Application and Simulation of Wavelet Packet Transform in Underwater Acoustic Signal De-noising

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***Abstract*—In this paper, wavelet packet algorithm was used in the de-noising of underwater acoustic signal processing since wavelet packet algorithm is suitable for non-stationary signal processing. The principle of wavelet packet de-noising algorithm was described and the MSE of underwater acoustic was improved by means of wavelet packet de-noising greatly .The result of computer simulation by Matlab illustrates that de-noising algorithm is an effective signal preprocessing method.**

Keywords-Signal de-noising, wavelet, wavelet packet, Matlab

I. INTRODUCTION

The ambient noise is unavoidable source of interference for the underwater acoustic instruments. With the low SNR of actual acoustic signal, the signal is often overwhelmed by the noise in the far field case. The wavelet packet transform(WPT) is an extension from wavelet transform. WPT is a kind of more detailed analysis and reconstruction methods, because it decomposes not only low-frequency but also high-frequency part of the signals .Compared with the wavelet analysis, wavelet packet analysis has better de-noising effect.

II. PRINCIPLE OF WAVELET PACKET TRANSFORM

In the wavelet packet frame, the de-noising idea is same as in the wavelet frame, except decomposing the low and high frequency part of spectrum at the same time. Apart from that the method of thresholding process in the wavelet packet is the same as in the wavelet analysis.

The continuous wavelet transform for signal $f(t) \in L^2(R)$ is $C_f(a,b) = < f , \psi_{a,b} > = |a|^{-\frac{1}{2}} \int_R f(t) \overline{\psi(\frac{t-b}{a})} dt$, and its reconstruction formula (inverse transform) is $f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{a^2} C_f(a,b) \psi(\frac{t-b}{a}) da db$, where $C_\psi = \int_0^\infty \frac{|\hat{\psi}(\bar{\omega})|}{|\bar{\omega}|} d\bar{\omega} < \infty$, $\psi(t)$ stands for wavelet function, $\overline{\psi(t)}$ for complex conjugate function, C_ψ and $f(t)$ is independent. $\psi(t)$ will become $\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi(\frac{t-b}{a})$ by dilating and translating, $a,b \in R$, $a \neq 0$ .If we set $a = 2j$, $b = 2^j k b_0$, k ranges over Z, and b_0 is normalized, then there exists $\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi(\frac{t-b}{a}) = a^{-\frac{j}{2}} \psi(2^{-j} t - k) = \psi_{j,k}(t)$, and we get the discrete wavelet transform coefficients $C_{j,k} = \int_{-\infty}^{\infty} f(t) \overline{\psi_{j,k}(t)} dt = < f, \psi_{j,k}(t) >$,its reconstruction formula is $f(t) = C \sum_j \sum_k C_{j,k} \psi_{j,k}(t)$, C is a constant independent of signals.

During the wavelet packet decomposition, the high-frequency and low- frequency band in the frequency domain occupied respectively half of the wide frequency band . When decomposed next time, low-frequency signal is divided into two equally frequency wide band, so did high-frequency .The decomposition next time is the same as before. The process of WPT by filters is similar to the WT. The difference between WPT and WT is that only low-frequency band is divided into two bands continuously in WT, while both a high low frequency band is divided in WPT. Finally, the entire band in WPT is divided into uniform frequency bands.

The coefficients of $f(t)$ in the wavelet transform is $c_k^{n,j}$,the Nth Wavelet coefficients is $c_k^{n,j} = 2^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(t) \overline{\psi_n(2^{-j} t - k)} dt$.

Wavelet packet decomposition algorithm is: with $c_k^{n,j}$, the nth and j th layer coefficients $c_k^{n,j-1}$ is decomposed into wavelet pack coefficients $\{c_l^{2n,j}, c_l^{2n+1,j} : k \in Z\}$, that is

$$\begin{cases} c_l^{2n,j} = \sum_k h_{l-2k} c_k^{n,j-1} \\ c_l^{2n+1,j} = \sum_k g_{l-2k} c_k^{n,j-1} \end{cases}.$$

As is shown in Figure 1, where ↓2 means down-sampling and ↑2 means up-sampling.

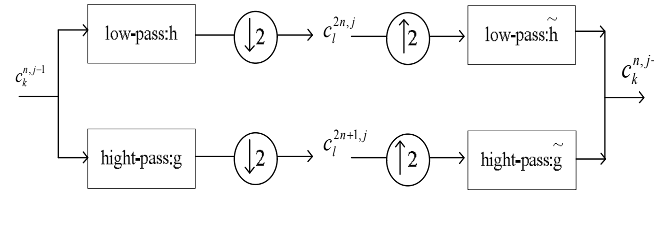

Figure 1. wavelet packet decomposition and reconstruction based on filters

Wavelet packet reconstruction algorithm is: with $c_k^{2n,j}$, $c_k^{2n+1,j}$, for $c_l^{n,j-1}$,that is $c_l^{n,j-1} = \sum_k h_{2k-l} c_k^{n,j} + \sum_k g_{2k-l} c_k^{n,j}$, where $g_k = (-1)^k \overline{h}_{-k+1}$. And up-sampling must be processed during reconstruction.

h can be taken as low-pass filter coefficients; g as high-pass filter coefficients. Response function of the low- pass filter is $H(\omega) = 2^{-0.5} \sum_{k \in z} h_k e^{-ik\omega}$, high pass filter response function is $G(\omega) = 2^{-0.5} \sum_{k \in z} g_k e^{-ik\omega} = 2^{-0.5} \sum_{k \in z} (-1)^k \overline{h}_{-k+1} e^{-ik\omega}$, wavelet function is $\psi(x) = 2^{-0.5} \sum_{k \in z} g_k \phi(2x-k)$, the scaling function $\phi(x) = 2^{-0.5} \sum_{k \in z} h_k \phi(2x-k)$, $\{c_k^{n,j}\}$ is called the wavelet packet on h_k. Following figure 2 is shown for three-layer space spatial analysis by wavelet packet decomposition.

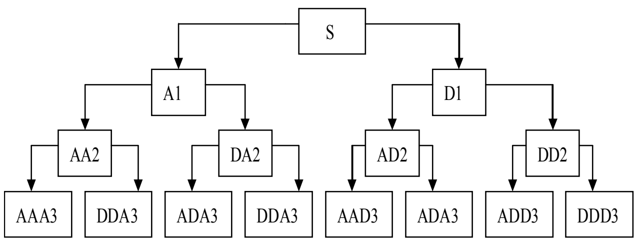

Figure 2. Wavelet packet decomposition tree

In figure 2, S is the signal, A is low frequency (approximation part), D is high- frequency (details part).The number is the order number of decomposition (the number of scales). The signal S can be expressed as a variety of decomposition methods, for example $S = AA2 + DA2 + D1$, $S = A1 + AD2 + ADD3 + DDD3$ and so on.

In figure 2 the first line represents the frequency band belongs to original signal, the following lines number represents the scale of wavelet decomposition, the number of columns is the frequency and location parameters. From the figure 2 we can see that after decomposition the signal frequency band is divided into two part, get the second layer which has a high and low frequency sub-band. Only low-frequency in WT is decomposed continuously, high-frequency sub-band unchanged. In WPT each sub-band is divided into another two lower levels sub-band. The whole frequency band is covered by layers of sub-band signals . WPT breaking limitations that the high frequency space can't be decomposed in WT although it can divide the low frequency limitations of space . So WPT can get more extensive time-frequency localization information, and is more suitable for transient signals, non- stationary signal analysis and detection.

III. THRESHOLDING PRINCIPLE

The main theory of thresholding is based on that orthogonal wavelet transform in particular has a strong decorrelation to the data. It enables the signal to focus the energy on some large wavelet coefficients in the wavelet domain, while the noise energy is distributed in the whole wavelet domain. Therefore, by the wavelet decomposition, the signal amplitude of the wavelet coefficients of magnitude is greater than the noise factor .That relatively large amplitude wavelet coefficients can be considered generally the main signal, while the smaller amplitude coefficient largely noise. By the threshold method the signal factor can be reserved, most of the noise factor can be decreased to zero. Finally, the effective signal can be restored through reconstruct wavelet coefficients obtained by inverse wavelet transform.

The choices of thresholds have two ways: hard and soft. For the soft threshold the absolute value of the signal is compared with the threshold value, when the data of absolute value is less than or equal to the threshold, it will reduce to zero; when greater than the threshold value ,the data becomes the difference value between the data and the threshold value. For the hard threshold the data less than or equal to the threshold point becomes zero, otherwise the data unchanged. These formulas are as following.

Hard threshold method:

$$\hat{W}_{j,k} = \begin{cases} W_{j,k}, |W_{j,k}| \geq \lambda \\ 0, |W_{j,k}| < \lambda \end{cases},$$

Soft threshold method:

$$\hat{W}_{j,k} = \begin{cases} \operatorname{sgn}(W_{j,k})(|W_{j,k}| - \lambda), |W_{j,k}| \geq \lambda \\ 0, |W_{j,k}| < \lambda \end{cases}$$

For hard threshold method, the department $\hat{W}_{j,k}$ is discontinuous at $W_{j,k} = \lambda$, which brings to the reconstructed signal oscillation; and soft threshold method to calculate $\hat{W}_{j,k}$ with overall good continuity. Soft threshold is chosen in this article.

IV. WAVELET PACKET DE-NOISING PROCESSING STEPS

Wavelet packet de-noising can be carried out according to the following four steps:

- Decompose signal by wavelet packet. Choose wavelet function, level N, then decompose the signal S to N layers wavelet packet;
- Calculate the optimal tree (ie to determine the best wavelet packet basis). Calculate the best tree with given Entropy standard. for Matlab GUI there is a special "Best Tree" button used to calculate the best tree;
- Threshold the wavelet packet coefficients. Select the appropriate threshold from 1 to N frequency coefficients and quantifying;
- Reconstruct wavelet packet. According to the Nth-low-frequency wavelet coefficients and Nth-high-frequency coefficients quantified, reconstructing wavelet packet.

Among these four steps, the most critical step is to select the threshold and how to quantify the threshold. To some extent, it is directly related to the quality of signal de-noising.

A strategy to further decomposition is generally determined by problems and signal energy distribution in actual process. Information entropy is the guider as characteristics function for Wavelet packet decomposition. Entropy is one measure of information regularity .There are several major entropy, Shannon entropy, P-order standard entropy, log energy entropy, entropy thresholding, SURE entropy. SURE entropy is chosen for this paper, no longer described for limited space.

V. DE-NOISING EVALUATION CRITERIA

Concentrated in the high-frequency WPT, details of the regional part of the signals will be removed as the noise component, finally the details of the smoothing phenomenon will appear obviously. So the evaluation criterion of information loss is needed to describe the relative degree of deviation from the original signal. These criteria are generally known as the fidelity criteria. Fidelity criteria in common use are: root mean square error (MSE), signal to noise ratio (SNR), peak signal to noise ratio (PSNR) and so on. Root mean square error (MSE) is used to describe similarity between the original signal and de-noised signal. MSE is defined as:

$$MSE = \frac{1}{N}\sum(\bar{x} - x)^2$$

x stands for original signal, $\bar{x}$ estimates signal de-noised by WPT , N for the signal length. The smaller the standard deviation after de-noising, the higher the degree of approximation between de-noising signals and the original signal, the better the de-noising.

VI. APPLICATION AND SIMULATION

In Matlab function of wpdencmp,wden, and soft threshold and the SURE best basis was selected for de-noising and function of wnoise for wavelet noise data in this paper. The simulation results are shown in figure 3.

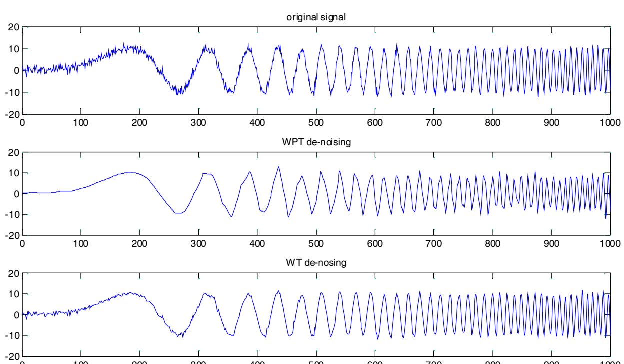

Figure 3. Simulation of the analog signal waveform wavelet packet de-noising

MSE of WPT is 0.8634, MSE of WT is 0.7814, and noise energy of WPT 17.9465 , noise energy of WT is 18.8899 with unit normalized. These data shows that WPT is better than WT in de-noising ability, because WPT analyzes not only the low frequency part of the decomposition but also high frequency part at the same time, so WPT has more accurate local analysis ability.

With some measured data Underwater 40m in Qingdao sea area, de-noising simulation results is shown in Figure 4. That simulation results indicate that the acoustic energy of waveform after de-noising significantly is smaller than the original. The figure 4 shows the soft thresholding significant effect .With the soft threshold de-noising effect is remarkable and better with smoothing.

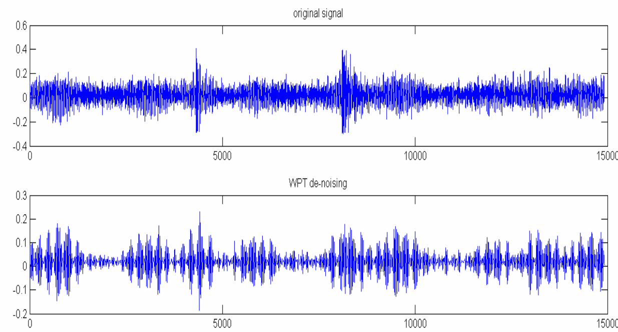

Figure 4. Wavelet packet de-noising of underwater acoustic signal waveform

VII. CONCLUSION

This article describes the principle of wavelet packet transform, thresholding and the general implementation steps, and by Matlab simulation results and related data prove that the signal de-noising based on wavelet packet is valuable. Introduced by the present theory and computer simulation results show that, wavelet packet transform in signal de-noising achieved good results with a wide range of application.

In the de-noising process by WPT, the key is to select the de-noising threshold and to select the WPT function based on experience. In addition, for wavelet packet de-noising, threshold selection layers does not have a precise theory used as a guide .These negative impact is worth further study to eliminate as much as possible.

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