Increasing and Decreasing with Fuzzy Time Series

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Abstract

There is a significant problem associated with the fuzzy time series. That is a strict increasing and decreasing case. Under the discussion case, fuzzy time series model arise a continuous increasing/decreasing forecasting value. From the illustrative example, we can see that our definition not only define the trend of the fuzzy numbers that represent the linguistic values of the linguistic variable more appropriately, but also can make the fuzzy time series reasonable.

Keywords: Fuzzy sets; Fuzzy forecasting.

1. Introduction

The main purpose of this article is to present a mathematical skill to construct a fuzzy time series model of a system. There has been a considerable of studies [1, 3-5] on fuzzy time series where fuzzy [6-9] implications are used to express forecasting rules. Many kinds of fuzzy time series model have been developed since Lee and Chou's [5] paper.

In this article, we present a new definition for forecasting with fuzzy time series based on Lee and Chou's method. The historical data of the Port of Kaohsiung used by Lee and Chou's method [5] are adopted to justify that our method is as reasonable as Lee and Chou's method but more complete than original method. In forecasting Port of Kaohsiung throught, there is a significant problem associated with the fuzzy time series. That is the historical data shown strict increasing and decreasing situation. According to Lee and Chou's method [5], we add a definition to conquer the attribution, based on the definition the fuzzy time series is more complete.

The remainder of the article is structured as follows. Section 2, presents the fuzzy time series definition. Section 3, we present a new definition for forecasting with fuzzy time series based on Lee and Chou's method, our method is also evaluated in section 4. Finally, the conclusions are made in section 5.

2. Fuzzy sets and Fuzzy time series

In 1965, Zadeh [6] introduced some basic concepts and terminology of "Fuzzy Sets". It makes the start of a scientific revolution related to uncertainty, and the effects of which are still being explored. In 1978, Dubios and Prade [2] defined the fuzzy number as a fuzzy subset of the real line. In 1975, Zadeh [9] introduced fuzzy arithmetic theory and its application.

Definition 2.1
A fuzzy number in real line $\Re$ is a fuzzy subset of $\Re$ that is normal and convex.

The fuzzy time series was first presented by Song and Chissom [3, 4]. There are a number of calculation methods to determine the fuzzy relation. Different methods will yield different results [3, 5, 10]. In their papers, they followed Mamdani's method [3] to determine the relations. The main difference between the fuzzy time series and traditional time series is that the values of the former are fuzzy sets while the values of the latter are real historical numbers. The definitions of fuzzy time series are reviewed as follows.

Definition 2.2 [3, 4]
Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of $\Re$, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1, 2, \ldots)$ are defined and $F(t)$ is the collection of $f_i(t)$ $(i = 1, 2, \ldots)$. Then $F(t)$ is called a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$.

$F(t)$ can be understood as a linguistic variable and $f_i(t)$ $(i = 1, 2, \ldots)$ as the possible linguistic values of $F(t)$. Because at different times, the values of $F(t)$ can be different, $F(t)$ is a function of time t. Also, since the universes of discourse can be different at different times, $Y(t)$ is used for the universe at time t.

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Definition 2.3 [3, 4]
Let I *and* J *be indices sets for* $F(t-1)$ *and* $F(t)$ *respectively. If for any* $f_j(t) \in F(t)$ *where* $j \in J$ *, there exists* $f_i(t-1) \in F(t-1)$ *where* $i \in I$ *such that there exists a fuzzy relation* $R_{ij}(t, t-1)$ *and* $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ *where '* $\circ$ *' is the max-min composition, then* $F(t)$ *is said to be caused by* $F(t-1)$ *only. Denote this as* $f_i(t-1) \rightarrow f_i(t)$ *or equivalently* $F(t-1) \rightarrow F(t)$.

Definition 2.4 [3, 4]
If for any $f_j(t) \in F(t)$ *where* $j \in J$ *, there exists* $f_i(t-1) \in F(t-1)$ *where* $i \in I$ *and a fuzzy relation* $R_{ij}(t, t-1)$ *such that* $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$ *, let* $R(t, t-1) = \bigcup_{i,j} R_{ij}(t, t-1)$ *where'* $\bigcup$ *'is the union operator. Then* $R(t, t-1)$ *is called the fuzzy relation between* $F(t)$ *and* $F(t-1)$ *and define this as the following fuzzy relational equation:* $F(t) = F(t-1) \circ R(t, t-1)$.

Definition 2.5 [3, 4]
Suppose $R_1(t, t-1) = \bigcup_{i,j} R^1_{ij}(t, t-1)$ *and* $R_2(t, t-1) = \bigcup_{i,j} R^2_{ij}(t, t-1)$ *are two fuzzy relations between* $F(t)$ *and* $F(t-1)$. *If for any* $f_j(t) \in F(t)$ *where* $j \in J$ *, there exists* $f_i(t-1) \in F(t-1)$ *where* $i \in I$ *and fuzzy relations* $R^1_{ij}(t, t-1)$ *and* $R^2_{ij}(t, t-1)$ *such that* $f_j(t) = f_i(t-1) \circ R^1_{ij}(t, t-1)$ *and* $f_j(t) = f_i(t-1) \circ R^2_{ij}(t, t-1)$ *, then define* $R_1(t, t-1) = R_2(t, t-1)$.

Definition 2.6 [3, 4]
Suppose $F(t)$ *is caused by* $F(t-1)$ *only or by* $F(t-1)$ *or* $F(t-2)$ *or...or* $F(t-m)$ $(m > 0)$. *This relation can be expressed as the following fuzzy relational equation:*

$$F(t) = F(t-1) \circ R_0(t, t-m) \qquad 2\text{-}1$$

or

$$F(t) = (F(t-1) \cup F(t-2) \cup \ldots \cup F(t-m)) \circ R_0(t, t-m) \qquad 2\text{-}2$$

Then equation 2-1 or 2-2 is called the first-order model of $F(t)$.

Definition 2.7 [3, 4]
Suppose $F(t)$ *is caused by* $F(t-1), F(t-2), \ldots,$ *and* $F(t-m)$ $(m > 0)$ *simultaneously. This relation can be expressed as the following fuzzy relation equation:*

$$F(t) = (F(t-1) \times F(t-2) \times \ldots \times F(t-m)) \circ R_a(t, t-m) \qquad 2\text{-}3$$

Then equation 2-3 is called the mth order model of $F(t)$.

To simplify the assumption, Chen assumed $F(t)$ is singleton. That is, $F(t)$ is a fuzzy set not a set of fuzzy sets. The simplified fuzzy time series is defined as follows.

Definition 2.8 [5]
$F(t)$ *is a fuzzy time series if* $F(t)$ *is a fuzzy set. The transition is denoted as* $F(t-1) \rightarrow F(t)$.

Definition 2.9 [5]
The universe of discourse $U = [D_L, D_U]$ *is defined such that* $D_L = D_{mim} - \frac{s}{\sqrt{n}} t_\alpha(n)$ *and* $D_U = D_{\max} + \frac{s}{\sqrt{n}} t_\alpha(n)$ *when* $n \le 30$ *or* $D_L = D_{\min} - \frac{s}{\sqrt{n}} z_\alpha$ *and* $D_U = D_{\max} + \frac{s}{\sqrt{n}} z_\alpha$ *when* $n > 30$*, where* $t_\alpha(n)$ *is* $100(1-\alpha)$ *percentile of the* t *distribution with* n *degrees of freedom and* z_α *is the* $100(1-\alpha)$ *percentile for the standard normal distribution, that is, if* Z *is* $N(0,1)$ *distribution then* $P(Z \ge z_\alpha) = \alpha$.

Definition 2.10 [5]
Assume there are m *linguistic values under consideration. Let* A_i *be the fuzzy number that represents the ith linguistic value of the linguistic variable where* $1 \le i \le m$ *. The support of* A_i *is defined to be*

$$\begin{cases} [D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}), 1 \le i \le m-1 \\ [D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}], i = m. \end{cases}$$

3. A new definition for forecasting with fuzzy time series

In the following, we propose a new definition to forecast by fuzzy time series. Generally, our method is a modification of the method proposed by Lee and Chou [5]. We add a new definition is more complete than Lee and Chou method due to the fact that we define stability concept of the fuzzy time series that represent the increasing and the decreasing case more appropriately.

In this article, we apply difference test to understand whether information are in stable state. If the information location is in unstable state, do the second difference to divide and continue assaying, recursion until the information are in stable state again. Let *d(t) – d(t-1)* is difference value, *d(t)* is t^{th} historical data, *d(t-1)* is $(t\text{-}1)^{th}$ historical data respectively.

Definition 3.1
For test H_0 : *non fuzzy trend against* H_1 : *fuzzy trend , the test given by*
$P(F(t) > F(t-1)) \ge 1 - \lambda$,
$P(F(t-1) > F(t)) \ge 1 - \lambda$,
$P(F(t) > F(t-1)) = P(F(t-1) > F(t)) = 0.5$,
$0 < \lambda < 0.5$.
Where Critical region:

$C^* = \left\{ C \left| C_2^k + C_2^{n-k} > C_\lambda = C_2^n \times (1-\lambda) \right. \right\}$, *α is significantly level, initial value vale is 0.2 or 0.3. This initial value is stricter if the value is near 0.5. Let k is the number of differential.*

Our forecasting algorithm based on fuzzy time series works as follows:

Step 1:

Let $d(t)$ be the history data under consideration and $F(t)$ be the fuzzy time series. Difference test to understand whether information are in stable state according to definition 3.1., recursion until the information are in stable state again. *Where critical region* $C^* = \left\{ C \left| C_2^k + C_2^{n-k} > C_\lambda = C_2^n \times (1-\lambda) \right. \right\}$.

Step 2:

Determine the universe of discourse $U = [D_L, D_U]$ according to definition 2.9.

Step 3:

Following the definition of 2.10, define A_i by letting its membership function

$$u_{A_i}(x) = \begin{cases} 1 \text{ for } x \in [D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}) \text{ where } 1 \le i \le m-1 \\ 1 \text{ for } x \in [D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}] \text{ where } i = m \\ 0 \text{ otherwise.} \end{cases}$$

Step 4:

Then $F(t) = A_i$ if $d(t) \in \text{supp}(A_i)$, where supp $(\cdot)$ denote the support.

Step 5:

Derive the transition rule from period $t-1$ to t and denote it as $F(t-1) \to F(t)$. Aggregate all transition rules. Let the set of rules be $R = \{ r_i | r_i : P_i \to Q_i \}$.

Step 6:

The value of $d(t)$ can be predicted by fuzzy time series $F(t)$ as follows:

Let $T(t) = \{ r_j | d(t) \in \text{supp}(P_j), where \ r_j \in R \}$ be the set of rules fired by $d(t)$, where $\text{supp}(P_j)$ is the support of P_j . Let $\text{supp}(P_j)$ be the median of $\text{supp}(P_j)$. The predicted value for $d(t)$ is

$$\sum_{r_j \in T(t-1)} \frac{\text{supp}(Q_j)}{|T(t-1)|}.$$

4. Forecasting history data with our method

We verify our method by forecasting the container throught by harbors in Taiwan area from 1973 to 2003 as shown in Table 1.

Step 1:

Let $d(t)$ be the history data under consideration and $F(t)$ be the fuzzy time series. Difference test to understand whether information are in stable state according to definition 3.1., recursion until the information are in stable state again.

Difference one =

$C^* = \left\{ C \left| C_2^{31-1} \times (1-0.2) \right. \right\} = 372$.

$\Rightarrow$ *do not reject* H_0

Difference two=

$C^{''} = \{ C \mid C = C_2^{17} + C_2^{13} \} = 214 < \{ C^{'} \mid C = C_2^{30} * (1-0.2) \} = 348$

$\Rightarrow$ *do not reject* H_0

Step 2:

Determine the universe of discourse $U = [D_L, D_U]$:

By definition 2.9, the discourse $U = [D_L, D_U]$ from Table 1, we have $D_{\min} = -83,048$, $D_{\max} = 1,180,920$, $s = 311,069.74$, and $n = 30$. Let $\alpha = 0.05$. Because n is less than 30, we use Student-t distribution with 30 degrees of freedom as a substitute for normal distribution. That is $U = [-179,430, 1,277,302]$.

Step 3:

Assume the following linguistic values are under consideration: extremely few, very few, few, some, many, very many, and extremely many. Following Definition 2.10, the supports of fuzzy numbers that represent the linguistic values are given by

$$u_{A_i}(x) = \begin{cases} 1, & \text{for } x \in [-179,430 + (i-1)(208,105), -179,430 + i(208,105)) \text{ where } 1 \le i \le m-1 \\ 1, & \text{for } x \in [-179,430 + (i-1)(208,105), -179,430 + i(208,105)] \text{ where } i = m \\ 0, & \text{otherwise} \end{cases}$$

such that A_1 ='extremely few', A_2 ='very few', A_3 ='few', A_4 ='some', A_5 ='many', A_6 ='very many', and A_7 ='extremely many'. Thus, we have

$\text{supp}(A_1) = [-179,430, 208,675)$

$\text{supp}(A_2) = [28,675, 236,780)$

$\text{supp}(A_3) = [236,780, 444,885)$

$\text{supp}(A_4) = [444,885, 652,990)$

$\text{supp}(A_5) = [652,990, 861,095)$

$\text{supp}(A_6) = [861,095, 1,069,200)$,

$\text{supp}(A_7) = [1,069,200, 1,277,302]$

Step 4:

The fuzzy time series $F(t)$ is given by $F(t) = A_i$ when $d(t) \in \text{supp}(A_i)$. Therefore, $F(1974) = A_1$, $F(1975) = A_2$, $F(1976) = A_2$, $F(1977) = A_2$,$\cdots$, and $F(2003) = A_4$

The correspondences between actual enrollments and fuzzy enrollments are shown in Table 1.

Step 5:

The transition rules are derived from Table 1. For example, $F(1974) \to F(1975)$ is $A_1 \to A_2$. All transition rules obtained from Table 1 are shown in Table 2.

Step 6:

We are going to forecast based on Table 2. The forecasting results from 1974 to 2003 are shown in Table 3. The forecasting process of year 2004 are illustrated in the following. The same process can be applied to other years.

Year 2004 To predict the enrollment of 2004, let t=2004. Since $d'(t-1) = d'(2003) = 486{,}119 \in \mathrm{supp}(A_4)$, $T(t-1) = T(2003) = \{r_{11} : A_4 \to A_2, r_{12} : A_4 \to A_3, r_{13} : A_4 \to A_4\}$. Thus the forecasted value for difference one $d'(t) = \overline{d'(1972)}$ is

$$\sum_{r_j \in T(2003)} \frac{\overline{\mathrm{supp}(Q_j)}}{|T(2003)|} = \frac{\overline{\mathrm{supp}(A_2) + \mathrm{supp}(A_3) + \mathrm{supp}(A_4)}}{3} \approx 340{,}832 .$$

Year 2004 of container throught by harbors in Taiwan area is $d(2003) + d'(2003) = 12{,}094{,}753 + 340{,}832 = 12{,}435{,}585$.

Table 1. Fuzzy historical TEUs[1]

Year (t)	TEUs d(t)	Difference one d'(t)	Difference two d''(t)	Fuzzy TEUs
1973	370,372			
1974	391,354	20,982		A_1
1975	473,353	81,999	61,017	A_2
1976	661,866	188,513	106,514	A_2
1977	755,258	93,392	-95,121	A_2
1978	1,062,561	307,303	213,911	A_3
1979	1,374,392	311,831	4,528	A_3
1980	1,644,376	269,984	-41,847	A_3
1981	1,787,758	143,382	-126,602	A_2
1982	1,902,264	114,506	-28,876	A_2
1983	2,429,310	527,046	412,540	A_4
1984	3,026,846	597,536	70,490	A_4
1985	3,075,151	48,305	-549,231	A_2
1986	4,104,953	1,029,802	981,497	A_6
1987	4,772,339	667,386	-362,416	A_5
1988	4,941,022	168,683	-498,703	A_2
1989	5,263,091	322,069	153,386	A_3
1990	5,463,566	200,475	-121,594	A_2
1991	6,129,667	666,101	465,626	A_5
1992	6,178,872	49,205	-616,896	A_2
1993	6,824,973	646,101	596,896	A_4
1994	7,307,304	482,331	-163,770	A_4
1995	7,665,178	357,874	-124,457	A_3
1996	7,866,995	201,817	-156,057	A_2
1997	8,520,199	653,204	451,387	A_5
1998	8,858,211	338,012	-315,192	A_3
1999	9,757,648	899,437	561,425	A_6
2000	10,510,762	753,114	-146,323	A_5
2001	10,427,714	-83,048	-836,162	A_1
2002	11,608,634	1,180,920	1,263,968	A_7
2003	12,094,753	486,119	-694,801	A_4

Table 2. Fuzzy transitions derived from Table 1.

$r_1 : A_1 \to A_2$	$r_7 : A_2 \to A_6$	$r_{13} : A_4 \to A_4$
$r_2 : A_1 \to A_7$	$r_8 : A_3 \to A_2$	$r_{14} : A_5 \to A_1$
$r_3 : A_2 \to A_2$	$r_9 : A_3 \to A_3$	$r_{15} : A_5 \to A_2$
$r_4 : A_2 \to A_3$	$r_{10} : A_3 \to A_6$	$r_{16} : A_5 \to A_3$
$r_5 : A_2 \to A_4$	$r_{11} : A_4 \to A_2$	$r_{17} : A_6 \to A_5$
$r_6 : A_2 \to A_5$	$r_{12} : A_4 \to A_3$	$r_{18} : A_7 \to A_4$

5. Conclusions

In this article, we add a useful definition to improve original method [4] to forecast the strict increasing and decreasing case based on fuzzy time series. From the illustrative example, we can see that our definition not only define the trend of the fuzzy numbers that represent the linguistic values of the linguistic variable more appropriately, but also can make the fuzzy time series reasonable. In summary, our method is as accurate as Lee and Chou method [4] but more reasonable in forecasting the strict increasing and decreasing case.

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7. References

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[1] TEUs=20-foot equivalent containers.