The Analysis of Noise Frequency Modulation Jamming Signal Based on Stochastic Differential

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Abstract-It is the most important problem in the radar countermeasures that the radar jamming effectiveness evaluation. The basis of the radar jamming effectiveness evaluation is the radar jamming signal processing. According to the intrinsic relations between the stochastic differential and the radar jamming signal processing, the stochastic calculus was used in the radar jamming signal processing in this paper. The noise frequency modulation signal was particularly analyzed. The Fokker-Planck equation of noise frequency modulation was presented and the Motion-Group Fourier Transform was used by converting the partial differential equation into the variable coefficient homogenous linear differential equations. Then the solutions were given by using the peano-baker series.

Keywords-Stochastic Differential, Fokker-Planck Equation, Motion-Group Fourier Transform (MGFT), Peano-Baker series

I. INTRODUCTION

It is the most important problem in the radar countermeasures that the radar jamming effectiveness evaluation. The basis of the radar jamming effectiveness evaluation is the radar jamming signal processing. That is how to get the probability density function (PDF) with the jamming signal. It is so difficult that many models are in the basis of statistic method which only described the relationship between PDF and signal noise ratio (SNR) in [1]~[5]. But PDF is connection with multi-parameters including noise variance, frequency of carrier wave and amplitude of carrier wave.

A new method that describe the relationship between phase noise (PN) and the filtered signal is found in [6]~[9]. According to this, we presented the models that describe the relationship between noise frequency modulation and pulse compression radar signal. The Fokker-Planck (FP) equation of noise frequency modulation is presented and the Motion-Group Fourier Transform (MGFT) is used by converting the partial differential equation into the variable coefficient homogenous linear differential equations. Also peano-baker series are used to solve the variable coefficient homogenous linear differential equations.

II. THE IMPACT OF NOISE FREQUENCY MODULATION JAMMING SIGNAL

A. The Statistical Character of Noise Frequency Modulation Jamming Signal

The noise frequency modulation jamming signal can be written as

$$J(t) = U_j \cos[2\pi f_j t + 2\pi K_{FM} \int_0^t u(s)ds] \quad (1)$$

where U_j is the amplitude of carrier wave, f_j is the frequency of jamming carrier wave, K_{FM} is the frequency modulation coefficient. $u(s)$ is the white noise with zero mean and variance of σ_n^2. The correlation function of $J(t)$ is

$$B_j(\tau) = \frac{U_j^2}{2} e^{-\frac{\sigma^2(\tau)}{2}} \cos 2\pi f_j \tau \quad (2)$$

where $e(t) = \int_0^t u(s)ds$ is coincidence with $N(0, \sigma_n^2 t)$, $\sigma^2(\tau)$ is the variance of frequency modulation $2\pi K_{FM}[e(t+\tau) - e(t)]$. Then

$$\begin{aligned}
\sigma^2(\tau) &= 4\pi^2 K_{FM}^2 E[e(t+\tau) - e(t)]^2 \\
&= 4\pi^2 K_{FM}^2 E[e^2(t+\tau) - 2e(t+\tau)e(t) + e^2(t)] \\
&= 4\pi^2 K_{FM}^2 \sigma_n^2 [(t+\tau) - 2t + t] \\
&= 4\pi^2 K_{FM}^2 \sigma_n^2 \tau
\end{aligned} \quad (3)$$

From (2) and (3), we can get

$$B_j(\tau) = \frac{U_j^2}{2} e^{-2\pi^2 K_{FM}^2 \sigma_n^2 \tau} \cos 2\pi f_j \tau \quad (4)$$

Obviously correlation function $B_j(\tau)$ is only relation with time difference τ and satisfies the condition of broad steady. In the following, we will derive the stochastic differential

equations (SDEs) first. Then we write the corresponding FP equations. At last we use the MGFT to solve the FP equations.

B. The Fokker-Planck Equation

The input noise frequency modulation jamming signal of radar receiver intermediate frequency (IF) filter is

$$s(t) = U_j e^{j(2\pi f_j t + 2\pi K_{FM} \int_0^t u(s)ds)} = U_j e^{j\phi(t)} \quad . \quad h(t)$$

is the impulse response of the matched IF filter. The output is

$$z(t) = h(t) * s(t) = \int_0^t h(\tau) U_j e^{j[\phi(t)-\phi(\tau)]} d\tau \quad (5)$$

Let write $\phi_{\Delta t} = \phi(t + \Delta t) - \phi(t)$, then

$$\begin{aligned} z(t + \Delta t) &= \int_0^{t+\Delta t} h(\tau) U_j e^{j[\phi(t+\Delta t)-\phi(\tau)]} d\tau \\ &= \int_0^t h(\tau) U_j e^{j[\phi(t)-\phi(\tau)]} e^{j\phi_{\Delta t}} d\tau \\ &\quad + \int_t^{t+\Delta t} h(\tau) U_j e^{j[\phi(t+\Delta t)-\phi(\tau)]} d\tau \\ &= e^{j\phi_{\Delta t}} \int_0^t h(\tau) U_j e^{j[\phi(t)-\phi(\tau)]} d\tau \\ &\quad + \int_t^{t+\Delta t} h(\tau) U_j e^{j[\phi(t+\Delta t)-\phi(\tau)]} d\tau \\ &= z(t) e^{j\phi_{\Delta t}} + \int_t^{t+\Delta t} h(\tau) U_j e^{j[\phi(t+\Delta t)-\phi(\tau)]} d\tau \end{aligned}$$

$$(6)$$

At the time interval $t \le \tau \le t + \Delta t$,

$$\phi(t + \Delta t) - \phi(\tau) \sim N(2\pi f_j(t + \Delta t - \tau), \quad (7)$$
$$4\pi^2 K_{FM}^2 \sigma_n^2 (t + \Delta t - \tau))$$

and $\phi(t + \Delta t) - \phi(\tau) = o(2\pi f_j \Delta t + 2\pi K_{FM}\sigma_n\sqrt{\Delta t})$. The integral in (6) is

$$\begin{aligned} & \int_t^{t+\Delta t} h(\tau) U_j e^{jo(2\pi f_j \Delta t + 2\pi K_{FM}\sigma_n\sqrt{\Delta t})} d\tau \\ &= U_j h(t) e^{jo(2\pi f_j \Delta t + 2\pi K_{FM}\sigma_n\sqrt{\Delta t})} \Delta t \\ &= U_j h(t)[1 + jo(2\pi f_j \Delta t + 2\pi K_{FM}\sigma_n\sqrt{\Delta t})]\Delta t \\ &= U_j h(t)[\Delta t + o(\Delta t)] \end{aligned}$$

$$(8)$$

Substitute (8) into (6), we have

$$z(t + \Delta t) = z(t) e^{j\phi_{\Delta t}} + U_j h(t)[\Delta t + o(\Delta t)] \quad (9)$$

So

$$\begin{aligned} dz(t) &= U_j h(t)dt + jz(t)[2\pi f_j dt + 2\pi K_{FM}\sigma_n dW(t)] \\ &= [U_j h(t) + j2\pi f_j z(t)]dt + j2\pi K_{FM}\sigma_n z(t)dW(t) \end{aligned}$$

$$(10)$$

Let $z(t) = r(t)e^{j\theta(t)}$, we rewrite (10) in polar coordinates as

$$dr(t)e^{j\theta(t)} = [U_j h(t) + j2\pi f_j r(t)e^{j\theta(t)}]dt \\ + j2\pi K_{FM}\sigma_n r(t)e^{j\theta(t)}dW(t) \quad (11)$$

The left of (11) is

$$\begin{aligned} & e^{j\theta(t)}dr(t) + jr(t)e^{j\theta(t)}d\theta(t) \\ &= [\cos\theta(t)dr(t) - r(t)\sin\theta(t)d\theta(t)] \quad (12) \\ &\quad + j[\sin\theta(t)dr(t) + r(t)\cos\theta(t)d\theta(t)] \end{aligned}$$

and the right is

$$\begin{aligned} & [U_j h(t) + j2\pi f_j r(t)e^{j\theta(t)}]dt + j2\pi K_{FM}\sigma_n r(t)e^{j\theta(t)}dW(t) \\ &= [U_j U_0 B\cos(2\pi f_0 t - \pi\frac{B}{T}t^2)dt - 2\pi f_j r(t)\sin\theta(t)dt \\ &\quad - 2\pi K_{FM}\sigma_n r(t)\sin\theta(t)dW(t)] \\ &\quad + j[U_j U_0 B\sin(2\pi f_0 t - \pi\frac{B}{T}t^2)dt + 2\pi f_j r(t)\cos\theta(t)dt \\ &\quad + 2\pi K_{FM}\sigma_n r(t)\cos\theta(t)dW(t)] \end{aligned}$$

$$(13)$$

From (11) to (13), we can get

$$\begin{bmatrix} dr(t) \\ d\theta(t) \end{bmatrix} = \begin{bmatrix} U_j U_0 B[\cos\varphi(t)\cos\theta(t) + \sin\varphi(t)\sin\theta(t)] \\ 2\pi f_j + \dfrac{U_j U_0 B[\sin\varphi(t)\cos\theta(t) - \cos\varphi(t)\sin\theta(t)]}{r(t)} \end{bmatrix} dt$$
$$+ \begin{bmatrix} 0 \\ 2\pi K_{FM}\sigma_n \end{bmatrix} dW(t)$$

$$(14)$$

where $\varphi(t) = 2\pi f_0 t - \pi\frac{B}{T}t^2$. we can derive the Fokker-Planck equation for (14)as:

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\sum_{i=1}^2 \frac{\partial}{\partial x_i}[a_i p(x,t)] + \frac{1}{2}\sum_{i,j=1}^2 \frac{\partial^2}{\partial x_i \partial x_j}[(HH^T)_{ij} p(x,t)] \\ &= -U_j U_0 B[\cos\varphi(t)\cos\theta(t) + \sin\varphi(t)\sin\theta(t)]\frac{\partial p}{\partial r} \\ &\quad - [2\pi f_j + \frac{U_j U_0 B[\sin\varphi(t)\cos\theta(t) - \cos\varphi(t)\sin\theta(t)]}{r(t)}]\frac{\partial p}{\partial\theta} \\ &\quad + \frac{(2\pi K_{FM}\sigma_n)^2}{2}\frac{\partial^2 p}{\partial\theta^2} \end{aligned}$$

$$(15)$$

C. Solving the Fokker-Planck Equation Using MGFT

Applying the MGFT to (15), we can convert it to an infinite system of linear ordinary differential equations (ODEs)

$$\frac{d\hat{p}}{dt} = -U_j U_0 B \cos\varphi(t)\hat{p}\eta(\tilde{X}_2, p) - U_j U_0 B \sin\varphi(t)\hat{p}\eta(\tilde{X}_3, p)$$

$$+2\pi f_j \eta(\tilde{X}_1, p)\hat{p} - 2\pi f_j \hat{p}\eta(\tilde{X}_1, p) + \frac{(2\pi K_{FM}\sigma_n)^2}{2}[\eta(\tilde{X}_1, p)]^2 \hat{p}$$

$$+\frac{(2\pi K_{FM}\sigma_n)^2}{2}\hat{p}[\eta(\tilde{X}_1, p)]^2 - (2\pi K_{FM}\sigma_n)^2 \eta(\tilde{X}_1, p)\hat{p}\eta(\tilde{X}_1, p)$$

$$(16)$$

Once we get the solution to the ODE (16), we can then substitute it into the Fourier inversion formula for the motion group to recover the PDF $p(r,\theta,\phi;t)$. To get the joint PDF $p(r,\theta;t)$ is just an integration, with respect to ϕ, as

$$p(r,\theta;t) = \frac{1}{2\pi}\int_0^{2\pi} p(r,\theta,\phi;t)d\phi$$
$$= \sum_{n\in Z} j^{-n} e^{-jn\theta}\int_0^\infty \hat{p}_{0,n}(p)J_{-n}(pr)pdp$$

$$(17)$$

Integrating (17) over θ will give us the marginal PDF of $z(t)$ as

$$p(r;t) = \int_0^\infty \hat{p}_{0,0}(p)J_0(pr)pdp \qquad (18)$$

D. Solving the Variable Coefficients Homogenous Linear Differential Equations

(16) is the variable coefficients homogenous linear differential equations. We use peano-baker series to solve it. The algorithm is as follows:

1) Dividing time domain $[0，t]$ into N intervals and Using peano-baker series to calculate the state transfer function at any time interval $[t_j，t_{j+1}]$

$$\Phi(t_j, t_{j+1}) = I + [A(t_j) + A(t_{j+1})]\tau/2 + A^2(t_j)\tau^2/2$$

$$(19)$$

where $\tau = t/N$, A is the coefficients matrix of variable coefficients homogenous linear differential equations.

2) Depending on the quality of state transfer function, the whole state transfer function is

$$\Phi(0,t) = \Phi(t_{N-1}, t_N)\Phi(t_{N-2}, t_{N-1})\cdots\Phi(t_0, t_1)$$

$$(20)$$

3) The formula

$$\hat{p}(t) = \Phi(0,t)\hat{p}(0) \qquad (21)$$

can be used to get the solution of variable coefficients homogenous linear differential equations.

III.   NUMERICAL RESULTS

Setting the experiment parameters $U_j = 1$, $U_0 = 1$, $T = 1\times 10^{-4}$, $f_0 = 1\times 10^5$, $f_j = f_0$, $B = 1\times 10^6$ and defining

the information entropy $H = -\int_0^\infty p(r)\ln p(r)dr$. The relationship between the carrier wave frequency f_j of noise frequency modulation jamming signal and the probability of detection information entropy H is described in Fig. 1.
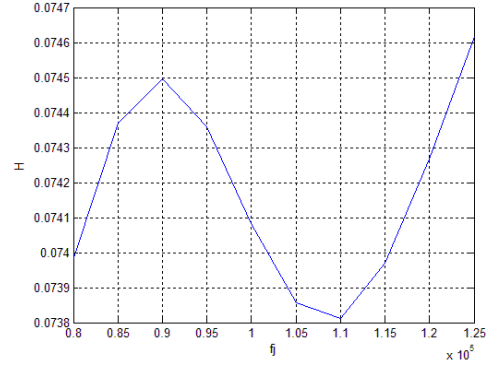

Figure 1. Probability of detection information entropy H for different jamming frequency f_j

IV.   SUMMARIES

We have discussed the application of stochastic differential theory in the domain of radar jamming signal processing. We have presented the Fokker-Planck equation for the noise frequency modulation jamming signal. And the MGFT has been used by converting the partial differential equation into the variable coefficient homogenous linear differential equations. Then the solutions are given by using the peano-baker series. So the probability density function of noise frequency modulation in the filter is given. The impact of noise frequency modulation jamming signal on the pulse compression radar is analyzed in the criterion of information entropy.

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