Analysis of Dynamic Mathematical Model of Bi-directional Contactless Inductive Power Transfer System Based on Generalized State-space Averaging Method

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Abstract—**Based on generalized state-space averaging method (GSSA) and selective modal analysis method, a dynamic mathematical model was built to depict the dynamic characteristic of the bi-directional contactless inductive power transfer (CIPT) system. The model has already been tested by Matlab, and the simulation results illustrate that the model can give a good description of characteristics of bi-directional CIPT system.**

Keywords-Bi-directional CIPT, Generalized state-space averaging method, Selective modal analysis method, System order reduction

I. Introduction

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The technology of CIPT has already been widely applied in a varity of power applications. Since it won't generate any electric spark and abrasion, it is safety and meaningfully to introduce it to practical application [1]. But at certain times, loads which has already been charged don't need to keep working continuely, and the power grid maybe suffers the peak of power consuming. So, it is necessary to find a way to transfer the energy from loads to the grid to buff the power-consuming peak and to reduce the negative impact on it. At the same time, the natural wastage of electric energy of the batteries can be reduced to a lower level.

The bi-directional CIPT system can transmit the energy in both directions so when it comes the power-consuming peak, the energy-storage devices can provide some support to the grid to reduce the negative concussion [2]. Now it is one of the research focuses of power transmission. In order to obtain the greatest possible efficiency of power transmission capacity, researchers tried so many ways to make the primary and secondary subsystem coordinate closer with each other. But as the coils are loosely coupled and eletromagic noise is inevitable, it's difficult to achieve an ideal control system via setting up a tranditional communication system.

This paper proposed a dynamic model to simulate the bi-directional system. To build the dynamic model, the mutual inductance electrical model should be set up, and then be analyzed with generalized state-space averaging method. The dynamic mathematical model of the system should be framed at the end.

II. THE ELECTRICAL MODEL OF THE SYSTEM

The schematic of the system is shown in Figure1.

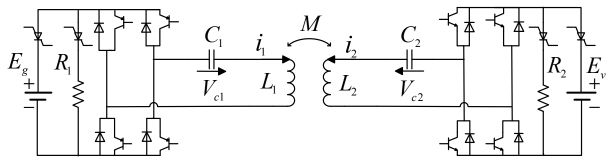

Figure 1. The schematic of the bi-directional CIPT system

The equations of the system listed below can be drafted on the basis of the Kirchhoff's voltage law and current law:

$$\begin{cases} s(t)E_g = L_1\dfrac{di_1}{dt} + j\omega M i_2 + V_{c1} \\ i_1 = C_1\dfrac{dV_{c1}}{dt} \\ i_2 R_2 = L_2\dfrac{di_2}{dt} + j\omega M i_1 + V_{c2} \\ i_2 = C_2\dfrac{dV_{c2}}{dt} \end{cases}$$

and

$$\begin{cases} i_1 R_1 = L_1 \dfrac{di_1}{dt} + j\omega M i_2 + V_{c1} \\ i_1 = C_1 \dfrac{dV_{c1}}{dt} \\ s(t) E_v = L_2 \dfrac{di_2}{dt} + j\omega M i_1 + V_{c2} \\ i_2 = C_2 \dfrac{dV_{c2}}{dt} \end{cases} \tag{1}$$

in which and stands for the current in primary and secondary coil, v_{c1} and v_{c2} are the voltage of the compensation capacitors in primary and secondary subsystem respectively. $S(t)$ is a switching function of the H-bridge resonant converter which can be shown as:

$$s(t) = \begin{cases} 1 mT \le t < (2m+1)T/2 \\ -1 \dfrac{(2m+1)T}{2} \le t \le (m+1)T \end{cases} \tag{2}$$

where m is a positive integer.

The GSSA method and improved-GSSA method are described in reference [3] and [4] respectively.

The actual state variables of the system can be depicted as:

$$x(t) = x_{-1} e^{-j\omega t} + x_0 + x_1 e^{j\omega t} \tag{3}$$

To describe i_1, V_{c1}, i_2 and V_{c2} with the model formed by zero-order and first-order fourier coefficient, formulas with 12 variables should be written as:

$$\begin{cases} i_{11} = x_1 + jx_2 \\ V_{c11} = x_3 + jx_4 \\ i_{21} = x_5 + jx_6 \\ V_{c21} = x_7 + jx_8 \end{cases} \tag{4}$$

$$i_{10} = x_9,\ V_{c10} = x_{10},\ i_{20} = x_{11},\ V_{c20} = x_{12} \tag{5}$$

And further more, the formula below can be drawn:

$$\begin{cases} i_{11} = i_{1-1}^{*} \\ V_{c11} = V_{c1-1}^{*} \\ i_{21} = i_{2-1}^{*} \\ V_{c21} = V_{c2-1}^{*} \end{cases} \tag{6}$$

The fourier form of formula (2) is

$$s(t)_0 = 0, s(t)_1 = -2j/\pi \tag{7}$$

Formulas listed above can get the state space equation of zero-order and

first-order couple with each other.

$$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} \langle \bullet \rangle_0 \\ \langle \bullet \rangle_1 \end{bmatrix} = \begin{bmatrix} A_1 A_2 \\ A_3 A_4 \end{bmatrix} \begin{bmatrix} \langle \bullet \rangle_0 \\ \langle \bullet \rangle_1 \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} V_1 \\ \dfrac{d}{dt}\begin{bmatrix} \langle \bullet \rangle_0 \\ \langle \bullet \rangle_1 \end{bmatrix} = \begin{bmatrix} A_5 A_6 \\ A_7 A_8 \end{bmatrix} \begin{bmatrix} \langle \bullet \rangle_0 \\ \langle \bullet \rangle_1 \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} V_2 \end{cases} \tag{8}$$

The twelve state variables should be divided into two groups, $\langle \bullet \rangle_0$ and $\langle \bullet \rangle_1$. They stand for state variables in zero-order and first-order state respectively.

$$\langle \bullet \rangle_0 = \begin{bmatrix} x_9 & x_{10} & x_{11} & x_{12} \end{bmatrix}^T \tag{9}$$

$$\langle \bullet \rangle_1 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T \tag{10}$$

Vector B,C and matrix $A_1 \sim A_8$ can be written as:

$$A_1 = \begin{bmatrix} 0 & -\dfrac{1}{L_1} & \dfrac{-j\omega M}{L_1} & 0 \\ \dfrac{1}{C_1} & 0 & 0 & 0 \\ \dfrac{-j\omega M}{L_2} & 0 & \dfrac{R_2}{L_2} & -\dfrac{1}{L_2} \\ 0 & 0 & \dfrac{1}{C_2} & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} \dfrac{R_1}{L_1} & -\dfrac{1}{L_1} & \dfrac{-j\omega M}{L_1} & 0 \\ \dfrac{1}{C_1} & 0 & 0 & 0 \\ \dfrac{-j\omega M}{L_2} & 0 & 0 & -\dfrac{1}{L_2} \\ 0 & 0 & \dfrac{1}{C_2} & 0 \end{bmatrix} \tag{11}$$

$$A_2 = A_3 = A_6 = A_7 = 0 \tag{12}$$

$$A_4 = \begin{bmatrix} 0 & \omega & -\dfrac{1}{L_1} & 0 & 0 & \dfrac{\omega M}{L_1} & 0 & 0 \\ -\omega & 0 & 0 & -\dfrac{1}{L_1} & -\dfrac{\omega M}{L_1} & 0 & 0 & 0 \\ \dfrac{1}{C_1} & 0 & 0 & \omega & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{C_1} & -\omega & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{\omega M}{L_1} & 0 & 0 & -\dfrac{R_2}{L_2} & \omega & -\dfrac{1}{L_2} & 0 \\ -\dfrac{\omega M}{L_1} & 0 & 0 & 0 & -\omega & -\dfrac{R_2}{L_2} & 0 & -\dfrac{1}{L_2} \\ 0 & 0 & 0 & 0 & \dfrac{1}{C_2} & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{C_2} & -\omega & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -\frac{R_1}{L_1} & \omega & -\frac{1}{L_1} & 0 & 0 & \frac{\omega M}{L_1} & 0 & 0 \\ -\omega & -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} & -\frac{\omega M}{L_1} & 0 & 0 & 0 \\ \frac{1}{C_1} & 0 & 0 & \omega & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_1} & -\omega & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\omega M}{L_1} & 0 & 0 & 0 & \omega & -\frac{1}{L_2} & 0 \\ -\frac{\omega M}{L_1} & 0 & 0 & 0 & -\omega & 0 & 0 & -\frac{1}{L_2} \\ 0 & 0 & 0 & 0 & \frac{1}{C_2} & 0 & 0 & \omega \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} & -\omega & 0 \end{bmatrix} \tag{13}$$

$$\begin{cases} B = \begin{bmatrix} 0 & \frac{-2}{\pi L_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{-2}{\pi L_1} & 0 & 0 \end{bmatrix}^T \end{cases} \tag{14}$$

Fomula (8) to (14) can formate a GSSA model of the bi-directional CIPT system.

The formula of the state variables can be written on the basis of formula (3) as:

$$\begin{cases} i_1 = 2x_1 \cos\cos(\omega t) - 2x_2 \sin\sin(\omega t) + x_9 \\ V_{c1} = 2x_3 \cos\cos(\omega t) - 2x_4 \sin\sin(\omega t) + x_{10} \\ i_2 = 2x_5 \cos\cos(\omega t) - 2x_6 \sin\sin(\omega t) + x_{11} \\ V_{C2} = 2x_7 \cos\cos(\omega t) - 2x_8 \sin\sin(\omega t) + x_{12} \end{cases} \tag{15}$$

III. SELECTIVE MODAL ANALYSIS METHOD

Selective modal analysis method is an accurate and effective theory which can execute selecting and analyzing operation on structure and behavior of the linear time-invariant system [5]. By applying this method, those parts which are associated with the dynamic characteristics of the system could be separated appropriately, then decoupling, order-reduction, modeling of input and output characteristics and model controlling could be executed.

The dynamic mathematic model is a twelfth-order system and it may increase the hardware requirements of the system, so the selective modal analysis method was introduced to reduce the order of (8).

Flow diagram shown in Figure2 gives the connection between zero-order model and first-order model:

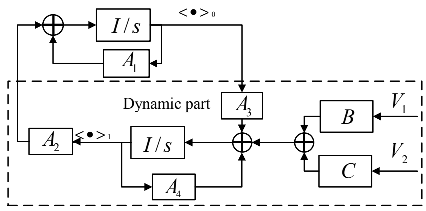

Figure 2. Diagram of interaction between the 0-order and 1-order

Considering matrix A_2 , A_3 , A_6 and A_7 are zero matrix, according to the selective modal analysis method, the dynamic model of fundamental components and zero-order average model can be decoupled directly. In addition, relationship between DC model represented by zero-order average model and dynamic model in the load-resonant power converter is weak. While the first-order system can represents the fundamental characteristics of the resonance output. Therefore, the dynamic model of the system can be set as:

$$\begin{cases} \frac{d}{dt}\langle \bullet \rangle_1 = A_4 \langle \bullet \rangle_1 + BV_1 \\ \frac{d}{dt}\langle \bullet \rangle_1 = A_8 \langle \bullet \rangle_1 + CV_2 \end{cases} \tag{16}$$

Simplified form of the actual formula of state variables are:

$$\begin{cases} i_1 = 2x_1 \cos\cos(\omega t) - 2x_2 \sin\sin(\omega t) \\ V_{c1} = 2x_3 \cos\cos(\omega t) - 2x_4 \sin\sin(\omega t) \\ i_2 = 2x_5 \cos\cos(\omega t) - 2x_6 \sin\sin(\omega t) \\ V_{c2} = 2x_7 \cos\cos(\omega t) - 2x_8 \sin\sin(\omega t) \end{cases} \tag{17}$$

IV. SIMULATION OF THE DYNAMIC MATHEMATICS MODEL

The simulation results of the model based on GSSA method and the model based on direct electric circuit topology are shown in figures below.

The parameters of system comparative simulation are: $L_1 = 130\mu H$, $L_2 = 130\mu H$, $M = 130\mu H$, $C_1 = 0.2034\mu F$, $C_2 = 0.2034\mu F$, $f = 40kHz$, $R_1 = 27\Omega, R_2 = 27\Omega$, $E_g = 100V$, $E_v = 100V$.

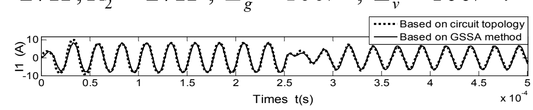

Figure 3. Simulation waveforms of i_1 based on GSSA method and circuit topology

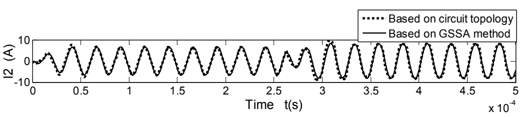

Figure 4. Simulation waveforms of i_2 based on GSSA method and circuit topology

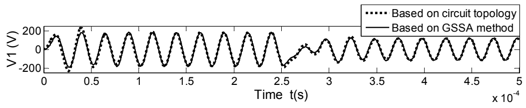

Figure 5. Simulation waveforms of V_1 based on GSSA method and circuit topology

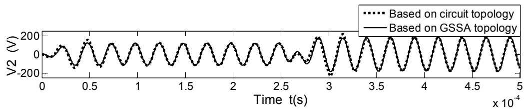

Figure 6. Simulation waveforms of V_2 based on GSSA method and circuit topology

According to the results illustrated by Figure3 to Figure6, the deviation is small between waves simulated by two different methods in transient process, and it is even negligible in steady-state region.

V. SUMMARIES

In this paper, the generalized state-space averaging method and the selective modal analysis method are applied in analyzing the technology of bi-directional contactless inductive power transfer. The representative system was fully analyzed by the method, and the simulation results were compared to results simulated by ciruit topology. Apparently, the deviation is acceptable. The models proposed in this paper can be used in system designing and stability assessing for a bi-directional CIPT system.

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