conducted by almost every ship. The wharf can provide the mooring ships with a refuge harbor, reduce the wave impact and the dynamic mooring load, keep the ships from damaging. One of the most effective measures for the navy to protect the submarines during the heavy weather is to moor them in the harbor. In order to properly arrange the ships and the mooring equipments, the dynamics of the mooring ships has been thoroughly investigated [1]. However, to the best of our knowledge, until now, there is hardly any practical and effective approach for the stability analysis of the mooring system.

The concept of Hybrid System was first proposed in 1986. During the last two decade, hybrid dynamical systems have become a major research topic. Hybrid System are dynamical systems that often consist of continuous time (CT) and discrete time (DT) processes [2]. Modern systems are often complex and cannot be simulated as a single model, many of them are interfaced with some logical or decision-making (LDM) process, such as the autonomous manufacturing, traffic control, and chemical process [3].

For the large-scale systems, it's an effective way to construct the model based on the Hybrid System theory. The motoring systems are basically noncontinuous, and their dynamic responses usually resemble those of the hybrid systems, especially when the sea states are high. So, it is essential to study mooring systems by introducing the hybrid system theory.

II. PRINCIPLE

A. Hybrid System

A given dynamical system should be considered a hybrid system if (and only if) it is impossible to deal with it either as a purely continuous–variable system or as a purely discrete–event system without ignoring important phenomena that result from the combination of continuous and discrete movements of this system [2]. The basic hybrid phenomenon is a combination of continuous state changes and abrupt state jumps. If both the continuous movement between the jumps and the state jumps must be considered when solving a given modeling, analysis or control task, the system has to be considered as hybrid [2].

Hybrid system includes mixed logical dynamical (MLD) systems, linear complementarity (LC) systems, piecewise affine (PWA) systems, and max-min-plus-scaling (MMPS) systems, etc. The linear complementarity systems are studied in this paper. A linear complementarity system is governed by the following simultaneous equations [4]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \\ 0 \le y(t) \perp u(t) \ge 0 \end{cases} \tag{1}$$

The notation $0 \le y(t) \perp u(t) \ge 0$ in equation (1) states that for every component i=1, 2,…, either $y_i(t) = 0$ or $u_i(t)$=0 .

B. Stability of Hybrid System

There are mainly two methods to investigate hybrid system's stability: common Lyapunov function approach and multiple Lyapunov function approach. The main idea of the Lyapunov function approach is to find one or a series of nonincreasing functions, from which we could conclude that the system is unstable, stable or asymptotically stable [5].

The common Lyapunov function approach can be viewed as a generalized Lyapunov function approach used in a hybrid system problem. If for all subsystems there exists an identical Lyapunov function, i.e. an aggregate summarizing function that continually decreases toward a minimum, the system is asymptotically stable [5]. The difficulty is to find such common Lyapunov function.

The main idea of multiple Lyapunov function approach is to find an auxiliary Lyapunov function for each subsystem. If all the auxiliary Lyapunov functions are nonincreasing at their switching times, the whole hybrid system is asymptotically stable [5].

The multiple Lyapunov function is easier to find, but the switching times are required in order to apply this approach. Therefore, multiple Lyapunov function approach is difficult to use for the linear complementarity systems. As a result, the common Lyapunov function approach for the linear complementarity system is analyzed.

C. Stability of LC System

If all the subsystems of a given LC system are smooth, then the following theorem holds [6].

Theorem: If all subsystems share a common positive definite radially unbounded Lyapunov function, the system is GUAS (global uniform asymptotic stability).

The above theorem indicates that the common Lyapunov functions are sufficient for the stability of the hybrid system. GUAS of linear system does not imply the existence of a common quadratic Lyapunov function. There is, however, a common piecewise quadratic Lyapunov function [6].

Even if there is no common Lyapunov function, stability follows if

$$V_{i(t_{k-1})}\left(x\left(t_k\right)\right)=V_{i(t_k)}\left(x\left(t_k\right)\right) \qquad \forall k=1,2,\cdots \tag{2}$$

where tk denote the switching times [6].

For the linear complementarity system governed by Equation (1), there exist two modes: "unconstrained" mode and "constrained" mode, as depicted in Table I.

TABLE I. TWO MODES OF THE LINEAR COMPLEMENTARITY SYSTEM

Unconstrained	Constrained
$\begin{cases}\dot{x}(t)=Ax(t)\\ y(t)=Cx(t)\end{cases}$	$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=0\end{cases}$
$y(t)\geq 0,\ \ u(t)=0$	$y(t)=0,\ \ u(t)\geq 0$

Aiming at the linear complementarity system, an approach to find the common Lyapunov function is presented as following:

1)The first step is to find an auxiliary Lyapunov function for the "unconstrained" subsystem:

- Select an arbitrary symmetric positive definite D1, for example, an identity matrix In.
- Solve the Lyapunov equation $A_1^{\mathrm{T}}\cdot P+P\cdot A_1=-D_1$, for P.
- If P is positive definite, the matrix A1 is asymptotically stable. If P is not positive definite, then A1 is not asymptotically stable.

2)Check that whether the "constrained" subsystem share the same Lyapunov function as the "unconstrained" one's.

- Solve the Lyapunov equation $A_2^{\mathrm{T}}\cdot P+P\cdot A_2=-D_2$ for D2, using the P calculated in Step (1);
- Check that whether D2 is positive definite: If D2 turns out to be positive definite, then A2 is asymptotically stable. If not, nothing can be said about asymptotically stable of A2 from the Lyapunov equation above. Switch to the first step and choose another arbitrary symmetric positive definite D1.

III. MODELING

A. LC Model for a Mooring System

Mooring system includes the mooring lines, fenders, ship and pier auxiliary equipments, etc. They are used to secure the ship to the pier, wharf or another ship. The mooring systems are usually systems subject to unilateral constraints, such as the mooring system depicted in Fig.1. The system illustrated in this figure is a typical mooring arrangement for submarines using the pneumatic fenders. Two submarines are moored side by side. The fenders are placed between the two submarines and the wharf. The submarine which lies away from the wharf is tied up with the cleat using mooring line.

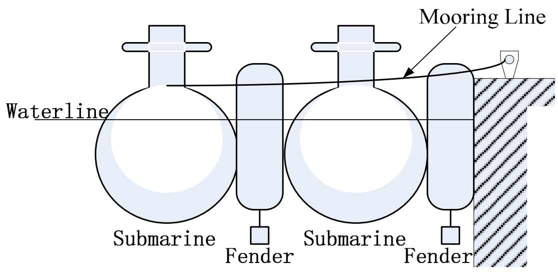

Figure 1. A mooring system

Supposing the system is stable when the mooring line reaches its longest length, and the line won't break. When affected by the wind and waves, the submarines sway. When the submarines move towards the wharf, the force on the mooring line equals zero. When they move outwards, the submarines were confined by the mooring line. When the submarine reaches the maximum length of the line, the force of the line will stop the submarine from moving outwards. According to the definition, the above mooring system is a typical linear complementarity system.

B. Stability of Mooring System

The mooring system illustrated in Fig.1 can be simplified as a two-cart system, as depicted in Fig.2. The springs and dampers correspond to the fenders, the carts correspond to the submarines. The left cart corresponds to the submarine which was moored away from the wharf, and the mooring line is simplified as a hook.

When the cart moves leftward and reaches the maximum length of the hook, the left cart must stop moving leftward, i.e. the motion of the left cart is constrained by an inelastic stop. Therefore, its rigid body dynamics with inelastic are impacted, and the velocities of the colliding bodies are changed instantaneously [7].

It is supposed that the carts are at their equilibrium positions when the hook stretches at its maximum length. The masses of the carts, the spring constants and the damping ratios are m, k and d, respectively.

x2 and x1 denote the deviations of the left and right cart from their equilibrium positions. $u(t)$ is the reaction force exerted by the hook, and it is nonzero only when the hook reaches its maximum length.

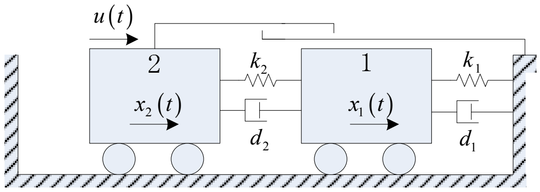

Figure 2. 2 A Simplified Two-Carts System

The mechanical system can be described as follows.

$$\begin{cases} m_1\ddot{x}_1(t) = k_2(x_2(t) - x_1(t)) - k_1 \cdot x_1(t) - d_1 \cdot \dot{x}_1(t) \\ m_2\ddot{x}_2(t) = u(t) - k_2(x_2(t) - x_1(t)) - d_2 \cdot (\dot{x}_2(t) - \dot{x}_1(t)) \end{cases} \quad (3)$$

Where $u(t)^{3}0$, if and only if $x_2(t)=0$.

Let $q_1 = (x_1\ \dot{x}_1)^T$, $q_2 = (x_2\ \dot{x}_2)^T$, $Q = (q_1,\ q_2)^T$ and $E = (1,0)$, Equation (3) can be written as:

$$\begin{cases} \dot{q}_1 = \begin{pmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -(k_1+k_2)/m_1 & -d_1/m_1 \end{bmatrix}}_{A_1} \cdot \begin{pmatrix} x_1 \\ \dot{x}_1 \end{pmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ k_2/m_1 & 0 \end{bmatrix}}_{A_2} \cdot \begin{pmatrix} x_2 \\ \dot{x}_2 \end{pmatrix} \\ \dot{q}_2 = \begin{pmatrix} \dot{x}_2 \\ \ddot{x}_2 \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ k_2/m_2 & d_2/m_2 \end{bmatrix}}_{A_3} \cdot \begin{pmatrix} x_1 \\ \dot{x}_1 \end{pmatrix} + \underbrace{\begin{bmatrix} 0 & 1 \\ -k_2/m_2 & -d_2/m_2 \end{bmatrix}}_{A_4} \cdot \begin{pmatrix} x_2 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 1/m_2 \end{bmatrix} u \end{cases} \quad (4)$$

$$\Rightarrow \begin{cases} \dot{Q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}_{Q} + \underbrace{\begin{bmatrix} 0 \\ I/m_2 \end{bmatrix}}_{B} \cdot u \\ Y = \underbrace{[0\ E]}_{C} \cdot \underbrace{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}_{Q}, \quad 0 \le Y \perp u \ge 0 \end{cases} \quad (5)$$

In order to verify the approach which was presented in the last section, we try to find a common Lyapunov function for a given linear complementarity system. For simplicity, we set the masses of the carts, the spring constants and the damping ratios to 1, i.e. $k_1 = k_2 = 1$, $d_1 = d_2 = 1$, $m_1 = m_2 = 1$.

After a few trials, we find that when choosing the arbitrary symmetric positive definite $D_1 = \begin{bmatrix} 9 & -1 & 0 & 1 \\ -1 & 9 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$, get

$P = \begin{bmatrix} 2 & -0.5 & 2 & 0.5 \\ -0.5 & 2.5 & -0.5 & 0 \\ 2 & -0.5 & 3.5 & -0.5 \\ 0.5 & 0 & -0.5 & 1.5 \end{bmatrix}$, P is positive definite.

After verification, we can conclude that $V_i = Q^T P Q$ is continuous Lyapunov function at the switching times. Therefore the LC system is asymptotically stable.

IV. Conclution

(1) The dynamic responses of the mooring structures are characterized by its noncontinuity. The coupling effects between the ships, fenders, wharf and the lines cannot be described as a purely continuous–variable system or as a purely discrete–event system, but they can be modeled as hybrid system. Based on the hybrid system theory, a linear complementarity model is constructed to describe the state jump in a typical submarine mooring system.

(2) An approach of finding the common Lyapunov function for the linear complementarity system is presented and verified by an example of a mooring system.

The mooring system is simplified in this paper. For instance, we didn't consider the situation in which the mooring line breaks, and the approach presented is far from effective for large-scale systems. The future research may focus on modeling the mooring systems with more accuracy, and finding an more effective way to find the Lyapunov function for complex systems.

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