Fuzzy Logic for Developments and Applications of Unbalance Three-Phase Fuzzy Feeder Models

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Abstract--Based on the fuzzy set theory, this study introduces three novel three-phase fuzzy feeder models for fast distribution system calculations. These fuzzy models are formulated using three lumped individual phase loads to represent single-phase, three-phase unbalanced, and nonuniformly distributed loads along a feeder. The proposed voltage-drop and line-loss fuzzy models are developed to accurately simulate the total series voltage drop at the end and the total copper loss of a given feeder, respectively. A hybrid fuzzy model is then developed to simulate both voltage drop and line loss accurately. The proposed fuzzy models are applied to a physical feeder. Simulation results show that it is possible to simplify complicated feeders to simple equivalent models in the calculations of voltage profiles and line losses with negligible error, even if there are various transformer connection schemes in the feeder.

Keywords: Distribution system, Feeder model, Fuzzy set theory, Three-phase system.

1. Introduction

Generally, in a modern distribution system various transformer connections are used to meet the requirements of different customers and to optimize system operation. These transformer connections could be single-phase, open delta-open delta, open wye-open delta, Scott and three-phase connections. Three-phase power flow programs, instead of traditional power flow programs, should be used to analyze distribution systems [1], [2]. Therefore, it is important to develop three-phase feeder models for distribution system planning, operation, and distribution automation and control (DAC).

The early studies on feeder modeling for total series voltage drop and copper loss calculations are concerned with uniformly distributed load [2]. Then, three simplified feeder models dealing with discrete distributed tapped-off loads were introduced [3]. These feeder models introduced are unidirectional, that is, the feeder models are accurate only if power is fed at the same end as specified in the model. Three bi-directional feeder models are developed [4]. These models simulate the feeders, laterals and feeder segments accurately, even when the power infeeds of given feeders or feeder segments are changed from one end to the other. Therefore, no data input need be changed when the system configuration is changed under service restoration and system reconfiguration.

The simplified models mentioned above are all based on the assumption that the discrete loads are equally distributed on the three phases along the feeder. This is usually not the case in a physical distribution system because distribution systems are inherently unbalanced. Three three-phase single-feeding-end feeder models introduced in [5].

A common trend for the feeder modelling in distribution systems in previous approaches has been toward the use of fixed (crisp) values for the input data. This assumption is in clear contrast to the real-life situation in which the uncertainty of input data is always present. Namely, in the planning stage of distribution systems, the decision maker is faced with the low precision and/or fuzziness of data. Fuzzy set theory derives from the fact that almost all natural classes and concepts are fuzzy rather than crisp in nature [6].

A novel method based on the fuzzy logic control to solve the feeder modelling problem is presented. Instead of the general assumption that discrete distributed tapped-off loads and feeder segment are known exactly [2]-[5], they are treated here as fuzzy variables. The proposed three-phase fuzzy feeder models use individual phase equivalent length $L_{\phi eqk}$ and equivalent load $S_{\phi eqk}$ of each tapped-off load point as crisp input data of a designed fuzzy feeder modelling controller (FFMC). Along a feeder, the individual phase voltage drop $\Delta v_{\phi k}$ at each tapped-off load point is considered to be crisp out data. Once $\Delta v_{\phi k}$ has been computed, the proposed fuzzy models can be generated using lumped loads to represent discretely distributed tapped-off loads along a feeder.

2. Implementation of Fuzzy Feeder Modelling Controller (FFMC)

A single-line diagram, shown in Fig. 1, illustrates a general three-phase feeder segment. There are n discrete loads distributed along the feeder, and the transformer impedance and resistance values are all represented in per units on their own KVA ratings.

A distribution transformer and its loads can be integrated and simply represented by its equivalent loads [7]. Hence, the sample feeder can be redrawn as Fig. 2. In Fig. 2, the individual phase equivalent length $L_{\phi eqk}$ and equivalent load $S_{\phi eqk}$ of the tapped-off load "*k*" are defined by

$$L_{\varphi eqk} = \sum_{i=k}^{n} L_i \qquad \phi = a,b,c; k = 1,2\cdots,n \tag{1}$$

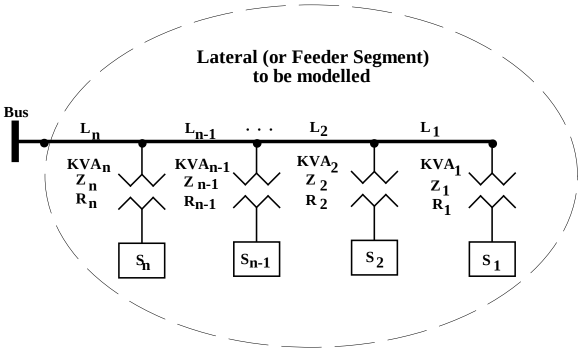

Fig.1: General feeder.

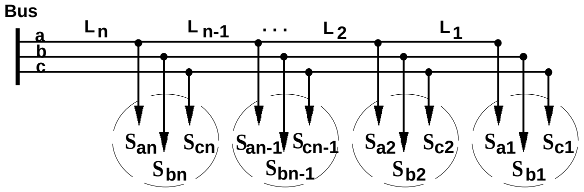

Fig.2: Simplified sample feeder for modeling.

$$S_{\varphi eqk} = \left| \sum_{i=k}^{n} \left[\left(\sum_{j=1}^{i} S_{\varphi j} \right) \cdot L_i \right] \right| / L_{\varphi eqk}$$

$$\phi = a, b, c; k = 1, 2 \cdots, n \qquad (2)$$

where L_i and $S_{\phi j}$ are the length of line segment *i* and the complex power consumption of load *j* respectively.

The main structure of the proposed FFMC is shown in Fig. 3. It comprises four principle components: a fuzzification interface, a knowledge base, process logic and a defuzzification interface. As shown in Fig. 3, the individual phase equivalent length $L_{\phi eqk}$ and the individual phase equivalent load $S_{\phi eqk}$, defined in (1) and (2), as the crisp input data. Then, along a feeder, the individual phase voltage drop $\Delta v_{\phi k}$ at each tapped-off load point is considered to be the crisp output data.

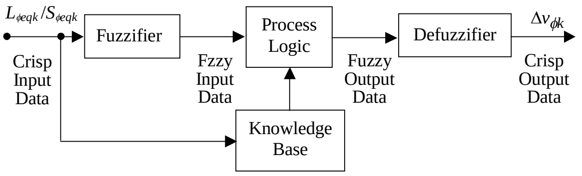

Fig. 3: Main structure of FFMC.

The procedures to design the FFMC, shown in Fig. 3, are as follows:

2.1. Specify the membership functions for input data

To express the input data, $L_{\phi eqk}$ and $S_{\phi eqk}$, in linguistic variables such as very small (VS), small (S), below average (BA), average (A) above average (AA), large (L), and very large (VL), they are first normalised based on previous experience of developing feeder models [4], [5]. Then, input data can be described by membership functions for the linguistic variables, as shown in Table I.

2.2. Establish the fuzzy relation matrix

A set of decision rules relating input data to output data are first compiled based on previous experience of developing feeder models. Those decision rules are expressed using linquistic variables. The most convenient way to present those decision rules is to use a decision table (Table II). Using fuzzy set notations, the decision table can be converted into a fuzzy relation matrix (Table III).

TABLE I: MEMBERSHIP FUNCTIONS FOR INPUT DATA

Normalized Input Data $L_{\phi eqk}/S_{\phi eqk}$	Membership Functions						
	VS	S	BA	A	AA	L	VL
-3	1	0.7	0.5	0.3	0	0	0
-2	1	0.9	0.7	0.5	0.2	0	0
-1	0.8	1	0.9	0.7	0.4	0.2	0
-0.3	0.6	0.8	1	0.9	0.6	0.4	0.2
0	0.4	0.6	0.8	1	0.8	0.6	0.4
0.3	0.2	0.4	0.6	0.9	1	0.8	0.6
1	0	0.2	0.4	0.7	0.9	1	0.8
2	0	0	0.2	0.5	0.7	0.9	1
3	0	0	0	0.3	0.5	0.7	1

2.3. Determine the membership function of output data

Use the composition rule of fuzzy set theory to determine the membership function of the output data.

TABLE II: DECISION TABLE

Voltage Drop $\Delta v_{\phi k}$		Equivalent Load, $S_{\phi eqk}$						
		VS	S	BA	A	AA	L	VL
Equivalent Length, $L_{\phi eqk}$	VS	VS	VS	VS	VS	S	S	S
	S	S	S	S	S	S	BA	BA
	BA	S	S	BA	BA	BA	A	A
	A	BA	BA	A	A	A	AA	AA
	AA	BA	A	A	AA	AA	L	L
	L	A	A	AA	L	L	VL	VL
	VL	A	AA	L	L	VL	VL	VL

TABLE III: FUZZY RELATION MATRIX

X_i	Equivalent Load and Length.	Voltage Drop						
		VS	S	BA	A	AA	L	VL
		Membership Value						
	$(S_{eqk\varphi}, L_{eqk\varphi})$	$\mu_R(X_i, VS)$	$\mu_R(X_i, S)$	$\mu_R(X_i, BA)$	$\mu_R(X_i, A)$	$\mu_R(X_i, AA)$	$\mu_R(X_i, L)$	$\mu_R(X_i, VL)$
X_1	(VS, VS)	1	0.5	0	0	0	0	0
X_2	(VS, S)	0.5	1	0.5	0	0	0	0
X_3	(VS, BA)	0.5	1	0.5	0	0	0	0
X_4	(VS, A)	0	0.5	1	0.5	0	0	0
X_5	(VS, AA)	0	0.5	1	0.5	0	0	0
X_6	(VS, L)	0	0	0.5	1	0.5	0	0
X_7	(VS, VL)	0	0	0.5	1	0.5	0	0
X_8	(S, VS)	1	0.5	0	0	0	0	0
X_9	(S, S)	0.5	1	0.5	0	0	0	0
X_{10}	(S, BA)	0.5	1	0.5	0	0	0	0
X_{11}	(S, A)	0	0.5	1	0.5	0	0	0
X_{12}	(S, AA)	0	0	0.5	1	0.5	0	0
X_{13}	(S, L)	0	0	0.5	1	0.5	0	0
X_{14}	(S, VL)	0	0	0	0.5	1	0.5	0
X_{15}	(BA, VS)	1	0.5	0	0	0	0	0
X_{16}	(BA, S)	0.5	1	0.5	0	0	0	0
X_{17}	(BA, BA)	0	0.5	1	0.5	0	0	0
X_{18}	(BA, A)	0	0	0.5	1	0.5	0	0
X_{19}	(BA, AA)	0	0	0.5	1	0.5	0	0
X_{20}	(BA, L)	0	0	0	0.5	1	0.5	0
X_{21}	(LS, VL)	0	0	0	0	0.5	1	0.5
X_{22}	(A, VS)	1	0.5	0	0	0	0	0
X_{23}	(A, S)	0.5	1	0.5	0	0	0	0
X_{24}	(A, BA)	0	0.5	1	0.5	0	0	0
X_{25}	(A, A)	0	0	0.5	1	0.5	0	0
X_{26}	(A, AA)	0	0	0	0.5	1	0.5	0
X_{27}	(A, L)	0	0	0	0	0.5	1	0.5
X_{28}	(A, VL)	0	0	0	0	0.5	1	0.5
X_{29}	(AA, VS)	0.5	1	0.5	0	0	0	0
X_{30}	(AA, S)	0.5	1	0.5	0	0	0	0
X_{31}	(AA, BA)	0	0.5	1	0.5	0	0	0
X_{32}	(AA, A)	0	0	0.5	1	0.5	0	0
X_{33}	(AA, AA)	0	0	0	0.5	1	0.5	0
X_{34}	(AA, L)	0	0	0	0	0.5	1	0.5
X_{35}	(AA, VL)	0	0	0	0	0	0.5	1
X_{36}	(L, VS)	0.5	1	0.5	0	0	0	0
X_{37}	(L, S)	0	0.5	1	0.5	0	0	0
X_{38}	(L, BA)	0	0	0.5	1	0.5	0	0
X_{39}	(L, A)	0	0	0	0.5	1	0.5	0
X_{40}	(L, AA)	0	0	0	0	0.5	1	0.5
X_{41}	(L, L)	0	0	0	0	0	0.5	1
X_{42}	(L, VL)	0	0	0	0	0	0.5	1
X_{43}	(VL, VS)	0.5	1	0.5	0	0	0	0
X_{44}	(VL, S)	0	0.5	1	0.5	0	0	0
X_{45}	(VL, BA)	0	0	0.5	1	0.5	0	0
X_{46}	(VL, A)	0	0	0	0.5	1	0.5	0
X_{47}	(VL, AA)	0	0	0	0	0.5	1	0.5
X_{48}	(VL, L)	0	0	0	0	0	0.5	1
X_{49}	(VL, VL)	0	0	0	0	0	0.5	1

2.4. Determine a proper output ($\Delta v_{\phi k}$) from the membership function of the output data

Use the maximum algorithm to choice the data with largest membership value as the output data ($\Delta v_{\phi k}$) [6]. Then, use a conversion table that has been compiled on previous experience to convert output data into numerical values.

3. Derivation of Unbalance Three–Phase Fuzzy Feeder Models

For combining discrete distributed loads, three pertinent calculations are required as follows:

$$L_t = \sum_{i=1}^{n} L_i \tag{3}$$

$$P_t = \sum_{\varphi=a}^{c} \sum_{i=1}^{n} P_{\varphi i} \tag{4}$$

$$Q_t = \sum_{\varphi=a}^{c} \sum_{i=1}^{n} Q_{\varphi i} \tag{5}$$

Where

L_t is the total length of the feeder

P_t is the total active load along the feeder

Q_t is the total reactive load along the feeder

Once the individual phase voltage drop ($\Delta v_{\phi k}$) at each of the tapped-off load point has been computed, along a feeder, each of the discrete load is considered to be an incremental load at its incremental distance as shown in Fig. 4. For per unit system, the incremental current is

$$I_{\varphi j} = (P_{\varphi j} - jQ_{\varphi j})/(1-\Delta v_{\varphi j}) \qquad \phi = a,b,c;\ j = 1,2\cdots,n \tag{6}$$

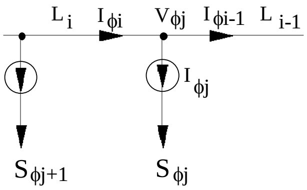

Fig. 4: A discrete load of feeder.

3.1. Three-phase voltage-drop fuzzy feeder model

A voltage-drop model was developed to maintain the voltage drop at the end of the feeder when the equivalent three-phase loads are connected and the loads are dispersed. The proposed unbalance three-phase voltage-drop fuzzy feeder model can be simplified and modeled as shown in Fig. 5. The three phase individual equivalent loads can be found as

$$P_{\varphi v} = \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{i} P_{\varphi j}/(1-\Delta v_{\varphi j}) \right) \cdot L_i \right] / L_t \quad \phi = a,b,c \tag{7}$$

$$Q_{\varphi v} = \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{i} Q_{\varphi j}/(1-\Delta v_{\varphi j}) \right) \cdot L_i \right] / L_t \quad \phi = a,b,c \tag{8}$$

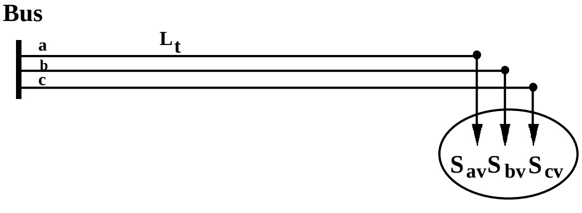

Fig. 5: Three-phase voltage-drop fuzzy feeder model.

3.2. Three-phase line-loss fuzzy feeder model

A line-loss model was proposed to maintain the same total copper loss in the feeder as in the original feeder. The proposed unbalance three-phase line-loss fuzzy model can be simplified and modeled as Fig. 6. The equivalent three phase individual loads that are connected at the end of feeder can be calculated as

$$P_{\varphi p} = \left\{ \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{i} P_{\varphi j}/(1-\Delta v_{\varphi j}) \right)^2 \cdot L_i \right] / L_t \right\}^{\frac{1}{2}} \quad \phi = a,b,c \tag{9}$$

$$Q_{\varphi p} = \left\{ \sum_{i=1}^{n} \left[\left(\sum_{j=1}^{i} Q_{\varphi j}/(1-\Delta v_{\varphi j}) \right)^2 \cdot L_i \right] / L_t \right\}^{\frac{1}{2}} \quad \phi = a,b,c \tag{10}$$

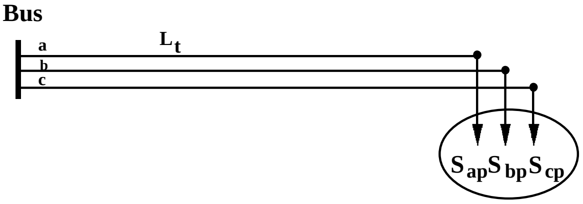

Fig. 6: Three-phase line-loss fuzzy feeder model.

3.3. Three-phase hybrid fuzzy feeder model

A hybrid model was developed to accurately represent both the total series voltage drop and total copper loss of the given feeder. The model shown in Fig. 7 combines the two foregoing models. For the assumption of balanced feeder, the proportion of the split line length can be shown as [5]

$$K = \frac{L_t \cdot L_p - L_v^2}{L_t^2 - 2L_t \cdot L_v + L_t \cdot L_p} \tag{11}$$

where the L_v *and* L_p are equivalent lengths of the voltage-drop model and line-loss model. They are represented as

$$L_v = \left| \sum_{i=1}^{n} \left[\left(\sum_{\varphi=a}^{c} \sum_{j=1}^{i} S_{\varphi j} \right) \cdot L_i \right] \right| / \hat{S}_t \tag{12}$$

$$L_p = \sum_{i=1}^{n} \left[\left| \left(\sum_{\varphi=a}^{c} \sum_{j=1}^{i} S_{\varphi j} \right) \right|^2 \cdot L_i \right] / \hat{S}_t^2 \tag{13}$$

The three phase individual equivalent loads, in the Fig.7, can be easily found as

$$P_{\phi h1} = \left[\frac{P_{\phi p}^2 - P_{\phi v}^2}{K \cdot (1-k)} \right]^{\frac{1}{2}} \qquad \phi = a,b,c \tag{14}$$

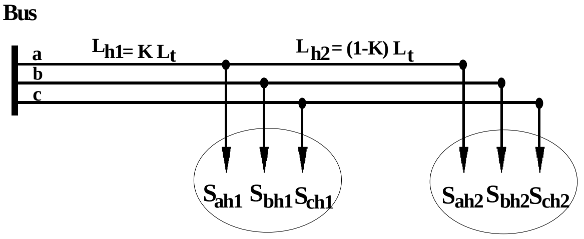

Fig. 7: Three-phase hybrid voltage-drop and line-loss fuzzy feeder model.

$$P_{\phi h2} = P_{\phi v} - \left[\frac{K \cdot \left(P_{\phi p}{}^{2} - P_{\phi v}{}^{2} \right)}{(1-k)} \right]^{\frac{1}{2}} \qquad \phi = a,b,c \quad (15)$$

$$Q_{\phi h1} = \left[\frac{Q_{\phi p}{}^{2} - Q_{\phi v}{}^{2}}{K \cdot (1-k)} \right]^{\frac{1}{2}} \qquad \phi = a,b,c \quad (16)$$

$$Q_{\phi h2} = Q_{\phi v} - \left[\frac{K \cdot \left(Q_{\phi p}{}^{2} - Q_{\phi v}{}^{2} \right)}{(1-k)} \right]^{\frac{1}{2}} \qquad \phi = a,b,c \quad (17)$$

4. Results and Conclusion

A physical feeder in the Taipower distribution system (Fig. 8) is adopted as a sample feeder to demonstrate the correctness of the proposed models. It is a radial-type feeder with three types of transformer connections: grounded wye-delta, open wye-open delta, and single-phase. The lengths of line sections are shown in Fig. 8. The conductor impedance of this feeder is 0.00202+j0.00271 p.u. per kilometer base on 1 MVA and $11.4/\sqrt{3}$ KV. A three-phase power flow program using the modeling and solution techniques presented in previous studies is used to simulate the following cases [1], [5].

The proposed unbalance three-phase voltage-drop fuzzy model (called the V-Drop Model in Table V); line-loss fuzzy model (Line-Loss); and hybrid fuzzy model (Hybrid) that combines the voltage-drop and line-loss models are compared for accuracy. A detailed model (Detailed) is made in great detail and is used here as the standard for comparison of study results. It is does not include any compromises and simulates every tapped-off load point as a bus.

The simulation results corroborate the correctness of each model and illustrate the three phase voltage at the end of the given feeder (End-Volt), the system line loss, and the relative accuracy of each of the models (Err %), expressed as a percentage of deviation from the detailed model of these results [3].

Table IV shows the individual phase loads of this sample feeder [5], [7]. The simulation results (Table V) show that the hybrid fuzzy model can simulate both the total series voltage drop and total copper loss accurately, and this is therefore considered to be the most acceptable model if both total series voltage drop and total copper loss are required to beaccurate. However, if only total series voltage drop or only total copper loss is required, the voltage-drop or line-loss fuzzy model should be used to simplify data input and reduce the computing effort because almost the same accuracy as that of the hybrid fuzzy model can be obtained by these simple models.

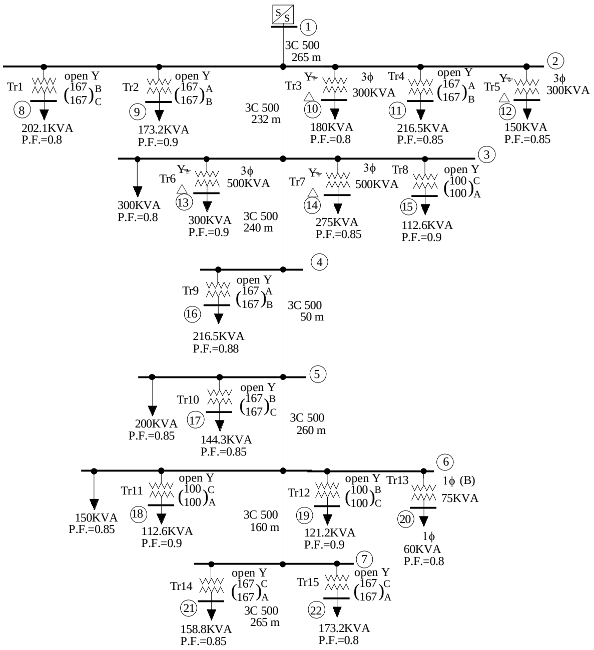

Fig. 8: Sample feeder.

The results also show that the number of iterations is dramatically reduced from 790 to 12 when the proposed models are used instead of the detailed model. This comparison shows that using the proposed unbalance three-phase fuzzy models are much more effecient than using the detailed model.

TABLE IV: INDIVIDUAL PHASE LOADS OF THE SAMPLE FEEDER

Node	Phase a	Phase b	Phase c
2	205.72+j255.25	361.01+j166.29	206.34+j76.31
3	312.76+j147.16	247.92+j151.86	284.43+j205.66
4	65.57+j106.42	124.95-j3.58	0
5	56.67+j35.12	96.05+j108.54	139.94+j37.72
6	107.34+j21.62	129.78+j120.24	148.78+j75.06
7	190.92+j14.83	0	82.62+j172.75

TABLE V: THE SIMULATION RESULTS

Model of feeder	End-Volt (pu)			Line-Loss (pu)	Iterations
	Phase a	Phase b	Phase c		
Detailed	0.9981	0.9990	0.9976	0.00303	790
V-Drop (Err %)	0.9981 (0)	0.9990 (0)	0.9977 (-0.0100)	0.00217 (28.383)	12
Line-Loss (Err %)	0.9977 (0.0401)	0.9984 (0.0601)	0.9977 (-0.0100)	0.00290 (4.290)	12
Hybrid (Err %)	0.9981 (0)	0.9990 (0)	0.9977 (-0.0100)	0.00300 (0.990)	12

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