

Consistent Fuzzy Linguistic Preference Relations for Computer Integrated Manufactory Systems Selection

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Abstract

Most decision-making and problem-solving tasks are too complex to be understood quantitatively. People perform such tasks using knowledge that is imprecise rather than precise. This study presents consistent fuzzy linguistic preference relations rather than fuzzy analytic hierarchy process (FAHP) to select computer integrated manufactory (CIM) systems. The proposed method yields decision matrices for making pairwise comparisons using additive transitivity. Only $n-1$ comparison judgments are required to ensure consistency on a level that contains n criteria.

Keywords: consistent fuzzy linguistic preference relations; FAHP; CIM; consistency

1. Introduction

Zadeh [1] introduced the fuzzy set theory in 1965, to rationalize uncertainty associated with impression or vagueness, and thus applicable to human thought. The AHP method can be used to express experts' opinions, but cannot model human thinking so fuzzy AHP, a fuzzy extension of AHP [2], was developed to solve hierarchical imprecise problems.

Consistency is crucial for achieving correct solutions in decision process. Due to each positive reciprocal matrix is described by fuzzy numbers in FAHP, so satisfying consistency is very difficult. Furthermore, establishing a fuzzy positive reciprocal matrix requires $\frac{n \times (n-1)}{2}$ judgments to be made for a level with n criteria. The numbers of comparisons increases with the number of criteria, so inconsistent conditions are likely to occur.

In order to solve the consistency problem, we adopt consistent fuzzy linguistic preference relations to construct fuzzy decision matrix instead of fuzzy positive reciprocal matrix. The proposed method yields decision matrices for making pairwise comparisons using additive transitivity. Only $n-1$ comparison judgments are required to ensure

consistency on a level that contains n criteria. Finally, the proposed method is demonstrated by applying it to select CIM systems [3].

2. Consistent fuzzy linguistic preference relations

The consistent fuzzy linguistic preference relations method was proposed by Wang and Chen [4] to deal with vagueness judgments. This study presents the proposed method to extend on consistency of Fuzzy AHP. The proposed method constructs fuzzy preference relations matrices using fuzzy linguistic assessment variables $\tilde{P} = (\tilde{p}_{ij}) = (p_{ij}^L, p_{ij}^M, p_{ij}^R)$ based on consistent fuzzy preference relations [5]. Table 1 lists the fuzzy linguistic assessment variables.

Table 1. Fuzzy linguistic assessment variables

Linguistic variables	Fuzzy numbers
Very Poor (VP)	$(0, p_{VP}^M, p_{VP}^R)$
\vdots	\vdots
Medium (M)	$(p_M^L, 0.5, p_M^R)$
\vdots	\vdots
Very Good (VG)	$(p_{VG}^L, p_{VG}^M, 1)$

Buckley [6] defines the consistency of a fuzzy positive reciprocal matrix as follows.

Definition 2.1. A fuzzy positive matrix $\tilde{A} = (\tilde{a}_{ij})$ is reciprocal if and only if $\tilde{a}_{ij} \otimes \tilde{a}_{ji} \approx \tilde{1}$, where $\tilde{1} = (1, 1, 1)$.

Definition 2.2. A fuzzy positive reciprocal matrix $\tilde{A} = (\tilde{a}_{ij})$ is consistent if and only if $\tilde{a}_{ij} \otimes \tilde{a}_{jk} \approx \tilde{a}_{ik}$.

Wang and Chen [4] proposed consistent fuzzy linguistic preference relations. These propositions are described as follows.

Proposition 2.1. Suppose a set of alternatives, $X = \{x_1, \dots, x_n\}$ associated with a fuzzy reciprocal multiplicative preference matrix $\tilde{A} = (\tilde{a}_{ij})$ with $\tilde{a}_{ij} \in [1/9, 9]$, then the corresponding fuzzy

reciprocal linguistic preference relation, $\tilde{P} = (\tilde{p}_{ij}) = \frac{1}{2}(1 + \log_9 \tilde{a}_{ij})$ with $\tilde{p}_{ij} \in [0,1]$, verifies the additive reciprocal, namely, the following statements are equivalent.

$$(1) p_{ij}^L + p_{ji}^R \approx 1 \quad \forall i, j \in \{1, \dots, n\}.$$

$$(2) p_{ij}^M + p_{ji}^M \approx 1 \quad \forall i, j \in \{1, \dots, n\}.$$

$$(3) p_{ij}^R + p_{ji}^L \approx 1 \quad \forall i, j \in \{1, \dots, n\}.$$

Proposition 2.2. For a reciprocal fuzzy linguistic preference relation $\tilde{P} = (\tilde{p}_{ij}) = (p_{ij}^L, p_{ij}^M, p_{ij}^R)$ to be consistent, the following statements must be equivalent:

$$(1) p_{ij}^L + p_{jk}^L + p_{ki}^R \approx \frac{3}{2}, \forall i < j < k$$

$$(2) p_{ij}^M + p_{jk}^M + p_{ki}^M \approx \frac{3}{2}, \forall i < j < k$$

$$(3) p_{ij}^R + p_{jk}^R + p_{ki}^L \approx \frac{3}{2}, \forall i < j < k$$

$$(4) p_{i(i+1)}^L + p_{(i+1)(i+2)}^L + \dots + p_{(j-1)j}^L + p_{ji}^R \approx \frac{j-i+1}{2}, \forall i < j$$

$$(5) p_{i(i+1)}^M + p_{(i+1)(i+2)}^M + \dots + p_{(j-1)j}^M + p_{ji}^M \approx \frac{j-i+1}{2}, \forall i < j$$

$$(6) p_{i(i+1)}^R + p_{(i+1)(i+2)}^R + \dots + p_{(j-1)j}^R + p_{ji}^L \approx \frac{j-i+1}{2}, \forall i < j$$

We note that if the values of the obtained matrix \tilde{P} with elements \tilde{p}_{ij} in the interval $[-c, 1+c]$ ($c > 0$) not in the interval $[0,1]$. In such a case, the obtained fuzzy numbers would need to be transformed using a transformation function which preserves reciprocity and additive consistency, namely a function $f: [-c, 1+c] \rightarrow [0,1]$, verifying

$$(1) f(-c) = 0.$$

$$(2) f(1+c) = 1.$$

$$(3) f(x^L) + f(x^R) = 1, \quad \forall x \in [-c, 1+c].$$

$$(4) f(x^M) + f(x^M) = 1, \quad \forall x \in [-c, 1+c].$$

$$(5) f(x^R) + f(x^L) = 1, \quad \forall x \in [-c, 1+c].$$

$$(6)$$

$$f(x^L) + f(y^L) + f(z^R) = \frac{3}{2}, \quad \forall x^L, y^L, z^R \in [-c, 1+c] \text{ such that } x^L + y^L + z^R = \frac{3}{2}$$

$$(7)$$

$$f(x^M) + f(y^M) + f(z^M) = \frac{3}{2}, \quad \forall x^M, y^M, z^M \in [-c, 1+c] \text{ such that } x^M + y^M + z^M = \frac{3}{2}$$

$$(8)$$

$$f(x^R) + f(y^R) + f(z^L) = \frac{3}{2}, \quad \forall x^R, y^R, z^L \in [-c, 1+c] \text{ such that } x^R + y^R + z^L = \frac{3}{2}$$

3. Illustrative example

To demonstrate the calculation of the proposed method, this study considers CIM systems selection originally presented by Bozdog et al. [3]. A big Turkish Motor Company wants to purchase a CIM system to produce some machinery parts. The top management determined four alternative CIM systems meeting the company's needs. A group of eight evaluators was

charged to choose the best CIM system (Goal). The criteria taken into account are better quality (C1), greater flexibility (C2), competition (C3), experience with new technology (C4), and expandability (C5). The alternatives are CIM1 (A), CIM2 (B), CIM3 (C), and CIM4 (D). The hierarchy of the problem is shown in Fig. 1.

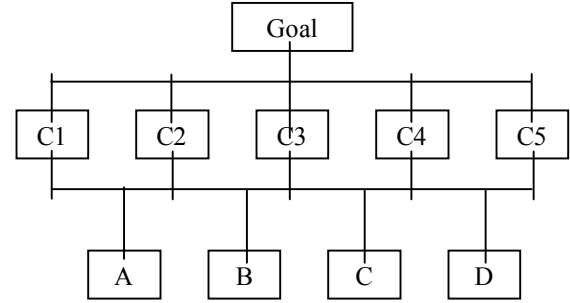


Fig1. The hierarchy structure

Fuzzy linguistic assessment variables are listed in Table 2 by converting Bozdog's Linguistic scale according to $p_{ij} = g(a_{ij}) = \frac{1}{2} \cdot (1 + \log_{9.5} a_{ij})$, $p_{ij} \in (0,1)$, $a_{ij} \in (1,9.5)$.

For example,

$$(0.1, 0.2, 0.3) = \left(\frac{1}{2} \cdot (1 + \log_{9.5} 5/2), \frac{1}{2} \cdot (1 + \log_{9.5} 3), \frac{1}{2} \cdot (1 + \log_{9.5} 7/2) \right)$$

Table 2. Fuzzy linguistic assessment variables

Linguistic variables	Bozdog's fuzzy scale	The proposed Reciprocal method's fuzzy scale	scale
Equally important	(1, 1, 1)	(0.5, 0.5, 0.5)	(0.5, 0.5, 0.5)
Weekly more important	(5/2, 3, 7/2)	(0.1, 0.2, 0.3)	(0.7, 0.8, 0.9)
Strongly more important	(9/2, 5, 11/2)	(0.2, 0.3, 0.4)	(0.6, 0.7, 0.8)
Very strongly more important	(13/2, 7, 15/2)	(0.3, 0.4, 0.5)	(0.5, 0.6, 0.7)
Absolutely more important	(17/2, 9, 19/2)	(0.4, 0.5, 0.6)	(0.4, 0.5, 0.6)

Tables 3 and 4 present the original group pairwise comparisons of all criteria and alternatives. The result of the FAHP method is A=0.635, B=0.060, C=0.000, and D=0.305. The ranks are A > D > B > C.

Table 3. Original pairwise group comparisons of the criteria

Goal	C1	C2	C3	C4	C5
C1	(1,1,1)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(11/2, 6, 13/2)
C2	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)	(1,1,1)	(9/2, 5, 11/2)
C3	(2/7, 1/3, 2/5)	(2/5, 1/2, 2/3)	(1,1,1)	(2/5, 1/2, 2/3)	(3/2, 2, 5/2)
C4	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)	(1,1,1)	(7/2, 4, 9/2)
C5	(2/13, 1/6, 2/11)	(2/11, 1/5, 2/9)	(2/5, 1/2, 2/3)	(2/9, 1/4, 2/7)	(1,1,1)

Table 4. Original group pairwise comparisons of the alternatives

Goal	A	B	C	D
C1				

A	(1,1,1)	(5/2, 3, 7/2)	(7/2, 4, 9/2)	(3/2, 2, 5/2)
B	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)
C	(2/7, 1/3, 2/5)	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)
D	(2/9, 1/4, 2/7)	(2/5, 1/2, 2/3)	(2/5, 1/2, 2/3)	(1,1,1)
C2				
A	(1,1,1)	(3/2, 2, 5/2)	(5/2, 3, 7/2)	(7/2, 4, 9/2)
B	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)
C	(2/7, 1/3, 2/5)	(2/5, 1/2, 2/3)	(1,1,1)	(3/2, 2, 5/2)
D	(2/9, 1/4, 2/7)	(2/5, 1/2, 2/3)	(2/5, 1/2, 2/3)	(1,1,1)
C3				
A	(1,1,1)	(2/5, 1/2, 2/3)	(2/7, 1/3, 2/5)	(2/9, 1/4, 2/7)
B	(3/2, 2, 5/2)	(1,1,1)	(2/5, 1/2, 2/3)	(2/7, 1/3, 2/5)
C	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(1,1,1)	(2/7, 1/3, 2/5)
D	(7/2, 4, 9/2)	(5/2, 3, 7/2)	(5/2, 3, 7/2)	(1,1,1)
C4				
A	(1,1,1)	(1,1,1)	(5/2, 3, 7/2)	((3/2, 2, 5/2)
B	(1,1,1)	(1,1,1)	(5/2, 3, 7/2)	(3/2, 2, 5/2)
C	(2/7, 1/3, 2/5)	(2/7, 1/3, 2/5)	(1,1,1)	(2/5, 1/2, 2/3)
D	(2/5, 1/2, 2/3)	(2/5, 1/2, 2/3)	(3/2, 2, 5/2)	(1,1,1)
C5				
A	(1,1,1)	(1,1,1)	(5/2, 3, 7/2)	(2/5, 1/2, 2/3)
B	(1,1,1)	(1,1,1)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
C	(2/7, 1/3, 2/5)	(2/5, 1/2, 2/3)	(1,1,1)	(2/9, 1/4, 2/7)
D	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(7/2, 4, 9/2)	(1,1,1)

The above example demonstrates how the decision matrices of the proposed method are constructed. The original data in Tables 3 and 4 yield the decision matrices of consistent fuzzy linguistic preference relations according to Propositions 2.1 and 2.2. For instance, Table 3 has five criteria. Only $(n-1=5-1=4)$ values $\{p_{12}, p_{23}, p_{34}, p_{45}\}$ are required to construct the decision matrices. The whole calculation is as follows:

$$\begin{aligned}
p_{12}^L &= \frac{1}{2}(1 + \log_{9.5} 3/2) = 0.59, \\
p_{12}^M &= \frac{1}{2}(1 + \log_{9.5} 2) = 0.65, \\
p_{12}^R &= \frac{1}{2}(1 + \log_{9.5} 5/2) = 0.70, \\
p_{23}^L &= \frac{1}{2}(1 + \log_{9.5} 3/2) = 0.59, \\
p_{23}^M &= \frac{1}{2}(1 + \log_{9.5} 2) = 0.65, \\
p_{23}^R &= \frac{1}{2}(1 + \log_{9.5} 5/2) = 0.70, \\
p_{34}^L &= \frac{1}{2}(1 + \log_{9.5} 2/5) = 0.30, \\
p_{34}^M &= \frac{1}{2}(1 + \log_{9.5} 1/2) = 0.35, \\
p_{34}^R &= \frac{1}{2}(1 + \log_{9.5} 2/3) = 0.41, \\
p_{45}^L &= \frac{1}{2}(1 + \log_{9.5} 7/2) = 0.78,
\end{aligned}$$

$$p_{45}^M = \frac{1}{2}(1 + \log_{9.5} 4) = 0.81,$$

$$p_{45}^R = \frac{1}{2}(1 + \log_{9.5} 9/2) = 0.83,$$

$$p_{31}^L = 1.5 - p_{12}^R - p_{23}^R = 0.09,$$

$$p_{31}^M = 1.5 - p_{12}^M - p_{23}^M = 0.19,$$

$$p_{31}^R = 1.5 - p_{12}^L - p_{23}^L = 0.32,$$

$$p_{41}^L = 2 - p_{12}^R - p_{23}^R - p_{34}^R = 0.18,$$

$$p_{41}^M = 2 - p_{12}^M - p_{23}^M - p_{34}^M = 0.35,$$

$$p_{41}^R = 2 - p_{12}^L - p_{23}^L - p_{34}^L = 0.52,$$

$$p_{42}^L = 1.5 - p_{23}^R - p_{34}^R = 0.39,$$

$$p_{42}^M = 1.5 - p_{23}^M - p_{34}^M = 0.50,$$

$$p_{42}^R = 1.5 - p_{23}^L - p_{34}^L = 0.61,$$

$$p_{51}^L = 2.5 - p_{12}^R - p_{23}^R - p_{34}^R - p_{45}^R = -0.15,$$

$$p_{51}^M = 2.5 - p_{12}^M - p_{23}^M - p_{34}^M - p_{45}^M = 0.04,$$

$$p_{51}^R = 2.5 - p_{12}^L - p_{23}^L - p_{34}^L - p_{45}^L = 0.25,$$

$$p_{52}^L = 2 - p_{23}^R - p_{34}^R - p_{45}^R = 0.05,$$

$$p_{52}^M = 2 - p_{23}^M - p_{34}^M - p_{45}^M = 0.19,$$

$$p_{52}^R = 2 - p_{23}^L - p_{34}^L - p_{45}^L = 0.34,$$

$$p_{53}^L = 1.5 - p_{34}^R - p_{45}^R = 0.26,$$

$$p_{53}^M = 1.5 - p_{34}^M - p_{45}^M = 0.35,$$

$$p_{53}^R = 1.5 - p_{34}^L - p_{45}^L = 0.43,$$

$$p_{32}^L = 1 - p_{23}^R = 0.30, p_{32}^M = 1 - p_{23}^M = 0.35,$$

$$p_{32}^R = 1 - p_{23}^L = 0.41, p_{43}^L = 1 - p_{34}^R = 0.59,$$

$$p_{43}^M = 1 - p_{34}^M = 0.65, p_{43}^R = 1 - p_{34}^L = 0.70,$$

$$p_{54}^L = 1 - p_{45}^R = 0.17, p_{54}^M = 1 - p_{45}^M = 0.19,$$

$$p_{54}^R = 1 - p_{45}^L = 0.22$$

Table 5 lists the consistent fuzzy linguistic preference relation matrix. The matrix has entries that are not contained in the interval $[0, 1]$, and thus these

transforming functions $f(x^L) = \frac{x^L + c}{1 + 2c}$,

$f(x^M) = \frac{x^M + c}{1 + 2c}$, $f(x^R) = \frac{x^R + c}{1 + 2c}$ are applied. Table 6

lists the Transforming matrix, in which, the weight w_i

was determined according to $w_i = A_i / \sum_{i=1}^n A_i$,

$A_i = \frac{1}{n} (\sum_{j=1}^n p_{ij})$. Similarly, Table 7 presents the decision matrices of alternatives determined by applying each criterion. Finally, adding the weights per CIM alternative multiplied by the weights of the corresponding criteria, a final score is obtained for each CIM alternative. Table 8 shows these scores. The final score employs the equation $W_i = \frac{1}{3}(p_{ij}^L + p_{ij}^M + p_{ij}^R)$ proposed by Yager [7]. The ranks are $A > D > B > C$. The result is the same as the original example.

Table 5. Consistent fuzzy linguistic preference relation matrix of criteria

Goal C1	C2	C3	C4	C5
C1 (0.5,0.5,0.5)	(0.59,0.65,0.7)	(0.68,0.81,0.91)	(0.48,0.65,0.82)	(0.75,0.96,1.15)
C2 (0.3,0.35,0.41)	(0.5,0.5,0.5)	(0.59,0.65,0.7))	(0.39,0.5,0.61)	(0.66,0.81,0.95)
C3 (0.09,0.19,0.32)	(0.3,0.35,0.41)	(0.5,0.5,0.5)	(0.3,0.35,0.41)	(0.57,0.65,0.74)
C4 (0.18,0.35,0.52)	(0.39,0.5,0.61)	(0.59,0.65,0.7)	(0.5,0.5,0.5)	(0.78,0.81,0.83)
C5 (-0.15,0.04,0.25)	(0.05,0.19,0.33)	(0.26,0.35,0.43)	(0.17,0.19,0.22)	(0.5,0.5,0.5)

Table 6. Transforming matrix from Table 5

Goal C1	C2	C3	C4	C5	Weight
C1 (0.5,0.5,0.5)	(0.57,0.62,0.66)	(0.64,0.74,0.81)	(0.48,0.62,0.74)	(0.7,0.86,1)	(0.21,0.27,0.34)
C2 (0.34,0.38,0.43)	(0.5,0.5,0.5)	(0.57,0.62,0.66))	(0.41,0.5,0.59)	(0.63,0.74,0.84)	(0.17,0.22,0.28)
C3 (0.19,0.26,0.36)	(0.34,0.38,0.43)	(0.5,0.5,0.5)	(0.34,0.38,0.43)	(0.56,0.62,0.69)	(0.14,0.17,0.22)
C4 (0.26,0.38,0.52)	(0.41,0.5,0.59)	(0.57,0.62,0.66)	(0.5,0.5,0.5)	(0.71,0.74,0.76)	(0.17,0.22,0.28)
C5 (0.0,0.14,0.3)	(0.16,0.26,0.39)	(0.31,0.38,0.44)	(0.24,0.26,0.29)	(0.5,0.5,0.5)	(0.09,0.12,0.17)

Table 7. Weight vectors for the alternatives

A	B	C	D
C1 (0.28,0.35,0.42)	(0.17,0.2,0.24)	(0.11,0.15,0.19)	(0.25,0.3,0.37)
C2 (0.28,0.37,0.48)	(0.22,0.27,0.34)	(0.17,0.22,0.29))	(0.09,0.14,0.22)
C3 (0.12,0.16,0.21)	(0.19,0.22,0.26)	(0.23,0.27,0.31)	(0.3,0.35,0.41)
C4 (0.26,0.3,0.34)	(0.24,0.27,0.31)	(0.15,0.18,0.21)	(0.2,0.25,0.31)
C5 (0.22,0.25,0.29)	(0.21,0.24,0.26)	(0.15,0.18,0.21)	(0.29,0.33,0.39)

Table 8. Final scores of the alternatives

	Fuzzy numbers	Final score
A	(0.19, 0.3, 0.47)	0.32
B	(0.16, 0.24, 0.37)	0.26
C	(0.12, 0.19, 0.31)	0.21
D	(0.17, 0.27, 0.43)	0.29

4. Conclusions

In this study, the consistent fuzzy preference relations are used to derive decision matrices, and the presented method is applied to the example employed by Bozdog (2003). This study reveals that the proposed method yields the same result as that of Bozdog. However, the proposed method can reduce the number of pairwise comparisons. The illustrated example involves five criteria and requires needs four comparisons. There are four alternatives and it only needs three comparisons. Therefore, the number of pairwise comparisons can be reduced by $(C_2^5 - 4) + 5 \times (C_2^4 - 3) = 21$ and consistency is ensured. The proposed method enhances the efficiency and accuracy of decisions. The increase in the number of criteria and alternatives reduces the number of comparison required by the proposed method. This study indicates that the proposed method simply and practically provides a ranking of alternative solutions to fuzzy decision-making problems

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