Study on Projective Synchronization in Different Chaotic Systems

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***Abstract*—A method is proposed to realize projective synchronization in different chaotic systems. The structure of the controllers is designed based on Lyapunov stability theory. Single-mode laser Lorenz system and Rössler system are taken as examples to verify the effectiveness of the proposed method. The method is proper to any chaotic systems, and it can be generally used.**

Keywords-projective synchronization, Single-mode laser Lorenz system, Rössler system, Lyapunov stability theory

I. INTRODUCTION

Since Pecora and Carroll realized synchronization in electronic circuits in 1990[1], chaos synchronization has attracted much attention for its great value in many fields of research, such as information and communication, auto-control, physical and ecological field and so on, and it has become a hotspot in modern science. Many synchronization methods have been proposed, such as Pecora Carroll (PC) method, active-passive method, variable coupling method, adaptive control method, variable feedback method and so on[2-6]. In recent years, the concept of chaotic synchronization has been further expanded, projective synchronization[7-8] and anti-synchronization[9-10] are also proposed. An outstanding advantage of projective synchronization is that the variables can be synchronized in any proportions, while in an anti-synchronization, the state variables of the two systems has equal amplitude and opposite directions which made the effect of synchronization more flexible and convenient. However, most of the methods mentioned above are mainly used in systems with the same structure since it is easy to realize. The identical chaotic systems have the same dynamic equations and system parameters, only the initial value is slightly different. Therefore, synchronization of the identical chaotic systems can be achieved more easily. While it's difficult to find identical chaotic systems in practical world, as a result, it's much more import to synchronize two different chaotic systems.

A method is proposed to realize projective synchronization in different chaotic systems. The structure of the controllers is designed based on Lyapunov stability theory. Single-mode laser Lorenz system and Rössler system are taken as examples to verify the effectiveness of the method. The method is proper to any chaotic systems, and it can be generally used. Meanwhile both projective synchronization and anti-synchronization of different chaotic systems can be realized by adjusting the scale factor, so it's more flexible and has a better prospect in the field of automatic control and other applications.

II. DESIGNING OF THE CONTROLLER

Single-mode laser Lorenz system and Rössler system are taken as examples to present the principle of the method and the structure of the controller.

Single-mode laser Lorenz system is taken as a target system, and the dynamic equation is as follows[11]

$$\begin{cases} \dot{x}_1 = -\sigma(x_1 - y_1) \\ \dot{y}_1 = x_1(\mu - z_1) - y_1 \\ \dot{z}_1 = x_1 y_1 - r z_1 \end{cases} \quad (1)$$

When the system parameters are given as $\sigma = 10, r = 8/3, \mu = 28$, the system is in chaos, and the phase map is shown in Fig. 1

Rössler system is taken as a response system, and it can be described as follows[12]

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 \\ \dot{y}_2 = x_2 + a y_2 \\ \dot{z}_2 = b + x_2 z_2 - c z_2 \end{cases} \quad (2)$$

When the parameters $a = b = 0.2, c = 5.7$,the system is in chaos, and the phase map is shown in Fig. 2.

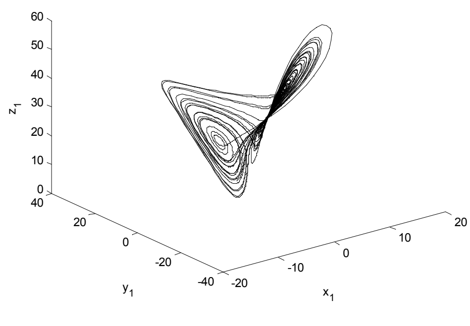

Figure 1. Phase map of system (1)

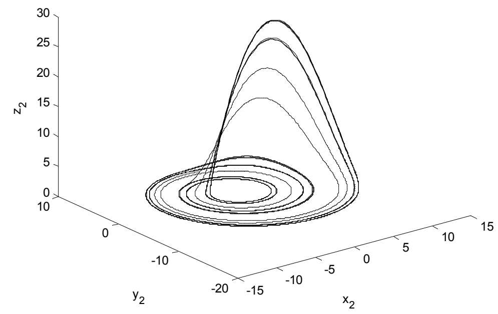

Figure 2. Phase map of system (2)

It's obviously seen that the nonlinear functions of the two systems are different, and the phase maps are also different in shape and model, they are two different chaotic systems. To achieve projective synchronization and projective anti-synchronization, the controller $u_i(t)$, $(i=1,2,3)$ are added on the response system, then Eq.(2) can be rewritten as the following

$$\begin{cases}\dot{x}_2 = -y_2 - z_2 + u_1(t)\\ \dot{y}_2 = x_2 + ay_2 + u_2(t)\\ \dot{z}_2 = b + x_2 z_2 - cz_2 + u_3(t)\end{cases} \quad (3)$$

The error variables between the two systems are defined as the following

$$\begin{cases}e_1 = kx_2 + x_1\\ e_2 = ky_2 + y_1\\ e_3 = kz_2 + z_1\end{cases} \quad (4)$$

Where, k is the scale factor. The Lyapunov function is constructed as

$$V = \frac{1}{2}e_1^{\,2} + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \quad (5)$$

Then we have

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3\\ &= -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} + e_1\{e_1 - \sigma(x_1 - y_1) + k[-y_2 - z_2 + u_1(t)]\}\\ &\quad + e_2\{e_2 + (\mu x_1 - x_1 z_1 - y_1) + k[x_2 + ay_2 + u_2(t)]\}\\ &\quad + e_3\{e_3 + (x_1 y_1 - rz_1) + k[b + x_2 z_2 - cz_2 + u_3(t)]\}\end{aligned} \quad (6)$$

If the controller is taken as

$$\begin{cases}u_1(t) = \dfrac{1}{k}[-e_1 + \sigma(x_1 - y_1) + k(y_2 + z_2)]\\ u_2(t) = \dfrac{1}{k}[-e_2 - \mu x_1 + x_1 z_1 + y_1 - k(x_2 + ay_2)]\\ u_3(t) = \dfrac{1}{k}[-e_3 - x_1 y_1 + rz_1 - k(b + x_2 z_2 - cz_2)]\end{cases} \quad (7)$$

then $\dot{V} = -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} \le 0$ (8)

According to Lyapunov stability theory[13], the state error converges to zero as time tends to infinity,

$$\lim_{t\to\infty} e_i(t) = 0 \qquad \text{(i=1, 2, 3)}$$

the projective synchronization is achieved.

Projective synchronization of two different chaotic systems with uncertain parameters is further studied.

Single-mode laser Lorenz system with uncertain parameters and Rössler system are taken as examples.

The error variables between the two systems are defined as the following

$$\begin{cases}e_1 = x_1 + kx_2\\ e_2 = y_1 + ky_2\\ e_3 = z_1 + kz_2\end{cases} \quad (9)$$

The parameter errors are defined as

$$\begin{cases}e_\sigma = \sigma - \hat{\sigma}\\ e_\mu = \mu - \hat{\mu}\\ e_r = r - \hat{r}\end{cases} \quad (10)$$

Where, $\hat{\sigma}, \hat{\mu}, \hat{r}$ are parameter identifications of σ, r, μ.

The Lyapunov function is constructed as

$$V = \frac{1}{2}e_1^{\,2} + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_\sigma^{\,2} + \frac{1}{2}e_\mu^2 + \frac{1}{2}e_r^2 \quad (11)$$

Then we have

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_\sigma\dot{e}_\sigma + e_\mu\dot{e}_\mu + e_r\dot{e}_r\\ &= -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} + e_1[e_1 - \sigma(x_1 - y_1) - ky_2 - kz_2 + ku_1(t)] - (\sigma - \hat{\sigma})\dot{\hat{\sigma}}\\ &\quad + e_2[(e_2 + \mu x_1 - x_1 z_1 - y_1 + kx_2 + kay_2 + ku_2(t)] - (\mu - \hat{\mu})\dot{\hat{\mu}}\\ &\quad + e_3[e_3 + x_1 y_1 - rz_1 + kb + kx_2 z_2 - kcz_2 + ku_3(t)] - (r - \hat{r})\dot{\hat{r}}\\ &= -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} + e_1[e_1 - \hat{\sigma}(x_1 - y_1) - ky_2 - kz_2 + ku_1(t) - (\sigma - \hat{\sigma})(x_1 - y_1)] - (\sigma - \hat{\sigma})\dot{\hat{\sigma}}\\ &\quad + e_2[e_2 + \hat{\mu}x_1 - x_1 z_1 - y_1 + kx_2 + kay_2 + ku_2(t) + (\mu - \hat{\mu})x] - (\mu - \hat{\mu})\dot{\hat{\mu}}\\ &\quad + e_3[e_3 + x_1 y_1 - \hat{r}z_1 + kb + kx_2 z_2 - kcz_2 + ku_3(t) - (r - \hat{r})z_1] - (r - \hat{r})\dot{\hat{r}}\end{aligned} \quad (12)$$

If the controller is taken as

$$\begin{cases}u_1(t) = [-e_1 - \hat{\sigma}(y_1 - x_1) + ky_2 + kz_2]/k\\ u_2(t) = [-e_2 - \hat{\mu}x_1 + x_1 z_1 + y_1 - kx_2 - kay_2]/k\\ u_3(t) = [-e_3 - x_1 y_1 + \hat{r}z_1 - kb - kx_2 z_2 + kcz_2]/k\end{cases} \quad (13)$$

The structure of the parameter identifications are taken as

$$\begin{cases}\dot{\hat{\sigma}} = (y_1 - x_1)e_1\\ \dot{\hat{\mu}} = xe_2\\ \dot{\hat{r}} = -z_1 e_3\end{cases} \quad (14)$$

then $\dot{V} = -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} \le 0$ (15)

According to Lyapunov stability theory,

$$\lim_{t\to\infty} e_i(t) = 0 \qquad \text{(i=1, 2, 3)}$$

The projective synchronization is achieved.

III. SIMULATION

Simulation is made according to the results above. During the simulation, initial values of the systems are taken respectively as $x_1(0)=0.2$, $y_1(0)=-1.0$, $z_1(0)=-1.0$, $x_2(0)=-4.0$, $y_2(0)=2.0$, $z_2(0)=1.1$. Keep the parameters unchanged, when the scale factor is taken as $k=-2$, after control is added, the simulation results are shown in Fig.3-5.

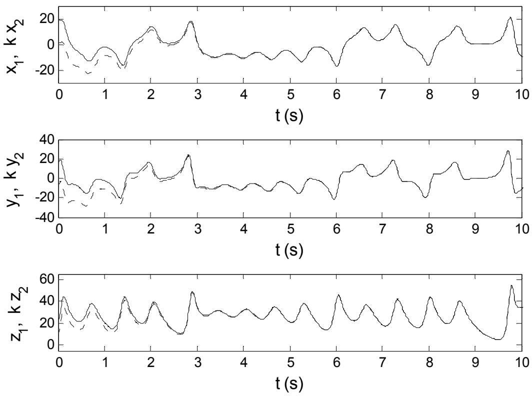

Figure 3. The evolution of the state variable vs time t $(k=-2)$

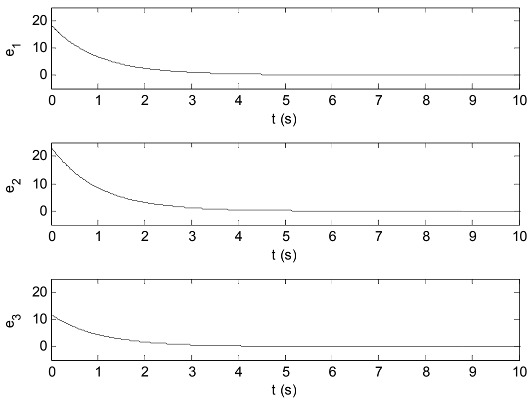

Figure 4. The evolution of the error variables vs time t $(k=-2)$

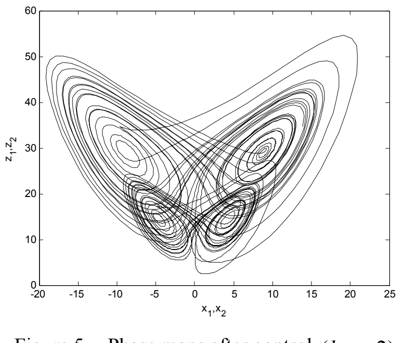

Figure 5. Phase maps after control $(k=-2)$

From Fig. 3-5, it's easy to see that when the scale factor is taken as $k=-2$, after the controller is added, the track of state variables of the two systems all reach synchronization and remain stable all the time, the error variables tend to zero, and as time evolves, the error curves keep steady without oscillating, which demonstrates the method to be effective and feasible. Meanwhile, the trends of the two systems are the same, and the variables of the two systems have a difference of 2 times in size, which means the projective synchronization of two systems is achieved.

It's also found that when the scale factor k is given as other values, synchronization can also be achieved, and when the value of k is given as $k=3$, the simulation results are shown in Fig.6-8.

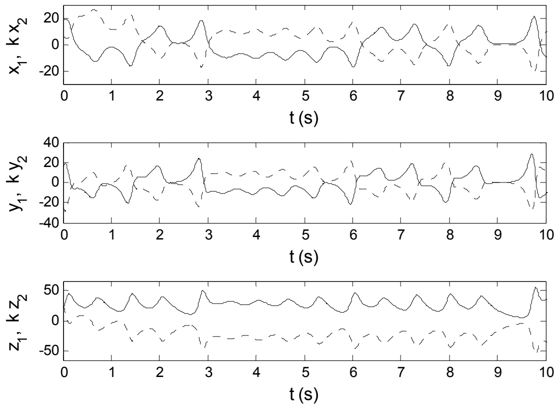

Figure 6. The evolution of the state variable vs time t $(k=3)$

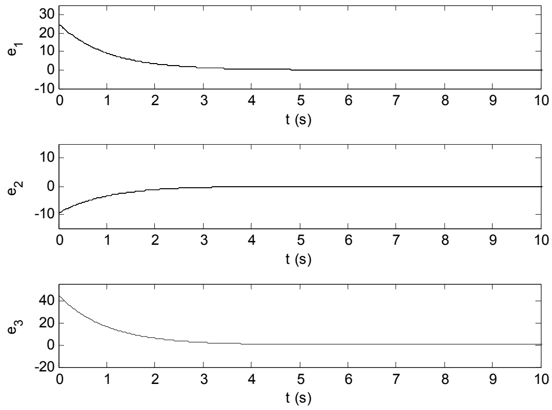

Figure 7. The evolution of the error variables vs time t $(k=3)$

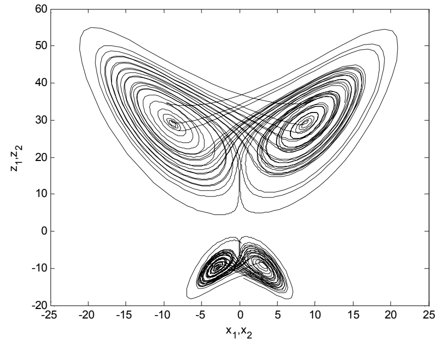

Figure 8. Phase maps after control $(k = 3)$

It's seen from Fig. 6-8 that when the scale factor k is given as $k = 3$, ,after the controller is added, the state variables of the two systems with concerning of the scale factors has equal amplitude and opposite directions, the error variables tend to zero, and as time evolves, the error curves keep steady without oscillating, Furthermore, the trends of the two systems are in opposite directions, and the variables of the two systems have a difference of 3 times in size, which means the projective anti-synchronization is realized.

When the Single-mode laser Lorenz system is with uncertain parameters, the initial value of the parameter identifications are given as $\hat{a}(0) = 10.5$, $\hat{b}(0) = 26.5$, $\hat{c}(0) = 4.5$, when the controller $u_i(t)$, ($i = 1,2,3$) and the parameter identifications are added, the scale factor is taken as $k = 1.1$, simulation results are shown in Fig.9-11.

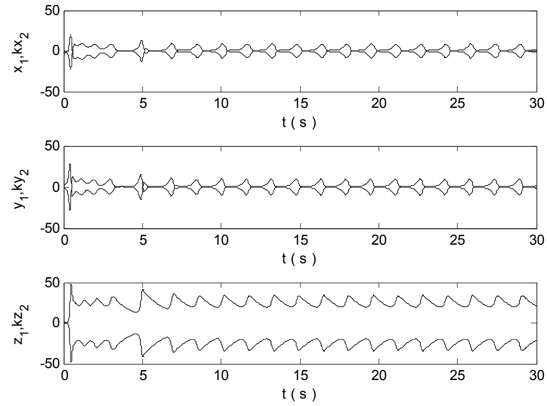

Figure 9. State variables vs time t

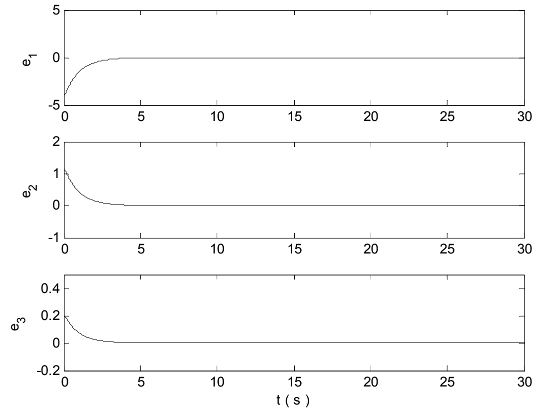

Figure 10. Error variables e_1, e_2, e_3 vs time t

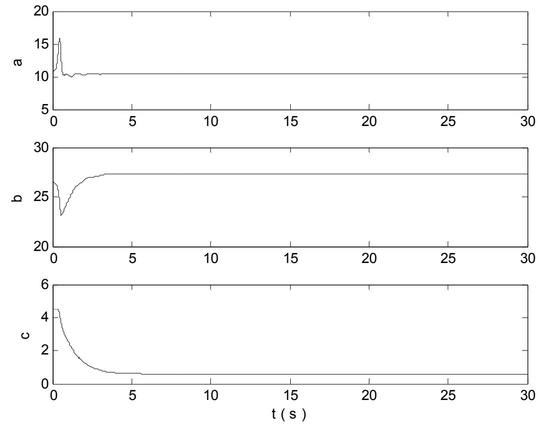

Figure 11. Uncertain parameters of system (1) vs time t

From Fig. 9-11, it's easy to see that after the controller is added, the state variables of the two systems with concerning of the scale factors has equal amplitude and opposite directions, the error variables tend to zero, and as time evolves, the error curves keep steady without oscillating. Meanwhile, the uncertain parameters approach to a fixed value, the unknown parameters are identified.

IV. CONCLUSION

A method is presented to realize projective synchronization in different chaotic systems. The structure of the controllers is given. Single-mode laser Lorenz system and Rössler system are taken as examples to verify the effectiveness of the controllers. The method is proper to not only the two systems we took in the paper, but also to any chaotic systems, and it can be generally used. Meanwhile, projective synchronization and projective anti-synchronization can be achieved with variables synchronized in any proportions, which is more flexible and has a better prospect in applications.

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