

## Modeling and Simulation of Lazy Eight Aircraft Maneuver

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**Abstract**—The Lazy Eight aircraft maneuver is researched through numerical simulation. First, a six DOF aircraft flight dynamics model is developed. To control the aircraft performing the maneuver, a basic flight control law is designed, employing classical PID controllers in both pitch and roll loop. Commands for maneuver simulation are derived from the descriptions of Lazy Eight maneuver flight procedure. By proper settings of the commands and PID controllers' gain values, Lazy Eight maneuver simulation is conducted exactly according to the maneuver descriptions. The model established in paper also provides researchers and a method to simulate other aircraft maneuvers.

**Keywords**—Flight dynamics, Flight simulation, Control law, PID controller, Lazy Eight

### I. INTRODUCTION

Lazy Eight maneuver is a standard flight training maneuver and also a maneuver used in the evaluation of flying qualities [1,2]. As a training maneuver, Lazy Eight has great value since it could help pilots develop subconscious feel, planning, orientation, coordination, and speed sense. It is challenging for most of the new pilots to perform a Lazy Eight maneuver, because it requires that the pilot plan and execute a complex maneuver involving pitch, bank, airspeed and altitude which are constantly in a state of change over an extended period of time. Therefore, training on ground based flight simulator before real flight is strongly recommended and modeling and simulation of Lazy Eight maneuver is needed. So far, there is little literatures referred simulation of the maneuver.

This paper adopts 6 DOF equations of motion while developing aircraft flight dynamics model, which could cover a wide flight envelop, other than linear equations of motion based on small disturbance theory [3]. PID controllers, where PID stands for proportional, integral and derivative control [4], are used in flight control law, partly because they are effective and partly because they are straightforward to design. The aim of this investigation is to develop a practical method to simulate flight training maneuvers.

### II. MANEUVER DESCRIPTION

The Lazy Eight maneuver is comprised of two consecutive 180-degree turns in opposite directions while making a climb and descent in a symmetrical pattern during each of the turns, as shown in Figure 1. This maneuver derives its name from the manner in which the extended longitudinal axis of the aircraft is made to trace a flight pattern in the form of a figure 8 lying on its side [1].

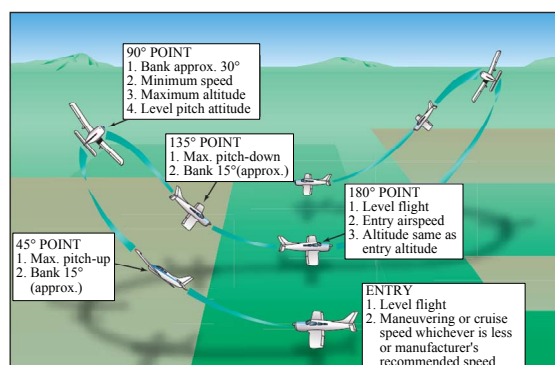


Figure 1. Lazy Eight maneuver

To help pilots accomplishing the maneuver, several key points can be selected on the horizon. Generally, the key points selected are 45°, 90°, 135° and 180° from the direction in which the maneuver is begun. Flight information such as altitude, airspeed and Euler angles of the aircraft at each reference point is demonstrated in Figure 1 in detail. Aircraft entries Lazy Eight maneuver flying straight and level at a specified airspeed. There are smooth and gradual changes in both pitch attitude and bank angle throughout the entire maneuver, which means the pilot must constantly operate longitudinal and lateral stick. After passing through the 180° point, the aircraft immediately starts to roll reversely to do the whole thing over in the opposite direction, completing the second half of the eight.

### III. NONLINEAR AIRCRAFT MOTION MODEL

Considering that there are remarkable variations of aircraft's motion parameters during a Lazy Eight maneuver, the six nonlinear rigid-body equations of motion were used to model the nonlinear aircraft dynamics in translational and rotational motion, which can represent the motion of aircraft more accurately.

Before derivation of the equations of motion, some assumptions have to be made. The aircraft is assumed to be a rigid-body with constant mass density which means there is no relative motion of the aircraft internal components, structural distortion. In addition, the aircraft is symmetry about the X-Z plane in body-axes, as shown in Figure 2. Curvature and rotation of the earth is neglected and the earth is regarded as an inertial reference. The gravity field dose not change with attitude. Atmosphere is considered to be stationary.

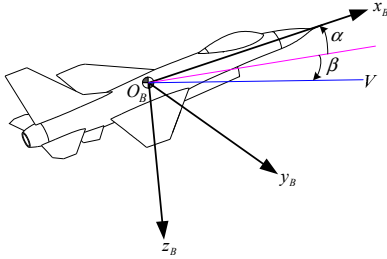


Figure 2. Aircraft body-fixed reference frame  $F_B$

Based on the assumptions above, the equations of motion for the aircraft then can be derived. These equations are collected as follows (12 equations in total):

$$\left. \begin{aligned} X - mg \sin \theta &= m(\dot{u} + qw - rv) \\ Y + mg \cos \theta \sin \phi &= m(\dot{v} + ru - pw) \\ Z + mg \cos \theta \cos \phi &= m(\dot{w} + pv - qu) \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} L &= I_x \dot{p} - I_{xz}(\dot{r} + pq) - (I_y - I_z)qr \\ M &= I_y \dot{q} - I_{xz}(r^2 - p^2) - (I_z - I_x)rp \\ N &= I_z \dot{r} - I_{xz}(\dot{p} - qr) - (I_x - I_y)pq \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} \dot{\phi} &= p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \\ \dot{\theta} &= q \cos \phi - r \sin \phi \\ \dot{\psi} &= (q \sin \phi + r \cos \phi) \sec \theta \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \dot{x}_E &= u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\ &\quad + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \dot{y}_E &= u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \\ &\quad + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ \dot{z}_E &= -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta \end{aligned} \right\} \quad (4)$$

Equations (1) ~ (4) are force equations, moment equations, kinematics equations and navigation equations respectively. Aerodynamic data used in these equations is derived from F-16 wind-tunnel tests results [5, 6].

It is more convenient to express the force equations in terms of  $V$ ,  $\alpha$  and  $\beta$  instead of  $u$ ,  $v$  and  $w$ , because  $V$ ,  $\alpha$  and  $\beta$  can be measured directly on real aircraft [7]. The relationship between  $V$ ,  $\alpha$ ,  $\beta$  and  $u$ ,  $v$ ,  $w$  are:

$$\left. \begin{aligned} V_T &= \sqrt{u^2 + v^2 + w^2} \\ \alpha &= \arctan(w/v) \\ \beta &= \arcsin(v/V_T) \end{aligned} \right\} \quad (5a)$$

$$\left. \begin{aligned} u &= V_T \cos \alpha \cos \beta \\ v &= V_T \sin \beta \\ w &= V_T \sin \alpha \cos \beta \end{aligned} \right\} \quad (5b)$$

Atmosphere model is an approximation of the International Standard Atmosphere (ISA), as described by equations (6) [8]. Given the aircraft's altitude, equations (6) return current temperature ( $T$  in Kelvin), air density ( $\rho$  in  $\text{kg/m}^3$ ) and speed of sound ( $a$  in m/s).

$$\left. \begin{aligned} T &= T_0 - 0.0065h \\ \rho &= \rho_0 e^{-\frac{g}{287.05T}h} \\ a &= \sqrt{1.4 \times 287.05T} \end{aligned} \right\} \quad (6)$$

### IV. FLIGHT CONTROL SYSTEM MODEL

Flight control system model includes control surface actuator models, engine model and a basic flight control law, Figure 3 shows the structure of aircraft flight control system.

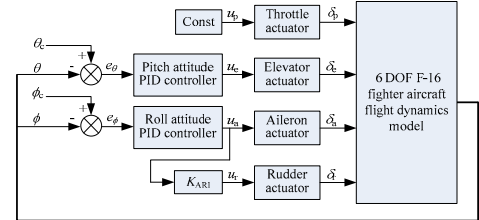


Figure 3. Aircraft flight control system

Control surfaces of the aircraft consist of elevator, ailerons, and rudder. Actuators for the control surfaces are model as first order filters with saturation in range and rate limiter. Structure of control surface actuator loop is shown in Figure 4 and parameters for each actuator are listed in Table I.

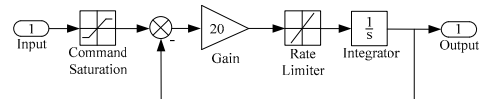


Figure 4. Control surface actuator loop

TABLE I. MAXIMUM VALUE AND RATE LIMIT OF CONTROL INPUT

Control	MIN.	MAX.	Rate Limit
Elevator	-25 deg	25 deg	60 deg/s
Ailerons	-25 deg	20 deg	90 deg/s
Rudder	-30 deg	30 deg	80 deg/s

The Engine is modeled as a simple first order lag in actual power level  $P_a$  to command power level  $P_c$ , equation (7). The engine power level time constant  $\tau_{eng}$  is chosen at 1 sec.  $P_c$  is a function of throttle position  $\delta_{th}$ , equation (8). Given actual power level, altitude and Mach number, engine thrust force then can be calculated by equation (9).

$$\dot{P}_a = \frac{1}{\tau_{eng}}(P_c - P_a) \quad (7)$$

$$P_c(\delta_{th}) = \begin{cases} 64.94\delta_{th} & \text{if } \delta_{th} \leq 0.77 \\ 217.38\delta_{th} - 117.38 & \text{if } \delta_{th} > 0.77 \end{cases} \quad (8)$$

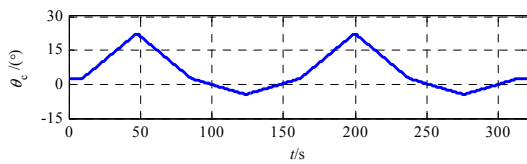
$$F_T = F_T(P_a, h, Ma) \quad (9)$$

Based upon the features of Lazy Eight maneuver, pitch and roll attitude are selected as control commands in flight control law design. Elevator-control channel and aileron-control channel are respectively used to control pitch and roll attitude. Each control channel is implemented by a PID controller. The aileron actuator input has a cross-connection to the rudder actuator via a gain KARI (known as the aileron-rudder interconnect [8]) to prevent the aircraft from sideslip. The throttle is set at a fixed position, usually close to the initial position, to control the aircraft holding at the same altitude when the maneuver is begun and completed.

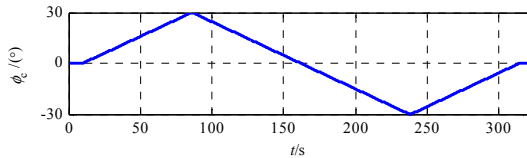
### V. MANEUVER SIMULATION

The initial flight condition for maneuver simulation is set to straight and level flight at the airspeed of 180m/s, 3000m above ground level. Time span of the simulation is 320s in total, with the first 10s and last 6s level flying.

Pitch command  $\theta_c$  and roll command  $\phi_c$  are shown in Figure 5, which are derived from Lazy Eight maneuver descriptions. The first half of the maneuver is between 10s and 162s. Max.  $\phi_c$  is 30° (at 90° PIONT). Max. and min.  $\theta_c$  are 20° (at 45° PIONT), -7° (at 135° PIONT) respectively. Time span for the second half of the maneuver is defined between 162s and 314s. Pitch command for the second half is just the same as the first half, while roll command the opposite.



(a) Pitch command



(b) Roll command

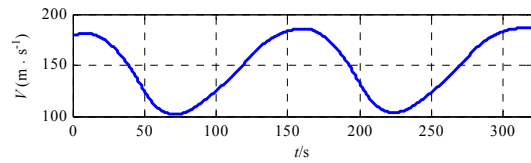
Figure 5. Commands for pitch and roll attitude control

Throttle position is fixed at 0.22, slightly greater than level flight throttle setting. The aileron-rudder interconnect gain KARI is set to 0.75. Through iterative tuning of the gains for PID controllers, a set of optimal gain values have finally been achieved, as listed in Table II.

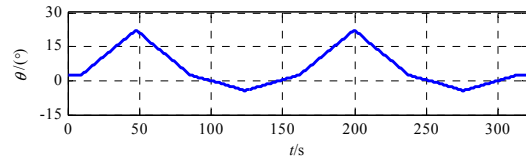
TABLE II. GAIN VALUES FOR PITCH AND ROLL PID CONTROLLER

Gain Type	Value (Pitch controller)	Value (Roll controller)
$K_p$	-1.8	-0.8
$K_i$	-0.4	-0.1
$K_d$	-0.4	-0.05

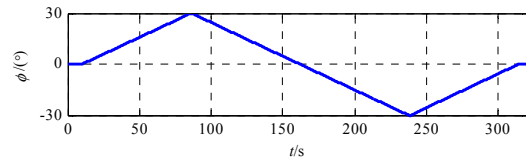
Figure 6-8 show numerical simulation results of Lazy Eight maneuver. Figure 6 shows airspeed and attitude variation of the aircraft. Since the throttle position is fixed, airspeed varies with the variation of altitude, decreasing as aircraft climbing and increasing as aircraft diving. Pitch and bank angle response match well with control commands input, which is primarily owing to the effective PID controllers. Yaw angle reaches 180° in the end of the first turn, and returns to the original 0° when the maneuver is completed.



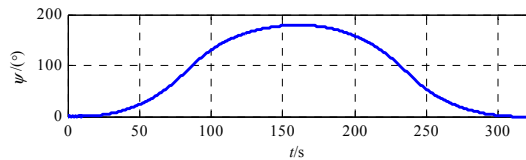
(a) Airspeed



(b) Pitch angle



(c) Bank angle

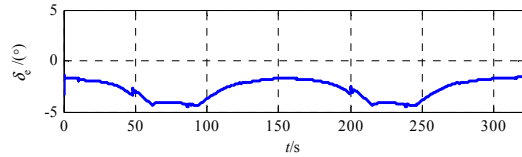


(d) Yaw angle

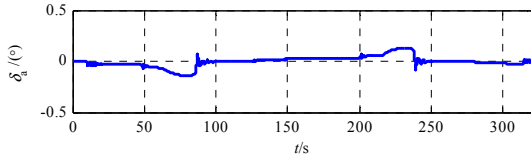
Figure 6. Airspeed and attitude variations

Figure 7 shows control surface deflections during maneuver simulation. All the control surfaces deflect smoothly and the deflections are small, mainly due to the high control effectiveness at high speed. Maximum

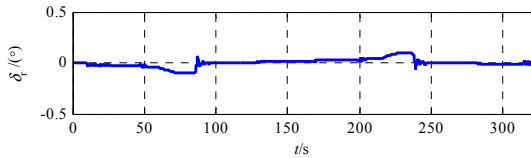
deflections get at lowest speed (90° and 270° PIONT). Ailerons and rudder deflect similarly because of the aileron-rudder interconnect control law.



(a) Elevator deflections



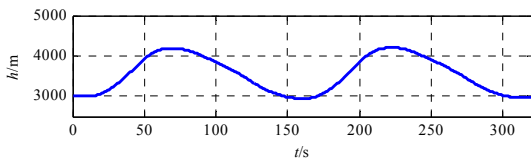
(b) Ailerons deflections



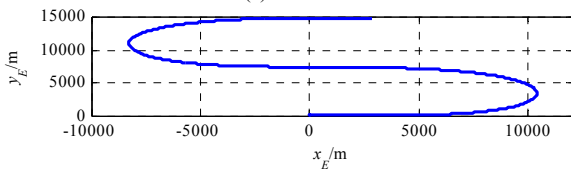
(c) Rudder deflections

Figure 7. Control surface deflections

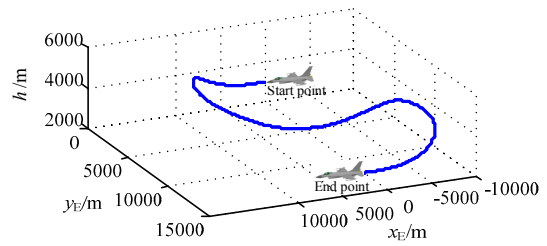
Figure 8 shows aircraft altitude and trajectory in different views. The horizontal plane trajectory consists of two strictly symmetrical turns. Altitudes at the end of each turn are very close to entry altitude which is consistent with the maneuver description. Three dimension trajectory is similar to that demonstrated in Figure 1.



(a) Altitude



(b) Horizontal plane trajectory



(c) Three dimension trajectory

Figure 8. Simulation results of aircraft altitude and trajectory

## VI. SUMMARIES

This paper presents the process of modeling and simulation of Lazy Eight maneuver. Classical PID controllers are applied in the flight control law and have shown well control effectiveness during the simulation. Result of Lazy Eight maneuver simulation has been successfully used in close formation flying qualities evaluation, which suggests an engineering application value of this investigation. The whole process described in this paper provides a convenient and rapid method of simulating aircraft maneuvers.

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