

## A New Design of AIC Structure for UWB Signal Sampling in the Application of Localization

Gang Yang

School of Information Engineering,  
Communication University of China,  
Beijing, China  
gangy@cuc.edu.cn

Xin Wan

Engineering center of Digital Audio & Video,  
Communication University of China,  
Beijing, China  
wallin82@cuc.edu.cn

**Abstract**-UWB signals are very suitable for localization, since their high time resolution can provide precise time of arrival (TOA) estimation. However, a major challenge in UWB signal processing is the requirement for high sampling rate. The theory of compressive sensing (CS) makes processing UWB signal at a low sampling rate possible via analog-to-information converter (AIC) if the signal has a sparse representation in a certain space. In this paper, a new framework called residual analog-to-information converter (RAIC) is proposed to improve AIC performance by adding a signal prediction/cancellation module in front of AIC. The structure of RAIC is designed and analyzed.

**Keywords**-residual analog-to-information converter (RAIC), UWB, CS, Localization

### I INTRODUCTION

UWB signals are very suitable for localization, since their high time resolution can provide very precise time of arrival (TOA) estimation, as suggested by the Cramer-Rao lower bound [1]. However, to digitize a UWB signal, very high sampling rate is required according to Nyquist sampling theorem which is typically on the order of tens of GHz. This ADC resolution "bottleneck" has been extensively studied and present solutions can be divided into two main approaches. The most straightforward approach is to divide the whole receiving frequency band into many subbands and utilize many parallel ADCs [2]. However, this approach faces the problems in combining the individual digitized signals. Another approach is to apply analog nonlinear signal compression techniques [3], such as logarithmic compression to reduce the dynamic range of the signal. However, the nonlinear distortion could reduce the SNR of the signal. Based on the above method, an adaptive prediction and cancellation digitization method is proposed in [4], which reduces the high dynamic range of wideband analog signals before digitization. However, the precondition of using this method is that the dynamic range of the signal should be very high and the power of one or several channels must be much higher than the power of the other signals.

All of the above methods are based on Nyquist theory and the sampling rates are high. Coming along with theorem called CS [5], it's approved that certain digital signals can be recovered from far fewer samples than ADC. However, CS is only applicable to discrete signals. In some important

situations the samples of analog signal may also be difficult to obtain such like UWB signals. In order to solve analog signals by using CS theorem, AIC [6-10] is developed which can be used to sample signals far below the Nyquist rate.

In this paper, a new approach called residual analog-to-information conversion (RAIC) is proposed to improve AIC performance by adding a signal prediction/cancellation module in front of AIC. The structure of RAIC is designed and analyzed.

### II COMPRESSIVE SENSING BACKGROUND

#### A. Basic theory of CS

CS theorem indicates that certain digital signals or images can be recovered from far fewer samples than traditional methods. CS theorem indicates that an  $N \times 1$  discrete-time signal vector  $x$  can be recovered from an  $M \times 1$  samples which is far fewer than  $N$ . To make this possible, CS relies on two principles: sparsity and incoherence[11].

Sparsity expresses the idea that the number of freedom degrees of a discrete time signal may be much smaller than its length. For example, in the equation  $x = \psi a$ , by  $K$ -sparse we mean that only  $K \leq N$  of the expansion coefficients  $a$  representing  $x = \psi a$  are nonzero. By compressible we mean that the entries of  $a$ , when sorted from largest to smallest, decay rapidly to zero. Such a signal is well approximated using a  $K$ -term representation.

Incoherent is talking about the coherence between the measurement matrix  $\psi$  and the sensing matrix  $\Phi$ . The sensing matrix is used to convert the original signal to fewer samples by using the transform  $y = \Phi x = \Phi \psi a$ . The definition

of coherence is 
$$\mu(\Phi, \Psi) = \sqrt{n} \cdot \max_{1 \leq k, j \leq n} |\langle \phi_k, \phi_j \rangle|$$
 It

follows from linear algebra that is  $\mu(\Phi, \Psi) \in [1, \sqrt{n}]$ . In CS, it concerns about low coherence pairs. The results in show that random matrices are largely incoherent with any fixed basis  $\Psi$ . Gaussian or  $\pm 1$  binaries will also exhibit a very low coherence with any fixed representation  $\Psi$ .

Since  $M < N$ , recovery of the signal  $x$  from the measurements  $y$  is ill-posed; however the additional assumption of signal sparsity in the basis  $\Psi$  makes recovery both possible and practical.

The signal can be recovered by solving the convex program below

$$\alpha = \arg \min \|\alpha\|_1 \quad \text{s.t. } y = \Phi\Psi\alpha \quad (1)$$

And  $M$  should obey  $M \geq C \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log N$ , where  $C$  is a small constant,  $K$  is the sparsity,  $N$  is the length of the original signal.

In its present form, CS is only applicable to discrete signals. AIC gives us the solution of how to apply CS on analog signals.

### B. Analog-to-information converter

Developing a framework for continuous CS will require defining new analog signal models for sparse signals and constructing an analog system that has CS-compatible properties. Consider a sparse analog signal  $x(t)$ , which can be represented by a finite number of continuous basic functions in a space

$$x(t) = \sum_{n=1}^N \alpha_n \psi_n(t) \quad t \in [0, T], \alpha_n \in \mathbf{R} \quad (2)$$

If there is only a few elements of vector  $\alpha$  is non-zero, the signal is compressible.

The output  $y[m]$  of AIC after signal compressing is expressed as

$$y[m] = \int_{-\infty}^{+\infty} x(\tau) p_c(\tau) h(t-\tau) d\tau \Big|_{t=m\Delta t}, \quad m \in \{1, 2, \dots, M\} \quad (3)$$

Where  $\Delta t$  is the sampling interval of low-rate ADC,  $p_c(t)$  is a pseudo-random PN sequence of  $\pm 1$ ,  $h(t)$  is a low pass filter impulse response. Substituting (3) from (2), (4) can be derived as

$$y[m] = \sum_{n=1}^N \alpha_n \int_{-\infty}^{+\infty} \psi_n(\tau) p_c(\tau) h(m\Delta t - \tau) d\tau, \quad m \in \{1, 2, \dots, M\} \quad (4)$$

Here the  $M \times N$  matrix  $V = \Phi\Psi$  is

$$v_{m,n} = \int_{-\infty}^{+\infty} \psi_n(\tau) p_c(\tau) h(m\Delta t - \tau) d\tau \quad (5)$$

With the output of AIC  $y[m]$  and matrix  $V$ , the non-zero magnitudes  $\alpha$  can be reconstructed with high probability by solving  $l_1$ -sparsest optimization problem.

### III RAIC MODEL DESCRIPTION

The UWB signal has both an ultra-wide bandwidth and a large dynamic range, which inspires us if the dynamic range could be reduced before AIC so that the sampling rate can be even lower than AIC. Based on the above idea, a new RAIC model is proposed (see Figure1).

As illustrated in Figure1, the AIC does not digitize the original received signal  $x(t)$ , but digitize the residual signal

$d(t) = x(t) - \hat{x}(t)$ . So, compared with the original AIC method, RAIC reduced the dynamic range of the received signal, and get even fewer bits than AIC. The RAIC method requires a signal predictor to predict  $\hat{x}(n)$  from the past samples of  $d(n)$ . Furthermore, DAC is used to convert the predicted signal  $\hat{x}(n)$  into an analog signal  $\hat{x}(t)$ . The function of automatic gain control is to keep the residual signal constant.

The SNR of the RAIC method can be expressed as

$$\begin{aligned} \text{SNR}_{\text{RAIC}} &= 10 \log_{10} \frac{E[x^2(t)]}{E[e^2(t)]} \\ &= 10 \log_{10} \frac{E[x^2(t)]E[d^2(t)]}{E[d^2(t)]E[e^2(t)]} \\ &= G_p + \text{SNR}_q \end{aligned} \quad (6)$$

Where  $E[\cdot]$  denotes the mean value,  $e(t)$  is the total digitization noise,  $\text{SNR}_q$  is the conventional ratio that its quantizer can achieve.  $G_p$  is the improvement gain of SNR. For a conventional quantizer, increasing it by 1 bit will increase SNR by 6 bit. For  $G_p=18$  dB, that means the RAIC method requires 3 bits less than AIC to achieve the same SNR.

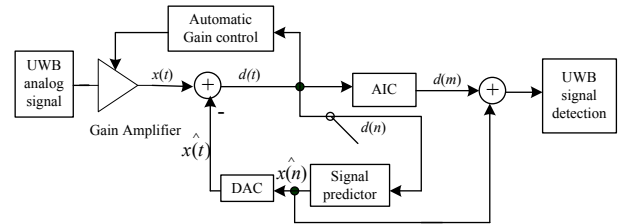


Figure 1. RAIC signal sampling system

### Signal prediction algorithms

The SNR gain achieved by RAIC are mainly determined by the signal predictor gain. In this section, two prediction algorithms are given. The algorithm of the signal predictor is strongly related to the statistical characteristics of the signal. When we have no prior knowledge about the signal, the signal is assumed to be stationary. Auto-Regressive (AR) predictor can be used to these signals.

In a  $p$ -order AR predictor, the estimation of  $\hat{x}(n)$  is

$$\hat{x}(n) = A(n)X^T(n-1) \quad (7)$$

where  $X(n-1) = [x(n-1), x(n-2), \dots, x(n-p)]$  is the latest  $P$  samples of  $x(n)$ ,  $T$  means the transposition, and  $A(n) = [a_1(n), a_2(n), \dots, a_p(n)]$  are the  $P$  coefficients at time  $n$ .

The mean square error of AR predictor can be expressed as

$$\begin{aligned}
 J &= E[e^2(n)] = E[(x(n) - \hat{x}(n))^2] \\
 &= E[x^2(n)] - 2E[x(n)A(n)X^T(n-1)] \\
 &\quad + (A(n)X^T(n-1))^2
 \end{aligned} \tag{8}$$

Let  $\partial J / \partial A(n) = 0$ , the optimal solution for each component of  $A(n)$  is

$$a_p(n) = \frac{E[x(n)]x(n-p)}{X^2(n-1)} \tag{9}$$

With the adaptive steepest-descent gradient algorithm, the prediction coefficients are adjusted by

$$A(n) = A(n+1) + ud(n)X(n-1) \tag{10}$$

Where  $d(n) = x(n) - \hat{x}(n)$ ,  $u$  is a small positive constant called step size.

#### IV UWB SIGNAL SAMPLING

UWB signal is a kind of signals which occupies several GHz of bandwidth by modulating an impulse-like waveform. A typical baseband UWB signal is Gaussian monocycle obtained by differentiation of the standard Gaussian waveform. A second derivative of Gaussian pulse is given by

$$p(t) = A \left[ 1 - 4\pi \left( \frac{t}{T_d} \right)^2 \right] e^{-2\pi \left( \frac{t}{T_d} \right)^2} \tag{11}$$

Where the amplitude  $A$  is used to normalize the pulse energy. Figure.2 shows the time domain waveform of (11). From Figure 2, we see that the duty cycle (the pulse duration divided by the pulse period) is really small. In other aspect of view, UWB signal is sparse in time domain. The Fourier transform (Figure 3) is occupied from near dc up to the system bandwidth  $BS \approx 1/T_d$ .

There are three key elements needed to be addressed in the use of CS theory into UWB signal sampling. 1) How to find a space in which UWB signals have sparse representation 2) How to choose random measurements as samples of sparse signal 3) How to reconstruct the signal.

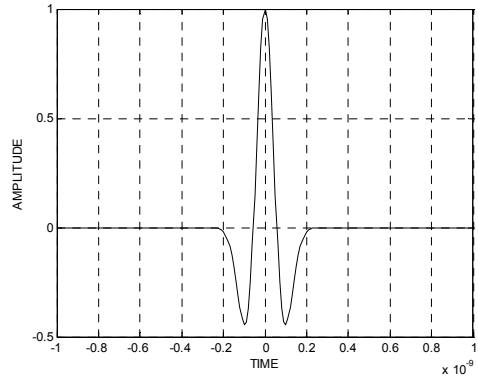


Figure 2. UWB signal in time domain

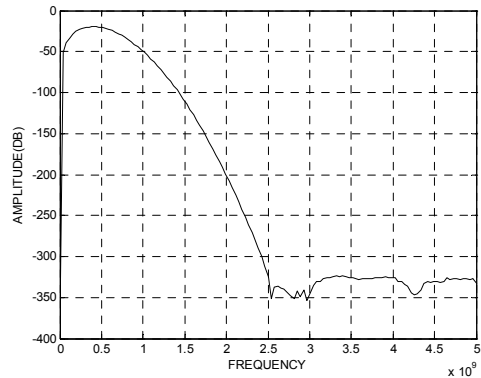


Figure 3. UWB signal in frequency domain

CS is mainly concerned with low coherent pairs. How to find a good pair of  $\Phi$  and  $\Psi$  in which UWB signals have sparse representation is the problem to be solved. Several tests are done to find a suitable pair for UWB signal. In the first test (see Figure 4),  $\Psi$  is spike basis  $\varphi_k(t) = \delta(t - k)$  and  $\Phi$  is random Gaussian matrix. The second test (see Figure 5) takes DCT basis for  $\Psi$  and random Gaussian matrix for  $\Phi$ . The third test (see Figure 6) takes wavelet basis for  $\Psi$  and random Gaussian matrix for  $\Phi$ . those figures demonstrate that the spike basis can recover the signal, while the others can not. The reason is that the UWB signal is sparse in time domain as we mention above. In DCT basis and wavelet basis, the UWB signal is not sparse.

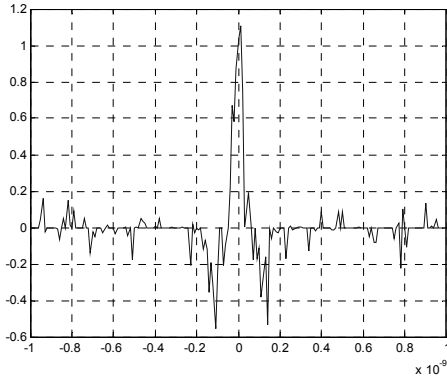


Figure 4. Reconstructed signal from measurements at 20% of the Nyquist rate

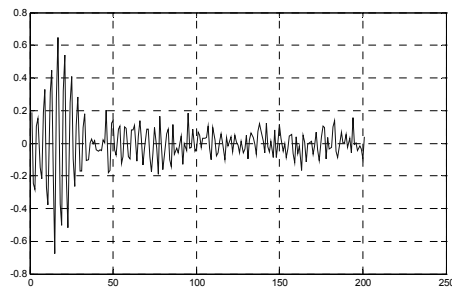


Figure 5. Reconstructed signal on DCT basis

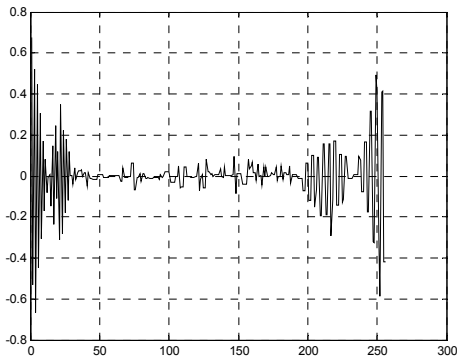


Figure 6. Reconstructed signal on wavelet basis

The mathematical principle can be formulated as

$$\mathbf{s} = \mathbf{G} \cdot \mathbf{E} \cdot p(t) \Big|_{t=kT} + \mathbf{n} \quad k=1 \dots N \quad (12)$$

Where  $\mathbf{s}$  is the sensing vector,  $\mathbf{G}$  is random Gaussian matrix,  $\mathbf{E}$  is spike matrix,  $p(t) \Big|_{t=kT}$  is the Nyquist samples with sample period  $T$ , total samples  $N$ .  $\mathbf{n}$  is the additive noise vector with bounded energy  $\|\mathbf{n}\|_2 \leq \varepsilon$ .

The coherence between measurement matrix  $\mathbf{E}$  and sensing matrix  $\mathbf{G}$  from (4) is near 1.  $\mathbf{G}$  matrix is largely incoherent with  $\mathbf{E}$ . Therefore, in our method, the precondition of sparsity and incoherence are satisfied.

$$u(\mathbf{G}, \mathbf{E}) = \sqrt{n} \cdot \max_{1 \leq k, j \leq n} \left| \langle \mathbf{g}_k, \mathbf{e}_j \rangle \right| \quad (13)$$

Since  $\mathbf{E}$  is spike matrix,  $\mathbf{G} \cdot \mathbf{E} = \mathbf{G}$ .

(12) can be simplified by

$$\mathbf{s} = \mathbf{G} \cdot p(t) \Big|_{t=kT} + \mathbf{n} \quad k=1 \dots N \quad (14)$$

In (14), the CS method is simplified, and the multiply complexity is reduced by  $MN^2$ . Therefore, the UWB signal is suitable for CS and it makes CS simpler and reduces the computation complexity.

The recovery algorithm is

$$\arg \min \|p(t) \Big|_{t=kT}\|_1 \quad \text{such that} \quad \|\mathbf{G} \cdot p(t) \Big|_{t=kT} - \mathbf{s}\|_2 \leq \varepsilon \quad (15)$$

The recovery multiply complexity is reduced by  $N^2$ .

Theorem:

Fix  $p(t) \Big|_{t=kT} \in \mathbb{R}^N$ , and it is  $K$  sparse on a certain basis  $\Psi$ . Select  $M$  measurements in the  $\Phi$  domain uniformly at random. Then if

$$M \geq c \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log N \quad (16)$$

for some positive constant  $c$ , the solution to (15) is success with high probability. From (16), we see that  $M$  is proportional to three factors:  $\mu$ ,  $K$  and  $N$ . If  $\mu$  and  $N$  are fixed, the sparser  $K$  can reduce the measurements needed to reconstruct the signal.

## V RAIC FOR WIRELESS LOCALIZATION

In order to further decrease the UWB signal sampling rate, this paper proposed a residual analog-to-information converter (RAIC). This method can decrease the sampling rate by differentiating  $t+1$  time signal from  $t$  time signal. Examples are done to show the comparison of the proposed method with traditional methods.

In all of the examples, the transmitted signal is demonstrated

as  $s(t) = (1 - 4\pi(\frac{t}{0.2 \times 10^{-9}})^2) \times \exp(-2\pi(\frac{t}{0.2 \times 10^{-9}})^2)$  is used. The bandwidth of the signal is 2.5GHz, and the traditional sampling frequency is 5GHz.

### A Example 1 (in LOS environment)

In the first example, we assume that the signal is passed through Rician channel, the number of multipath is six and the channel is time-invariant. In the first simulation (see Figure 7), the observed time is set to 0.2um. Figure 7(1) shows the UWB signal passing through Rician channel interference at time  $t$  moment. Figure 7(2) shows the UWB signal passing through rician channel interference at time  $t+1$  moment. Figure 7(3) shows the differential signal by subtracting time  $t$  signal from  $t+1$  moment. Figure 7(4) shows the reconstructed signal by CS theory at the sampling rate of 10% of the Nyquist sampling rate. The time delay error of both methods is about 1nm.

In the second simulation (see Figure 8), we shorten the observed time to 0.02 $\mu$ m, all of the other parameters are the same. Figure 8(4) shows the measurement we need to reconstruct the signals is up to 30%. And much more details of the signal can be seen. And the time delay error is 1nm.

Comparing these two simulations, we get the conclusion 1) the receiver could potentially achieve TOA estimation performance with the receiver that works with full Nyquist rates while using 10% of Nyquist sampling rate, although the signal is not recovered precisely. 2) by enlarging the sampling rate by 30%, more detail information of the signal can be recovered. However, for TOA estimation, we don't need to recover the full signal but the peak location of the signal which makes the use of 10% Nyquist sampling rate possible.

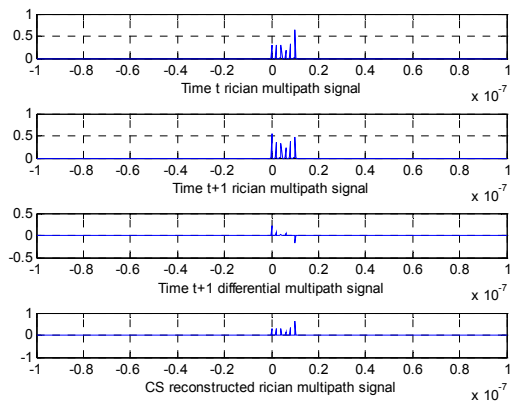


Figure 7. UWB signal sampling based on RAIC (Rician channel)

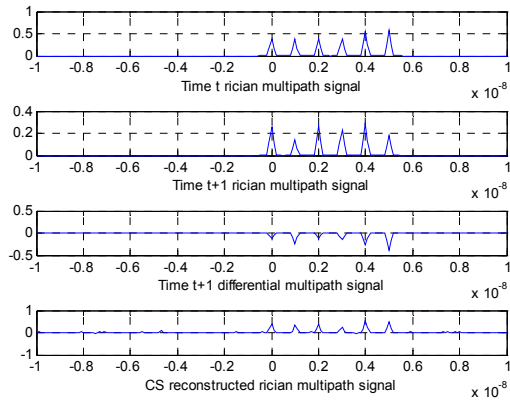


Figure 8. UWB signal sampling based on RAIC (Rician channel)

**B. Example 2 (in NLOS environment)**

In the second example, we simulate the TOA estimation of UWB signal in NLOS environment (the number of multipath is set to six).

In this example, we assume that the signal is passed through Rayleigh channel, the number of multipath is six and

the channel is time-invariant. In the first simulation (see Figure9), the observed time is set to 0.2 $\mu$ m. Figure9(1) shows the UWB signal passing through rician channel interference at time t moment. Figure9(2) shows the UWB signal passing through rician channel interference at time t+1 moment. Figure9(3) shows the differential signal by subtracting time t signal from t+1 moment. Figure9(4) shows the reconstructed signal by CS theory at the sampling rate of 10% of the Nyquist sampling rate. The time delay error of both methods is about 1nm.

At the second simulation(see Figure10), we shorten the observed time to 0.02 $\mu$ m, all of the other parameters are the same. It is shown in Figure10(4) that the measurement we need to reconstruct the signals is 35% and much more details of the signal can be seen compared with Figure9(4). And the time delay error is 1nm.

Comparing these two simulations, the conclusion is that 1) the accuracy of TOA estimation achieved by 11% Nyquist sampling rate is the same as full Nyquist sampling rate. 2) when more sampling rate is used, more detail information can be recovered. However, in TOA estimation, we don't need to recover the whole signal but the peak location of the signal. Finally, we can get the TOA estimation by 11% Nyquist sampling rate and the drawback is that some detail informations of the signal is lost .

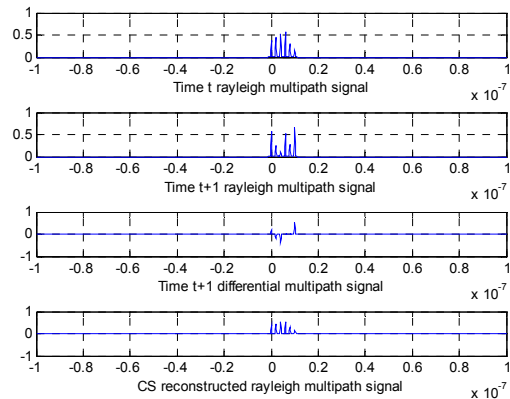


Figure9. CS UWB signal sampling based on RAIC (Rayleigh channel)

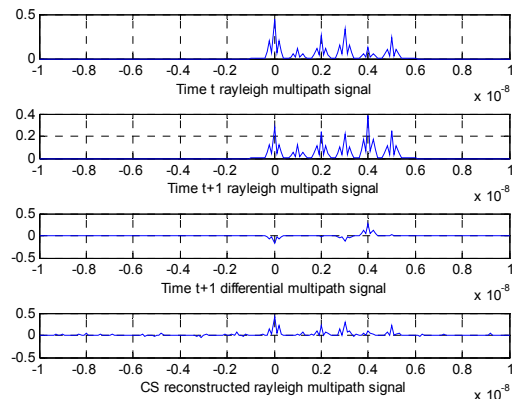


Figure10 CS UWB signal sampling based on RAIC (Rayleigh channel)

## VI CONCLUSION

In this paper, a new approach for UWB impulse radio sampling is proposed. The residual analog-to-information converter (RAIC) is based on the concept of compressive sensing (CS). RAIC gives the lower sampling rate than analog-to-information converter (AIC) by adding a signal prediction/cancellation module into AIC. The advantages of RAIC are: 1) reduce the computational complexity. When the observed time interval is small, the change of these two signals is not much, therefore the differential signal is even more sparse. Since the detect sample to recover the signal is proportional to sparse K factor, the sample of RAIC is smaller than AIC. 2) RAIC is based on the differentiation of two signals, therefore it can denoise and cancel interference.

## REFERENCES

- [1] S. Gezici, Z. Tian, G. Giannakis. et al. "Localization via ultra-wideband radios", IEEE Signal Process. Magazine, vol.22, no.4, pp.70-84, Jul. 2005.
- [2] J. Mitola. "Technical challenges in the globalization of software radio", IEEE Commun. Mag., vol.37, no.2, pp.84-89, 1999.
- [3] R. Arnott, S. Ponnekanti, et.al. "Advanced base station technology", IEEE Commun. Mag., vol.36, no.2, pp.96-101, 1998.
- [4] H. Nie, P. Takis Mathiopoulos. "Adaptive prediction and cancellation digitization method for wideband multistandard software radio base-station receivers", IEEE Trans. On Vehicular Technology, vol.55, no.3, 2006.
- [5] E. J. Candès, J. Romberg, T. Tao. "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information", IEEE Trans. Inform. Theory, vol.2, no.2, pp.489-509, Feb. 2006.
- [6] S. Kirolos, J. Laska, et.al, "Analog-to-Information Conversion via Random Demodulation", IEEE Dallas/CAS Workshop on Design, Applications, Integration and Software, pp.71-74, Oct. 2006.
- [7] T. Ragheb, S. Kirolos, "Implementation models for analog-to-information conversion via random sampling", 50th Midwest Symposium on Circuits and Systems, pp.325-328, Aug. 2007.
- [8] S. Kirolos, T. Ragheb, "Practical Issues in Implementing Analog-to-Information Converters" The 6th International Workshop on System-on-Chip for Real-Time Applications, pp.141-144, Dec. 2006
- [9] J. Laska, S. Kirolos, et. al, "Random Sampling for Analog-to-Information Conversion of Wideband Signals" 2006 IEEE Dallas/CAS Workshop on Design, Applications, Integration and Software, pp.119-121, Oct. 2006
- [10] J. Laska, S. Kirolos, "Theory and Implementation of an Analog-to-Information Converter using Random Demodulation" IEEE International Symposium on Circuits and Systems, pp.1959-1962, May 2007
- [11] E. J. Candès, M. B. Wakin. "An introduction to compressive sampling", IEEE Signal Processing Magazine, vol.25, no.2, pp. 21-30, Mar. 2008.