Information Fusion Algorithms in Ins/Smns Integrated Navigation System

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***Abstract*-As the math model and the noise statistical information are difficult to be addressed precisely and the Scene Matching Navigation System (SMNS) output is stochastic, limited and probably mismatching, only a few algorithms suitable for Inertial Navigation System /Scene Matching Navigation System (INS/SMNS) integrated navigation system for information fusion were developed. In order to find new algorithms suitable for this system, this paper studies the issue as follows. Firstly, this paper presents the improved Kalman filter by applying the methods of extrapolation, discretizing system model in unequal interval and eliminating the measurement output delay to the common Kalman filter. Secondly, this paper studies the variable step-size LMS algorithm and normalized LMS algorithm basing on H^{∞} optimal estimation. Lastly this paper applies them to the INS/SMNS integrated navigation. Simulation results demonstrate that they are suitable for INS/SMNS integrated navigation.**

Keywords-Information fusion, Improved Kalman filter, Variable step-size LMS, Normalized LMS

I. INTRODUCTION

INS/SMNS integrated navigation system can provide aircraft position information with high accuracy, so it is preferred by many countries [1]. However, only a few algorithms suitable for INS/SMNS integrated navigation system for information fusion were developed [2] by reason that the math model and the noise statistical information are difficult to be addressed precisely and the SMNS output is stochastic, limited and probably mismatching. So finding new information fusion algorithms suitable for INS/SMNS integrated navigation system is very necessary and meaningful.

One popular information fusion algorithm applied in navigation systems is Kalman filter. But the problems of time alignment, multi-rate filter and output measurement delay will be faced when the common Kalman filter is applied in INS/SMNS integrated navigation system which results in the filter accuracy too low to be applied in this system. To solve these problems, the common Kalman filter is improved for INS/SMNS integrated navigation system and addressed in this paper.

Another popular information fusion algorithm is LMS (Least Mean Square), especially the variable step-size LMS and the normalized LMS. The reasons why they are popular are that: on one hand, there is no need for any assumption on model's uncertainty and noise statistical information; on the other hand, they are optimal estimation algorithms based on the H^{∞} optimal estimation theory presented in the reference paper [3]. Based on this theory, the variable step-size LMS and normalized LMS are applied in INS/SMNS integrated navigation system and addressed in this paper.

At the end of this paper, the simulation results demonstrate that the improved Kalman filter, the variable step-size LMS and the normalized LMS are suitable for INS/SMNS integrated navigation.

II. APPLICATION OF IMPROVED KALMAN FILTER IN INS/SMNS INTEGRATED NAVIGATION SYSTEM

A. INS/SMNS Integrated Navigation System Model

The state equation of INS/SMNS integrated navigation system is the same as the error equation of INS in this paper as follows [1]:

$$\dot{X}(t) = F(t)\cdot X(t) + G(t)\cdot W(t) \tag{1}$$

The measurement equation that will produce the measurement is as follows:

$$Z = \begin{bmatrix} (L_I - L_S)\cdot(R_m + h) \\ (\lambda_I - \lambda_S)\cdot(R_n + h)\cdot\cos L \end{bmatrix} \tag{2}$$

where L_I and λ_I are longitude and latitude output from INS. L_S and λ_S are longitude and latitude output from SMNS. R_m is the radius of curvature in meridian plane. R_n is the radius of curvature in prime vertical. N_N and N_E are north and east matching error of SMNS.

After discretization of the equation (1) and (2), the following discrete equations will be obtained:

$$\left.\begin{aligned} X_{k+1} &= \Phi_{k+1,k}X_k + \Gamma_{k+1,k}W_k \\ Z_k &= H_k X_k + V_k \end{aligned}\right\} \tag{3}$$

B. Common Kalman Filter Process

The process of common Kalman filter is as follows:

$$\left.\begin{aligned} \hat{X}_{k+1,k} &= \Phi_{k+1,k}\hat{X}_{k,k} \\ \hat{X}_{k+1,k+1} &= \hat{X}_{k+1,k} + K_{k+1}[Z_{k+1} - H_{k+1}\hat{X}_{k+1},_k] \\ P_{k+1,k} &= \Phi_{k+1,k}P_{k,k}\Phi_{k+1,k}^T + \Gamma_{k+1,k}Q_k\Gamma_{k+1,k}^T \\ K_{k+1} &= P_{k+1,k}H_{k+1}^T[H_{k+1}P_{k+1,k}H_{k+1}^T + R_{k+1}]^{-1} \\ P_{k+1,k+1} &= P_{k+1,k}P_{k+1,k} - H_{k+1}^T[H_{k+1}P_{k+1,k}H_{k+1}^T + R_{k+1}]^{-1}H_{k+1}P_{k+1,k} \end{aligned}\right\} \quad (4)$$

When applying the common Kalman filter in INS/SMNS integrated navigation system, the problems faced when the Kalman filter is applied in INS/SMNS integrated navigation system are time alignment, multi-rate filter and measurement output delay. Because of these problems, the filter accuracy of the common Kalman filter is too low to be applied in this system. To solve these problems, the common Kalman filter is improved in the following section in this paper.

C. Improved Kalman Filter

1) Improvement 1- extrapolation method for time alignment

The output of INS and SMNS is asynchronous which will bring error in the measurement equation. So before using the output of INS and SMNS in measurement equation, the time when INS and SMNS output information must be aligned. Extrapolation method is applied for time alignment in this paper.

As shown in figure 1, T_i is the update rate of INS output, t_{i0} , t_{i1} and t_{i2} are the time when the INS outputs the related aircraft information, t_{s1} , t_{s2} and t_{s3} are the time when the SMNS outputs aircraft position (longitude and latitude). When the aircraft flies into the scene matching area, the computer in aircraft records the longitude, latitude and the corresponding time that INS outputs.

Once a correct matching point is obtained at the time (e.g. t_{s2} shown in figure 3) no output from INS, the following method will be used to calculate the INS output value at time t_{s2} : Find the INS output time t_{i1} and t_{i2} which are nearest time to t_{s2}, and the longitude and latitude that INS outputs at time t_{i1} and t_{i2} , denoted as (L_{i1},λ_{i1}) and (L_{i2},λ_{i2}) . Then substitute the t_{i1} , t_{i2} , (L_{i1},λ_{i1}) and (L_{i2},λ_{i2}) into the formula (5) to get the longitude and latitude denoted as (L_i,λ_i) which is considered as INS outputs at time t_{s2} .

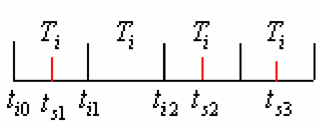

Figure 1. Time Relationship

$$\left.\begin{aligned} L_i &= L_{i1} + (L_{i2} - L_{i1})(t_{s2} - t_{i1})/(t_{i2} - t_{i1}) \\ \lambda_i &= \lambda_{i1} + (\lambda_{i2} - \lambda_{i1})(t_{s2} - t_{i1})/(t_{i2} - t_{i1}) \end{aligned}\right\} \quad (5)$$

Now, the (L_i,λ_i) and (L_{s2},λ_{s2}) are the output of INS and SMNS at the same time t_{s2} . They can be used in measurement equation (2) to obtain the measurement at time t_{s2} .

2) Improvement 2- system model is discretized in unequal interval

The sampling rate denoted as T_d used by the discretization of equation (1) and (2) is usually equal to the measurement rate. The measurement rate is equal to the interval between two matching points in this paper. Because the matching point arrival is random, the measurement rate may be very long which will lead to incorrectness of state transition matrix if the sampling rate is still equal to the measurement rate. Some improvement shall be taken to avoid the incorrectness of state transition matrix.

It can be seen from the equation (4) that the equations for the Kalman filter fall into two groups: time update equations and measurement update equations. According to this characteristic, the improvement can be taken as follows. The figure 2 will be used during the following description where T_i is the update rate of INS output, t_{i1} , ... , t_{ik} are the time when the INS outputs the related aircraft information, t_{s1} and t_{s2} are the time when the SMNS outputs aircraft position and INS has no output.

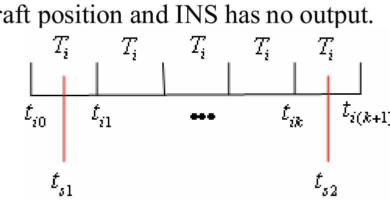

Figure 2. variable rate filter

（1） At the time of t_{i1} ,…, t_{ik} , INS outputs the related aircraft information, SMNS has no output, so there is no measurement. Sampling rate T_d is equal to T_i , and filter is processed according to following time update equations (6).

$$\left.\begin{aligned} \hat{X}_{k+1,k+1} &= \Phi_{k+1,k}\hat{X}_{k,k} \\ P_{k+1,k+1} &= \Phi_{k+1,k}P_{k,k}\Phi_{k+1,k}^T + \Gamma_{k+1,k}Q_k\Gamma_{k+1,k}^T \end{aligned}\right\} \quad (6)$$

（2） At time t_{s2} , SMNS outputs aircraft position, INS output can be produced according to formula (5), so a measurement can be produced by equation (2). Sampling rate is equal to $T' = t_{s2} - t_{ik}$. The filter is processed according to equation (4) which includes the time update equations and the measurement update equations.

（3） At time $t_{i(k+)}$, sampling rate is equal to $T^{''} = t_{i(k+1)} - t_{s2}$. Filter is processed according to the equations (6) which are the time update equations.

3) Improvement 3- Eliminate the measurement output delay

In SMNS, the images matching position needs the unequal matching calculation time, which results the delay characters of measurement output. The paper [4] applies a method to eliminate the measurement output delay in INS/SAR (Synthetic Aperture Radar) integrated navigation system. Based on the method presented in paper [4], the characteristic of INS/SMNS integrated navigation system and improvement 1 and improvement 2 of Kalman filter presented in this paper, this section applies the following solution to eliminate the measurement output delay.

In figure 2, it is supposed that the aircraft get an image at the time of t_{s1}. However, considering the matching calculation time, the image matching position is outputted until time t_{s2}. The solution is as follows:

（1） At time t_{s1} , apply the Improvement 1-extrapolation method to obtain the longitude and latitude (L_i, λ_i) that is considered as the INS output at time t_{s1}.

（2） At time t_{s2} , the image matching position (L_s, λ_s) is outputted and its corresponding time is t_{s1}. Substitute the (L_s, λ_s) and the (L_i, λ_i) in equation (2) to obtain a measurement. Let sampling rate be equal to $T^{'} = t_{s1} - t_{i0}$. The filter is processed according to equation (4) to obtain the state estimate of time t_{s1}.

（3） Let sampling rate T_d be equal to $T^{'} = t_{i1} - t_{s1}$ 、 $T = T_i$ 、 $T^{'} = t_{s2} - t_{ik}$ respectively, and the filter is processed according to time update equations (6), then obtain the state estimate corresponding to time t_{s2} without the effect of the measurement output delay.

III. APPLICATION OF IMPROVED LMS IN INS/SMNS INTEGRATED NAVIGATION SYSTEM

A. INS/SMNS Integrated Navigation System Model

The output of SMNS is the aircraft position (longitude and latitude). So only the system parameters related to position information are estimated in this paper. The position error equation of INS can be simplified as formula (7):

$$[\delta L(t) \quad \delta\lambda(t)] = [1 \quad t]\begin{bmatrix} \delta L(0) & \delta\lambda(0) \\ \dfrac{\delta v_N}{R_M + h} & \dfrac{(\delta v_E + v_E tgL)}{R_N + h}\sec L \end{bmatrix} \tag{7}$$

where δL 、 $\delta\lambda$ are the longitude and latitude error of INS output. $\delta L(0)$ 、 $\delta\lambda(0)$ are the initial longitude and latitude error of INS output. δv_E 、 δv_N are the east and north speed error of INS output. v_E is east speed of aircraft . L is the aircraft's latitude. R_M 、 R_N are the radius of curvature in meridian plane and the radius of curvature in prime vertical. h is the altitude of the aircraft.

The parameter matrix to be estimated is as follows:

$$\theta = \begin{bmatrix} \delta L(0) & \delta\lambda(0) \\ \dfrac{\delta v_N}{R_M + h} & \dfrac{(\delta v_E + v_E tgL)}{R_N + h}\sec L \end{bmatrix} \tag{8}$$

Measurement equation is as follows:

$$Z = \begin{bmatrix} L_I - L_S \\ \lambda_I - \lambda_S \end{bmatrix}^T = \begin{bmatrix} \delta L + N_N \\ \delta\lambda + N_E \end{bmatrix}^T = H\theta + V \tag{9}$$

where L_I, λ_I are longitude and latitude of INS output. L_S, λ_S are longitude and latitude of SMNS output. N_N 、 N_E are north and east matching error.

When a measurement is obtained according to equation (9), parameter matrix θ can be estimated according to the algorithm described in the next section which is denoted as $\hat{\theta}$. When INS needs to be corrected, substitute $\hat{\theta}$ into formula (7), then obtain the INS error.

B. Improved LMS Algorithm

Firstly, the H^{∞} norm of transfer operator T is defined as:

$$\| T \|_{\infty} = \sup_{u \neq 0, u \in H_2} \frac{\| y \|_2}{\| u \|_2} \tag{10}$$

where the T is a transfer operator that maps an input u to an output y, the notation $\| u \|_2$ denotes the H_2 norm of the sequence $\{u\}$.

Consider the system model as follows:

$$Z_i = H_i\theta + V_i \ , \ i \geqslant 0 \tag{11}$$

where $Z_i = [z_{i1} \quad \dots \quad z_{in}]$ is the measured output vector (No.i). $H_i = [h_{i1} \quad \cdots \quad h_{im}]$ is the measurement matrix (No.i) that has been known. $\theta_{m\times n}$ is the system parameter to be estimated. $V_i = [v_{i1} \ \cdots \ v_{in}]$ is the measurement noise (No.i).

The H^∞ adaptive filter is: Find a $F_f(\cdot)$ to update $\hat{\theta}_i = F_f(Z_1...Z_i; H_1...H_i)$ leading to $\| T_f(F) \|_\infty = \min$. Find a $F_p(\cdot)$ to update $\hat{\theta}_i = F_p(Z_1,...,Z_i; H_1,...,H_i)$ leading to $\| T_p(F) \|_\infty = \min$. Where $T_f(F)$ and $T_p(F)$ are transfer operators that map the disturbances to the estimation errors.

It can be known from the paper [2]:

When H_i is exciting and the estimation strategy $F_f(\cdot)$ is the same as the normalized LMS algorithm as follows:

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \frac{\mu H^T(n+1)e(n+1)}{1+\mu H(n+1)H^T(n+1)} \qquad (12)$$

the $\| T_f(F) \|_\infty = \min = 1$. In other words, the normalized LMS algorithm is a H^∞ filter. In the formula (12), $e(n+1) = Z(n+1) - H(n+1)\hat{\theta}(n)$ is the prediction error for one step, μ is the step parameter.

When $0 < \mu < \inf_n \frac{1}{H(n)H^T(n)}$, H_i is exciting and the estimation strategy $F_p(\cdot)$ is the same as the variable step-size LMS algorithm as follows:

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \mu H^T(n+1)e(n+1) \qquad (13)$$

the $\| T_p(F) \|_\infty = \min = 1$. In other words, the variable step-size LMS algorithm is a H^∞ filter. In the formula (13), $e(n+1) = Z(n+1) - H(n+1)\hat{\theta}(n)$ is the prediction error for one step, μ is the learning rate.

IV. SIMULATION

The simulation conditions are as follows:

(1) Flight trajectory is generated by the trajectory generator. The simulation lasts 60s. The north and west speed are both 350m/s. The initial latitude is 39.9^0.The initial longitude is 116.3^0. The initial altitude is 1km. When aircraft flights into the scene matching area, the speed error is 0.5m/s, the longitude error and altitude error are both 60m.

(2) In INS, the update rate of INS output is 100ms. The gyroscope constant error and random drift are both 0.2(0)/h. The accelerometer constant error is 1x10-4g, the accelerometer random error is 5x10-5g, where "g" is the acceleration of gravity.

(3) In the whole scene matching area, there are10 available matching points. The interval between them is1~2s for random. Scene matching error is 3~5m for random. The delay of measurement output is 0~1s for random.

Figure 3 and 4 show the simulation result that the improved Kalman filter is applied in INS/SMNS integrated navigation system.

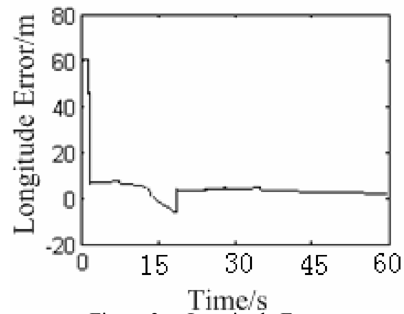

Figure 3. Longitude Error

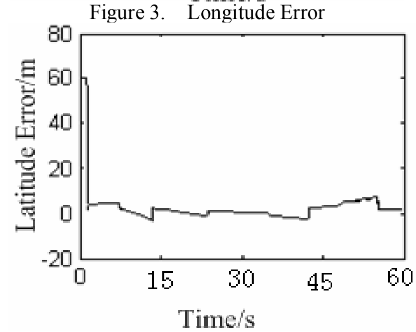

Figure 4. Latitude Error

It can be seen from figure 3 and 4 that the filter accuracy of improved Kalman filter is 6m which totally satisfy the meter-level positioning requirement of INS/SMNS integrated navigation system. So the design of the improved Kalman filter presented in this paper has considered the characteristic of INS/SMNS integrated navigation system correctly, solved the problems of time alignment, multi-rate filter and measurement output delay and improved the filter accuracy effectively. All of these work let the improved Kalman filter be totally suitable for INS/SMNS system.

Figure 5 and 6 show the simulation result that the normalized LMS and the variable step-size LMS are applied in INS/SMNS integrated navigation system, in which the dot line is the simulation result of RLS, and the solid line is the simulation result of the normalized LMS and the variable step-size LMS.

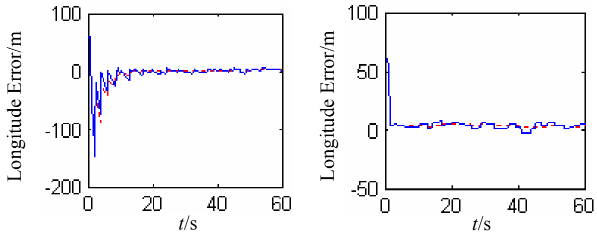

Figure 5. Longitude and latitude error of RLS and normalized LMS algorithm

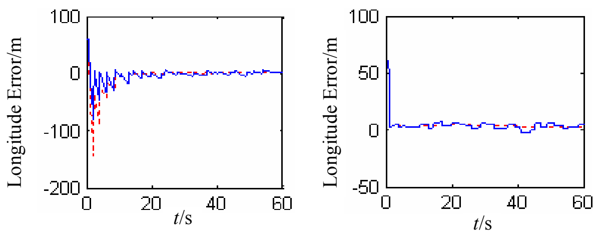

Figure 6. Longitude and latitude error of RLS and variable step-size LMS algorithm

It can be seen from the figure 5 and 6 that the filter accuracy and convergent rate of the normalized LMS and the variable step-size LMS are both the same as that of RLS which is proved to be suitable for INS/SMNS integrated navigation system in the reference paper [5], so the normalized LMS and the variable step-size LMS are suitable for INS/SMNS integrated navigation too.

V. CONCLUSIONS

In general, in order to find new information fusion algorithms that are suitable for INS/SMNS integrated navigation system, this paper studies the issue as follows. Firstly, this paper presents the improved Kalman filter by applying the methods of extrapolation, discretizing system model in unequal interval and eliminating the measurement output delay to the common Kalman filter. Secondly, this paper studies the variable step-size LMS algorithm and normalized LMS algorithm basing on H^{∞} optimal estimation. Lastly this paper applies them to the INS/SMNS integrated navigation. Simulation results demonstrate that:

（1） For the improved Kalman filter, its filter accuracy satisfies the meter-level positioning requirement of INS/SMNS integrated navigation system. So the improved Kalman filter presented in this paper is suitable for INS/SMNS integrated navigation system.

（2） For the step-size LMS and the normalized LMS algorithms, they have the same filtering accuracy with RLS (Recursive Least Square) which is proved to be suitable for INS/SMNS integrated navigation system in the reference paper [5]. So they are suitable for INS/SMNS integrated navigation too.

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