Optimization of a Fuzzy Controller Using Taguchi Methods With Multiple Performance Characteristics

Cheng-Hsiung Chiang[1] and Liang-Hsuan Chen[2]

[1] Department of Computer Science, Hsuan Chuang University, Hsinchu, Taiwan, R.O.C.
[2] Department of Industrial and Information Management, National Cheng Kung University, Tainan, Taiwan, R.O.C.

Abstract

Fuzzy controllers in general are designed using the experts' knowledge or human experience in order to decide the settings of a number of system factors. However, the relations between the factors and system performance are not easy to be determined. Usually, more than one system performance characteristic is considered. For determining the settings of factors in a systematical way, we apply Taguchi methods in this paper to design the structure of a fuzzy controller with multiple performance characteristics. A cart-pole system is exemplified to illustrate the design processes. An aggregated performance index is also proposed to measure the response of the cart-pole system. The simulated results showed that the proposed method can find the efficient design for fuzzy controller.

Keywords: Fuzzy control; Taguchi method; Multiple performance characteristics; Cart-pole system.

1. Introduction

Fuzzy controllers have a number of successful applications in a wide variety of fields, such as automatic control, data classification, expert system, robot control, pattern recognition, and so on. Many studies have addressed the applications of fuzzy controllers. Amongst them, some proposals of design procedures in order to develop the optimal fuzzy controllers with the assistance of some useful approaches, such as fuzzy neural network (FNN) and genetic algorithms (GAs), are important. In general, fuzzy controllers may be classified into two types according to fuzzy controller input. For the first type, the fuzzy controller is based on the traditional control theory, e.g. fuzzy PID controller [1]; regarding the second type, the controller is constructed according to the soft computing based approaches, such as fuzzy set theory, neural networks (NNs), GAs, etc. With regard to the applicability and flexibility, the second performs better efficiency and can be properly used in the uncertainly environment; therefore, this study deals with the second type.

Three important components should be determined in designing the structure of a fuzzy controller, namely, fuzzy control rules, membership functions (MFs) representing the linguistic terms in the rules, and the operator types in the rules. Concerning the design of fuzzy rules, several literatures, such as [2], have reported that GA is a well approach. Regarding the construction of MFs for a fuzzy controller, a number of studies, e.g. [2], have been done through the algorithms of GA, NN or FNN. In addition, several literatures have done the investigation of the design of fuzzy operators, such as [3].

Although GA or FNN in the existing studies can reach the approximate solutions, a lot of computational efforts are needed. For finding out the optimal factor level in the control problems, experimental design was used in the fields of artificial intelligence recently. Similar to the traditional experimental designs, Taguchi's methods are applied to determine the robust design of product or system through the orthogonal arrays. Such methods were employed to design NN [4] or fuzzy controllers [5], or optimization of industrial process [6]. Following the methods, recently several researchers, such as [7], considered more than one performance characteristic in the system design.

With the help of Taguchi's orthogonal arrays, this paper uses seven factors with two or three levels to express the factors of fuzzy controller, such as the number of linguistic terms for an input variable, the number of linguistic terms for an output variable, and so on. Considering multiple performance characteristics, an aggregated performance index is presented to find significant factors that have influences to the aggregated system performance, and then the factors' settings can be determined. We finally use a classical cart-pole control example to demonstrate the applicability of the proposed design approach.

2. Procedure of Taguchi method

A fuzzy controller is generally consisted of four modules, i.e. fuzzy rule base, fuzzy inference engine, fuzzification, and defuzzification. In order to design an optimal fuzzy controller, a series of experimental runs are conducted to obtain the system measures under various level combinations of factors. Subsequently, the settings of optimal factors are determined.

The design procedure based on the Taguchi method can be described as follows.

Step 1. *Identify performance measure of the fuzzy controller.* The determined performance measure should be able to characterize the performance characteristic of a control system.

Step 2. *Specify design factors.* A number of factors can affect system performance. These factors include:

(1) Input/output MFs. The types and numbers of MFs for the fuzzy set of each input and output variable are influential to the output of a fuzzy controller. Some common MFs are used by researchers including triangular shape, Gaussian function, or bell-shape MFs. In addition, the number of MFs for input and output variables will determine the space scale of the fuzzy rules.

(2) Fuzzy inference rules. The fuzzy rule has the **IF-THEN** form to express the control rule. Different definitions of and/or may produce different outcomes of antecedents in the **IF** clause. Moreover, fuzzy implication operators are also important to encode the knowledge of the rule for fuzzy inference.

(3) Defuzzification methods. A few defuzzification methods are developed in the engineering fields, such as centroid of area (COA) and mean of maximum (MOM). The use of inappropriate defuzzification methods may produce unstable control effects such that the control system cannot be robust.

Step 3. *Determine an appropriate data collection plan and experiments.* In order to collect data to measure the control effects, a data collection plan should be developed in advance. The well-known experimental design matrices, orthogonal arrays, recommended by Taguchi [8] are used in this study. Experimental runs are conducted according to various level combinations of design factors.

Step 4. *Determine a proper performance index to measure system performance.* A performance index is usually applied by using the performance measures based on the same factor level combinations.

3. Aggregated performance index

Usually, only one performance characteristic is considered in the design process. However, for real applications, the requirement of incorporating several distinct performance characteristics into the design is important. Thus, an aggregated performance index has to be defined for the purpose. The procedure is described as follows. Suppose n performance characteristics are considered in the design, and m experiments are conducted based on factor level combinations for collecting the data of associated performance measures. For the jth performance characteristic in the ith factor-level-combination experiment, the loss function of the smaller-the-better problem, L_{ij}, is calculated as

$$L_{ij} = \frac{1}{K}\sum_{k=1}^{K} y_{ijk}^2 \tag{1}$$

where K is the number of runs (repeated experiments at the same level combination), and y_{ijk} is the performance measure at the kth run. Using the loss function, the performance index is determined as [8]

$$\eta_{ij} = -10 \cdot \log_{10}(L_{ij}),\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n, \tag{2}$$

where η_{ij} is the S/N ratio. For aggregating more than one performance index, we first standardize the S/N value in each experiment by

$$z_{ij} = \frac{\eta_{ij} - \overline{\eta}_j}{S_{\eta_j}},\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{3}$$

where z_{ij} is the standardized S/N ratio and $\overline{\eta}_j$ (S_{η_j}) is the average S/N ratio (standard deviation) of the jth performance characteristic. The aggregated performance index is then determined as

$$\varphi_i = \sum_{j=1}^{n} w_j \cdot z_{ij},\ i = 1,2,\ldots,m \tag{4}$$

where w_j is the weight assigned to the jth performance characteristic.

4. Application to a cart-pole system

For demonstrations, we apply the proposed design approach to a cart-pole system as follows.

4.1. Structure of cart-pole system

A cart-pole system, which is well known in classical control problems, is a highly nonlinear dynamical system. The movable pole is attached to the cart by a frictionless hinge that rotates in the vertical plane aligned with the track. The problem is to stabilize the pole vertically with appropriate horizontal driving forces on the cart. The mathematical model is [9]

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = \frac{g\sin(x_1) - \dfrac{\cos(x_1)}{m_p + m_c}\left(m_p l (x_2)^2 \sin(x_1) + F\right)}{\dfrac{4}{3}l - \dfrac{m_p l \cos^2(x_1)}{m_p + m_c}}, \tag{5}$$

where the state variable x_1 represents the angular position (in radians) of the pole and x_2 is the first derivative of x_1 and represents the angular velocity of the pole in rad./sec. F is the driving force on the cart. l is the half-length of the pole. $m_p(m_c)$ is the mass of the pole (cart), and g specifies the gravitational acceleration constant.

To the control of the cart-pole system and to keep the pole in the vertical position, two input variables, angular position (x_1) and angular velocity (x_2), and one output variable, the driving force (F), are included in the fuzzy controller.

4.2. Design of fuzzy controller

The ranges of angular position, angular velocity and the driving force are $x_1 \in [-1.5, 1.5]$ rad., $x_2 \in [-1.5, 1.5]$ rad. and $F \in [-50, 50]$ newton, respectively. Table 1 exemplifies the symmetric fuzzy inference rules with five linguistic states used in this paper. The design factors and their levels are shown in Table 2.We have to determine the optimal level combination for the seven factors.

Table 1: A symmetric rule table

Velocity (x_2)	Angular position (x_1)				
	NB	NS	ZE	PS	PB
PB	ZE	PS	**PB**	**PB**	**PB**
PS	NS	ZE	PS	**PB**	**PB**
ZE	**NB**	NS	ZE	PS	**PB**
NS	**NB**	**NB**	NS	ZE	PS
NB	**NB**	**NB**	**NB**	NS	ZE

As shown in Table 2, this study applies three different types of fuzzy MFs, namely triangular shape, Gaussian function, and bell-shape, and the three different numbers of fuzzy MFs are five, seven, and nine. Two ***and*** operator functions, ***min*** and ***algebraic product***, are employed to aggregate the two input variables. For encoding the knowledge of the rules, two fuzzy implication functions are investigated, i.e., ***min*** and ***algebraic product***. Two defuzzification methods are investigated, namely COA and MOM.

Table 2: Design factors and their levels

Factor		Level		
		1	2	3
A	Input fuzzy MFs	Triangular	Gaussian	Bell
B	Output fuzzy MFs	Triangular	Gaussian	Bell
C	No. of input MFs	5	7	9
D	No. of output MFs	5	7	9
E	*and* operator	*Min.*	*Algebraic Product*	
F	Implication function	*Min.*	*Algebraic Product*	
G	Defuzzification	COA	MOM	

For performing experimental design, a data collection plan has to be determined. The orthogonal arrays $L_{18}(2^1 \times 3^7)$, suggested by Taguchi [8], is used to accommodate the factors listed in Table 2. We considered one of the three levels in the last three columns as the dummy level for assigning the three factors E, F, and G to these columns. Eighteen experiments based on different factor level combinations of design factors have to be performed in this experimental layout.

4.3. Performance characteristics

Two kinds of performance characteristics for x_1 are considered, namely, the smoothness degree and the convergence degree.

(1) The smoothness degree: f_s

In the cart-pole system, we emphasize x_1 the transformation process of the angular position of the pole. Firstly a polynomial function $g(t)$ is fitted, considering all observation values (the sampling points) of x_1. The smoothness degree is defined as

$$f_s = \sum_{i=1}^{N} (g(t_i) - x_{1,i}), \tag{6}$$

where N is the total sampling number, $t_i$is the ith sampling time in the simulation, and $x_{1,j}$ is the observation value of x_1 at the ith sampling time.

(2) The degree of convergence: f_c

In the cart-pole system, we expect that the value of x_1 could converge to zero as soon as possible. The convergent criterion is defined as

$$\delta_i = \begin{cases} 1, & |x_{1,i} - T| - \varepsilon > 0 \\ 0, & \text{otherwise} \end{cases}, \tag{7}$$

where T is the target value set as zero and $\varepsilon = 0.005$ in this example. The degree of convergence is defined as

$$f_c = \frac{\sum_{i=1}^{N} \delta_i}{N}, \tag{8}$$

where N is the total sampling number.

5. Simulation results

Firstly, we assign the weight for convergence degree (w_c) and the weight for smoothness degree (w_s) as 0.6 and 0.4, respectively. We constructed the simulation program by using the MATLAB software with SIMULINK toolbox to simulate the cart-pole dynamic system described as Eq. (5), and applied FUZZY LOGIC toolbox to build up the fuzzy controller. For executing the simulations, some parameters in Eq. (5) are specified as l=0.5m, m_p=0.1kg and g=9.8m/s^2.

Eighteen experiments are performed based on eighteen different level combinations of design factors, since the orthogonal arrays L_{18} is used. For obtaining more reliable results, 10 experimental runs are made for each experiment. For each run, the initial value of state variable $x_1(t=0)$ is assigned randomly, and the initial $x_2(t=0)$ is specified zero. The simulation time and the sampling time are 20 sec. and 0.1 sec., respectively. After the simulations, the z_c, z_s and φ can be evaluated for the 18 experiments. For example, (z_c, z_s, φ) are calculated as (-0.5051, -0.2012, -0.3835) for 2nd experiment and (2.0766, -0.7063, 0.9634) for 12th experiment.

Using the simulation data calculated in the above, the effect of each factor level can be obtained based on the experiment layout of L_{18} and is illustrated in Fig. 1. The optimal level combination, $\boldsymbol{A_1B_1C_3D_3E_2F_1G_1}$, can then be found, which represents the combination of *triangular* MFs for input/output variables, *nine* MFs for input/output variables, ***algebraic product*** for ***and*** operator, ***min*** operator for implication function, *COA* for defuzzification method, respectively. Once the optimal level combination is determined, confirmation experiments are conducted for verifying the combination. The performance measures of the two performance characteristics, convergence degree and smoothness degree, are 8.9026 db and 34.0708 db, respectively, which are the best among those in the previous 18 experiments. This indicates that the optimum level combination is verified and the additive model is applicable in this example.

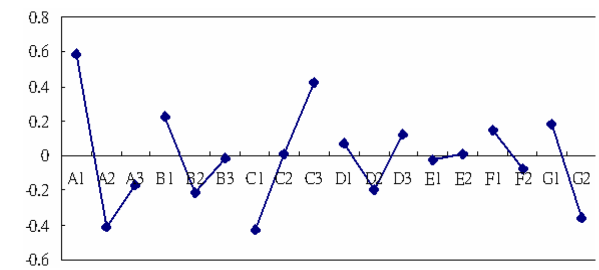

Fig. 1: The effect of factor level for aggregated performance.

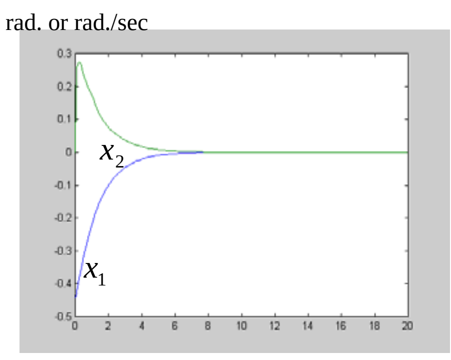

Fig. 2: The response curves for $x_1(t=0)$ = -0.441 rad.

For demonstrations, using initial values of x_1 = -0.441 rad., Fig. 2 illustrate the response curves of x_1 and x_2 during the simulation time. The two response curves smoothly converge to zero earlier than 10 seconds.

6. Concluding remarks

The design of a fuzzy controller is an important task in the relevant applications. Instead of the existing methods, this study employs Taguchi method to investigate the optimization of a fuzzy controller. For considering multiple performance characteristics, we presented the aggregated performance index based on Taguchi method. A cart-pole system is applied to demonstrate the proposed procedure. An optimum factor level combination is found, namely triangular MFs for input/output variables, nine MFs for input/output variables, ***algebraic product*** for ***and*** operator, ***min*** operator for implication function, COA for defuzzification method. The optimal factor level has been verified as the best among all level combinations.

7. References

[1] H.-J. Cho, K.-B. Cho, and B.-H. Wang, "Fuzzy-PID hybrid control: Automatic rule generation using genetic algorithms," *Fuzzy Sets and Systems*, vol. 92, pp. 305-316, 1997.

[2] O. Cordon, F. Gomide, F. Herrera, F. Hoffmann, and L. Magdalena, "Ten years of genetic fuzzy systems: current framework and new trends," *Fuzzy Sets and Systems*, vol. 141, pp. 5-31, 2004.

[3] O. Valenzuela, I. Rojas, and F. Rojas, "Statistical analysis of the different operator involved in the fuzzy inference process," *Lecture Notes In Artificial Intelligence*, vol. 2955, pp. 63-71, 2006.

[4] F.-J.C. Khaw, B.S. Lim, and E.-L.N. Lim, "Optimal design of neural networks using the Taguchi method," *Neurocomputing*, vol. 7, pp. 225-245, 1995.

[5] C.L. Chen and M.H. Chang, "Optimal design of fuzzy sliding mode control: A comparative study," *Fuzzy Sets and Systems*, vol. 93, pp. 37-48, 1998.

[6] C.C. Huang and T.T. Tang, "Optimizing multiple qualities in As-spun polypropylene yarn by neural networks and genetic algorithms," *Journal of Applied Polymer Science*, vol. 100, pp. 2532-2541, 2006.

[7] H.C. Liao, "Multi-response optimization using weighted principal component," *International Journal of Advanced Manufacturing Technology*, vol. 27, pp. 720-725, 2006.

[8] P.J. Ross, *Taguchi Techniques for Quality Engineering*, McGraw-Hill, Singapore, 1996.

[9] J.E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, Englewood Cliffs, N.J., 1991.