Projective Chaos Synchronization Between the Different Chaotic Systems

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***Abstract*—The projective synchronization between the different chaotic systems is investigated based on the Lyapunov stability theory. The nonlinear feedback controllers are designed through constructing a Lyapunov function. The chaotic systems are synchronized by adjusting the feedback gain coefficients. This method can be applied for identical systems. Numerical simulations show that the synchronization method works well.**

Keywords-Projective synchronization, Chaos , Lyapunov function, Nonlinear feedback

I. INTRODUCTION

Since the projective synchronization method was initially proposed by Mainieri and Rehacek[1], much attention has been devoted to research on projective synchronization of chaos due to its potential applications, and many synchronization methods have been proposed, such as, a modified projective synchronization(MPS) method is proposed and a general kind of proportional relationship between the drive system and response system is obtained[2-4]. An adaptive control scheme for the modified projective synchronization(AMPS) with uncertain parameters is presented[5-7,9]. The function projective synchronization(FPS) method is proposed to synchronize two chaotic systems based on the Lyapunov stability theory[8,10,11].

In this paper, the projective synchronization between the different chaotic systems and between two identical systems are investigated based on the Lyapunov stability theory. The nonlinear feedback controllers are designed through constructing a Lyapunov function. The two different chaotic systems are synchronized by adjusting the feedback gain coefficients. Numerical simulations show that the synchronization method works well.

II. PROJECTIVE SYNCHRONIZATION OF DIFFERENT CHAOTIC SYSTEMS

Rössler system is taken as the drive system, which is described as follow:

$$\begin{aligned} \dot{x}_1 &= -(y_1 + z_1) \\ \dot{y}_1 &= x_1 + a y_1 \\ \dot{z}_1 &= x_1 z_1 - c z_1 + b \end{aligned} \tag{1}$$

and Lorenz system as the response system is given by

$$\begin{cases} \dot{x}_2 = a_1 (y_2 - x_2) + u_1 \\ \dot{y}_2 = b_1 x_2 - x_2 z_2 - y_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - c_1 z_2 + u_3 \end{cases} \tag{2}$$

where x_1, y_1, z_1 and x_2, y_2, z_2 are the state variables of the two systems. Rössler system and Lorenz system exhibit chaotic behaviour when system parameters are given by $a = b = 0.2$, $c = 5.7$ and $a_1 = 10$, $b_1 = 28$, $c_1 = 8/3$ respectively. The u_i ($i = 1,2,3$) are the nonlinear feedback controllers to be designed such that two chaotic systems can be synchronized in the following sense:

$$\begin{cases} \lim_{t \to \infty} \| x_2 - \alpha x_1 \| = 0, \\ \lim_{t \to \infty} \| y_2 - \beta y_1 \| = 0, \\ \lim_{t \to \infty} \| z_2 - \gamma z_1 \| = 0. \end{cases} \tag{3}$$

Without the controllers u_i ($i = 1,2,3$), the trajectories of the two different systems will quickly separate each other independently. But, the trajectories of the two controlled systems will approach synchronization in the above sense for any different initial condition by adjusting appropriate gain coefficients.

Theorem. For given scaling factors α, β, γ , the propective synchronization between Rössler system and Lorenz system will be achieved by following controllers and the domain of the appropriate gain coefficients ki($i = 1,2,3$).

$$\begin{cases} u_1 = f_1(x_1, y_1, z_1) - k_1 e_1 \\ u_2 = f_2(x_1, y_1, z_1) - k_2 e_2 \\ u_3 = f_3(x_1, y_1, z_1) - k_3 e_3 \end{cases} \tag{4}$$

where $f_i(x_1, y_1, z_1)$ ($i = 1,2,3$) are the functions of the state variables in drive system as follows:

$$\begin{cases} f_1(x_1, y_1, z_1) = \alpha a_1 x_1 - (\alpha + \beta a_1) y_1 - \alpha z_1 \\ f_2(x_1, y_1, z_1) = (\beta - \alpha b_1) x_1 + (1 + a) \beta y_1 + \alpha \gamma x_1 z_1 \\ f_3(x_1, y_1, z_1) = -\alpha \beta x_1 y_1 - (c - c_1) \gamma z_1 + \gamma x_1 z_1 + \gamma b \end{cases} \tag{5}$$

The domain of the feedback gain coefficients is given by

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$$\begin{cases} k_1 \geq -a_1 \\ k_2 \geq \dfrac{(a_1+b_1-\gamma z_1)^2}{2(a_1+k_1)} - 1 \\ k_3 \geq \dfrac{\beta^2 {y_1}^2}{2(a_1+k_1)} - c_1 \end{cases} \tag{6}$$

Proof.Define

$e_1 = x_2 - \alpha x_1$, $e_2 = y_2 - \beta y_1$, $e_3 = z_2 - \gamma z_1$,then, the error dynamical system is given by

$$\begin{cases} \dot{e}_1 = \dot{x}_2 - \alpha \dot{x}_1 \\ \dot{e}_2 = \dot{y}_2 - \beta \dot{y}_1 \\ \dot{e}_3 = \dot{z}_2 - \gamma \dot{z}_1 \end{cases} \tag{7}$$

The Lyapunov function is constructed as :

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)$$

The differential of the Lyapunov function along the trajectory of error dynamical system Eq.(7) is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1[-a_1e_1 + a_1e_2 + \alpha a_1x_1 + (\beta a_1 + \alpha)y_1 + \alpha z_1 + u_1] \\ &\quad + e_2[(b_1 - \gamma z_1)e_1 - e_2 - \alpha x_1e_3 - e_1e_3 + (\alpha b_1 - \beta)x_1 - (1+a)\beta y_1 - \alpha\gamma x_1z_1 + u_2] \\ &\quad + e_3[e_1e_2 + \beta y_1e_1 + \alpha x_1e_2 - c_1e_3 + \alpha\beta x_1y_1 + (c - c_1)\gamma z_1 - \gamma b - \gamma x_1z_1 + u_3] \end{aligned} \tag{8}$$

considering Eq.(4), (5) and (6), we can obtain the derivative form of as follows:

$$\begin{aligned} \dot{V} = &-\frac{a_1+k_1}{2}\left(e_1 - \frac{a_1+b_1-\gamma z_1}{a_1+k_1}e_2\right)^2 - \frac{a_1+k_1}{2}\left(e_1 - \frac{\beta y_1}{a_1+k_1}e_3\right)^2 \\ &-\left(1 + k_2 - \frac{(a_1+b_1-\gamma z_1)^2}{2(a_1+k_1)}\right)e_2^2 - \left(c_1 + k_3 - \frac{\beta^2 {y_1}^2}{2(a_1+k_1)}\right)e_3^2 \end{aligned} \tag{9}$$

Therefore, due to that a chaotic system is bounded, we can chose a set of appropriate feedback gain coefficients ki in Eq.(6) makes $\dot{V} < 0$. The error states variables e_i will become zero based on the Lyapunov stability theory. The projective synchronization is achieved.

III. Projective Synchronization Of Identical Chaotic Systems

By same method, the projective chaotic synchronization can be achieved between two identical systems also. Here the Rössler system is handled as the example. The drive system and response system are described as follow:

$$\begin{cases} \dot{x}_2 = -(y_2 + z_2) \\ \dot{y}_2 = x_2 + ay_2 \\ \dot{z}_2 = x_2z_2 - cz_2 + b \end{cases}$$

$$\begin{cases} \dot{x}_2 = -(y_2 + z_2) + u_1 \\ \dot{y}_2 = x_2 + ay_2 + u_2 \\ \dot{z}_2 = x_2z_2 - cz_2 + b + u_3 \end{cases}$$

where u_1, u_2, u_3 is nonlinear controllers. The error vectors are defined as

$$\begin{cases} e_1 = x_2 - \alpha x_1 \\ e_2 = y_2 - \beta y_1 \\ e_3 = z_2 - \gamma z_1 \end{cases}$$

Where the α, β, γ are the scaling factors. Then the following Lyapunov function is chosen

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2)$$

We can take the controllers as

$$\begin{cases} u_1 = f_1(x_1, y_1, z_1) - k_1e_1 \\ u_2 = f_2(x_1, y_1, z_1) - k_2e_2 \\ u_3 = f_3(x_1, y_1, z_1) - k_3e_3 \end{cases}$$

$$\begin{cases} f_1(x_1, y_1, z_1) = (\beta - \alpha)y_1 + (\gamma - \alpha)z_1 \\ f_2(x_1, y_1, z_1) = (\beta - \alpha)x_1 \\ f_3(x_1, y_1, z_1) = (1 - \alpha)\gamma x_1z_1 + (\gamma - 1)b \end{cases}$$

Where the f_i's are the function of the state variables And the k_1, k_2, k_3 are the feedback gain coefficients.

Therefore ,we can obtain the time derivation of the Lyapunov function along the state trajectory

$$\begin{aligned} \dot{V} &= e_1\left[-e_2 - e_3 + (\alpha - \beta)y_1 + (\alpha - \gamma)z_1 + u_1\right] \\ &+ e_2\left[e_1 + ae_2 + (\alpha - \beta)x_1 + u_2\right] \\ &+ e_3\left[e_1e_3 + \gamma z_1e_1 + (\alpha x_1 - c)e_3 + (\alpha - 1)\gamma x_1z_1 + (1 - \gamma)b + u_3\right] \\ &= e_1\left[-k_1e_1 - e_2 - e_3\right] + e_2\left[e_1 - (k_2 - a)e_2\right] + e_3\left[\gamma z_1e_1 - (k_3 + c - \alpha x_1)e_3\right] \\ &= -k_1e_1^2 - (k_2 - a)e_2^2 - (k_3 + c - \alpha x_1)e_3^2 + (\gamma z_1 - 1)e_1e_3 \\ &= -\frac{k_1}{2}e_1^2 - (k_2 - a)e_2^2 - \left(k_3 + c - \alpha x_1 - \frac{(\gamma z_1 - 1)^2}{2k_1}\right)e_3^2 \\ &- \frac{k_1}{2}\left(e_1 - \frac{\gamma z_1 - 1}{k_1}e_3\right)^2 \end{aligned}$$

For $\dot{V} \leq 0$, the domain of those feedback factors k_i are obtained

$$\begin{cases} k_1 \geq 0 \\ k_2 \geq a \\ k_3 \geq \alpha x_1 - c + \dfrac{(\gamma z_1 - 1)^2}{2k_1} \end{cases}$$

IV. Numerical simulations

In this section, the numerical examples is given to verify and illustrate the effectiveness of the proposed method.

Firstly, for the chaotic synchronization between the different systems, the domain of the y_1 and z_1 in the drive system are computed from the attractor of Lorenz system, which are $-10 < x_1 < 13$, $-12 < y_1 < 8$ and $0 < z_1 < 24$

shown in Fig. 1, Fig.2 and Fig.3. Selecting $k_1 = 20$, the $k_2 \geq 23.1$ and $k_3 \geq -2.2$ are attained, we take $k_2 = 28$ and $k_3 = 3$ here. During the simulation, the initial values of the drive and response systems are taken as $x_1(0) = 3$, $y_1(0) = -5$, $z_1(0) = 3$ and $x_2(0) = -4$, $y_2(0) = 5$, $z_2(0) = -3$, the scaling factors are given by $\alpha = 2, \beta = 0.5, \gamma = 3$,the system parameters are chosen as $a = 0.2$, $b = 0.2$, $c = 5.7$ and $a_1 = 10$, $b_1 = 28$,$c_1 = 8/3$. So the simulation results between the different systems are shown in Figure 1—Figure 4.

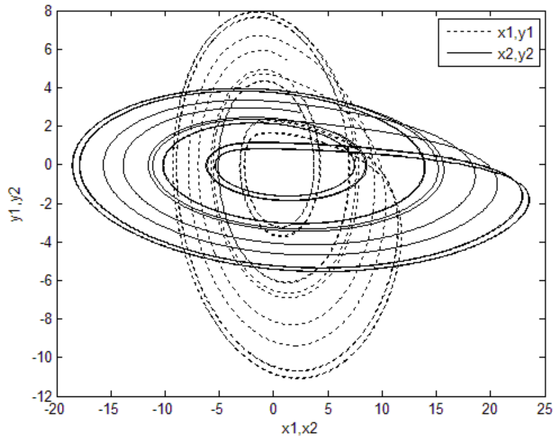

Figure 1. The attractors in projective synchronization for different systems.(1)

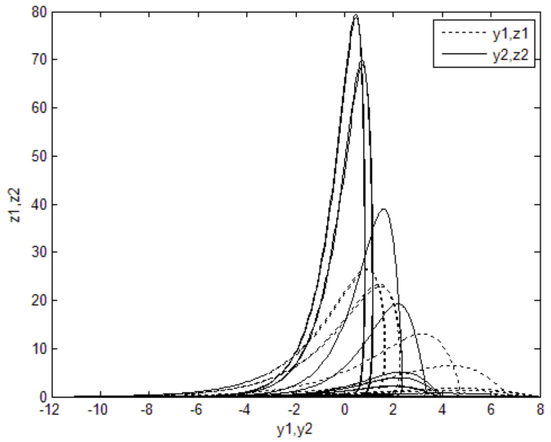

Figure 2. The attractors in projective synchronization for different systems.(2)

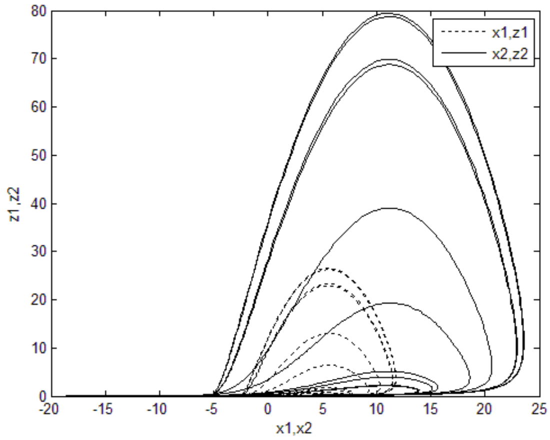

Figure 3. The attractors in projective synchronization for different systems.(3)

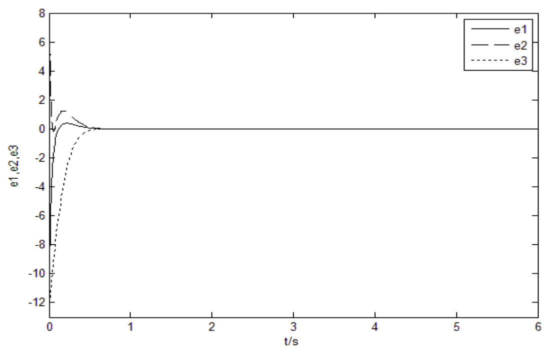

Figure 4. The synchronization errors between the different systems

Next, the simulation result for the projective synchronization between two identical chaotic systems is shown briefly. Where the system parameters is given by $a = 0.2, b = 0.2, c = 5.7$. The initial values of the state variables for the two system are (3,-5,3) and (-4,5,-3). The scaling factors and the feedback gain coefficients are taken as $\alpha = 1$, $\beta = \gamma = 0.6$, $k_1 = 10$, $k_2 = 3$, $k_3 = 7$ respectively. Then the attractors in projective synchronization for the identical systems and the synchronization errors between the state variables are shown in figure 5—figure 8.

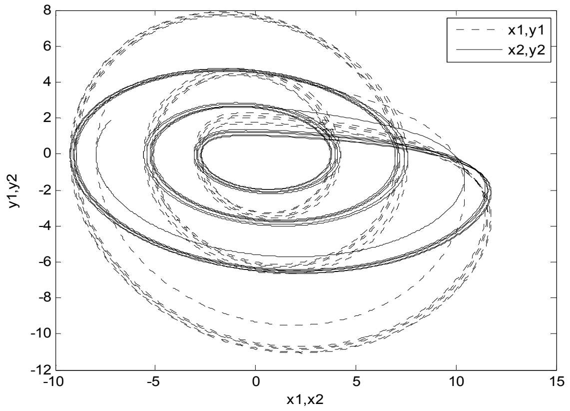

Figure 5. The attractors in projective synchronization for identical systems (1)

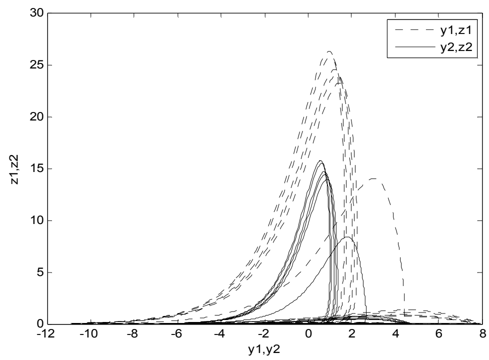

Figure 6. The attractors in projective synchronization for identical systems (2)

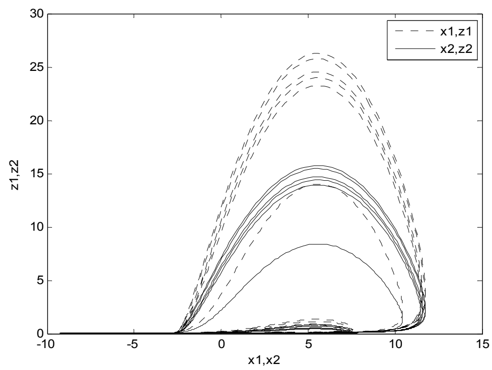

Figure 7. The attractors in projective synchronization for identical systems (3)

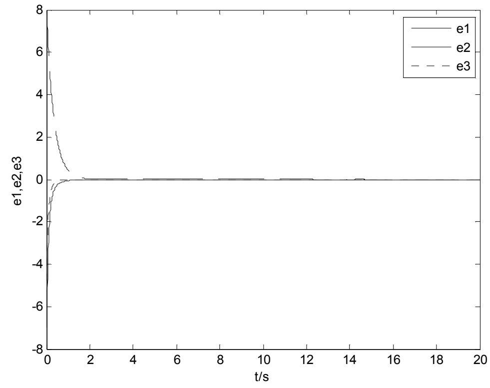

Figure 8. The synchronization errors between the identical systems

V. SUMMARIES

This paper is concerned with the projective synchronization for different chaotic systems and identical systems via a nonlinear feedback control. The domain of the feedback gain coefficients which associated with the nonlinear controllers are computed by estimating the value of the state variables from the attractor of the drive system. In numerical simulations, the effectiveness and the feasibility of the proposed scheme have been demonstrated using Rössler and Lorenz chaotic systems. This method can be also applied to other chaotic system as a general approach.

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